ALGEBRA

A DEFINITIONS

Definition Constant

A **constant** is a number.

Ex: 0, 3, π

Definition Variable

A **variable** is a quantity which we represent by a letter.

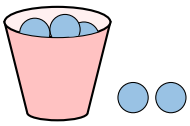
Ex:

The variable x is the number of marbles inside the cup.

Definition Expression

An **expression** is an algebraic form consisting of constants, variables, and operation signs such as $+, -, \times, \div$ and $\sqrt{}$.

Ex:



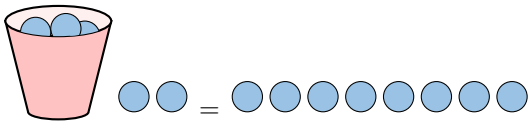
A cup contains x marbles. Next to the cup, there are 2 marbles outside. The expression for the number of marbles is

$$x + 2$$

Definition Equation

An **equation** is a mathematical statement consisting of two expressions, the **left-hand side** and the **right-hand side**, separated by an equal sign $=$.

Ex:

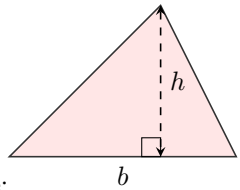


A cup contains x marbles. The equation for the number of marbles is

$$x + 2 = 8$$

Definition Formula

A **formula** is an equation, often related to the real world, to physics or to geometry.

Ex: For a triangle: 

, $A = \dfrac{b \times h}{2}$ is the formula for the area.

B NOTATIONS

Definition Product notation

We can omit the $\times$ sign when it is followed by a variable or a parenthesis.

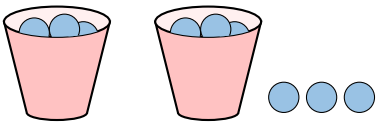
Ex:

- $2 \times x = 2x$
- $2 \times (L + l) = 2(L + l)$

Definition Repeated addition

$$\overbrace{x + x + \ldots + x}^{n \text{ terms}} = n \times x$$

Ex:



Each cup contains x marbles. Simplify the expression for the number of marbles:

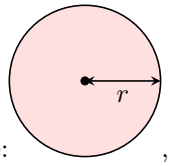
$$x + x + 1 + 1 + 1$$

Answer:

$$x + x + 1 + 1 + 1 = 2x + 3$$

Definition Repeated multiplication

$$\overbrace{x \times x \times \ldots \times x}^{n \text{ factors}} = x^n$$

Ex: For a circle: 

, simplify the formula for the area $A = \pi \times r \times r$.

Answer:

$$\begin{aligned} A &= \pi \times r \times r \\ &= \pi r^2 \end{aligned}$$

C IDENTITY

Definition Identity

An **identity** is an equality between two expressions such that their evaluations produce the same value for all values of the variables.

Identities are fundamental in algebra: they allow us to transform and simplify expressions and are the foundation for solving equations and manipulating formulas.

Proposition 0 and 1 Identities

$$1 \times x = x \qquad \text{and} \qquad 0 \times x = 0$$

Proposition Commutativity Identities

$$a + b = b + a \qquad \text{and} \qquad a \times b = b \times a$$

Proposition **Associativity Identities**

$$(a+b)+c=a+(b+c) \qquad \text{and} \qquad (a\times b)\times c=a\times(b\times c)$$

Ex: Show that $l+L+l+L=2l+2L$.

Answer:

$$\begin{aligned} l+L+l+L &= l+l+L+L && \text{(collecting terms)}\\ &= 2l+2L && \text{(repeated addition)} \end{aligned}$$

Method **Simplifying by Collecting Like Terms**

Simplifying an expression by collecting like terms involves combining terms that have the same variables raised to the same powers.

1. **Identify like terms**: Like terms are terms that have the same variable(s) raised to the same power. For example, $3x$ and $5x$ are like terms, but $3x$ and $3x^2$ are not.
2. **Regroupe like terms**: Add or subtract the coefficients (numerical parts) of the like terms. The variable part remains the same.

Ex: Simplify the expression: $2x+4+x-2$

Answer:

$$\begin{aligned} 2x+4+x-2 &= 2x+4+x-2 && \text{(identifying like terms)}\\ &= (2+1)x+4-2 && \text{(combining like terms)}\\ &= 3x+2 && \text{(simplifying)} \end{aligned}$$

D SUBSTITUTING

Definition **Substituting**

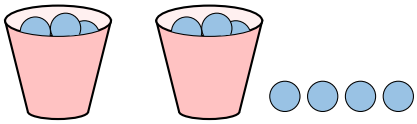
Substituting is replacing a variable in an expression or equation with a specific value.

To avoid confusion with signs, especially when substituting negative values, we usually write substitutions in parentheses.

Method **Evaluating**

To **evaluate** an expression, substitute a number for each variable and perform the arithmetic operations.

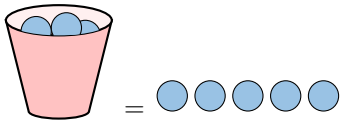
Ex:



Each cup contains x marbles. The expression for the number of marbles is

$$2x+4$$

Evaluate this expression when $x=5$ (meaning there are 5 marbles in each cup):



Answer:

$$\begin{aligned} 2x+4 &= 2\times(5)+4 && \text{(substituting } x=5)\\ &= 10+4\\ &= 14 \end{aligned}$$

There are 14 marbles.