# APPROXIMATIONS AND ERRORS

## A ROUNDING

### A.1 ROUNDING

**Ex 1:** Round 12.3456 to 2 decimal places (2 dp).

**Ex 2:** Round 0.004567 to 2 significant figures (2 sf).

**Ex 3:** Round 98765 to 3 significant figures (3 sf).

**Ex 4:** Round 3.14159 to 3 decimal places (3 dp).

**Ex 5:** Round the number $N = 459.982$ to:

| Rounded to | Answer |
|---|---|
| nearest whole number | |
| 1 decimal place | |
| 2 significant figures | |

**Ex 6:** Complete the table by rounding each number as indicated.

| Number | to 3 s.f. | to 2 d.p. |
|---|---|---|
| 34.052 | | |
| 0.08961 | | |
| 109.99 | | |

### A.2 ESTIMATING VALUES

**Ex 7:** A group of 5 friends go to a restaurant. The price of a meal is \$ 9.99 per person.
Estimate the total bill by rounding the price to **1 significant figure**.

\$

**Ex 8:** A person has a monthly revenue of \$ 1957.
Estimate the annual revenue by rounding the monthly amount to **1 significant figure**.

\$

**Ex 9:** A theatre sold 495 tickets at a price of \$19.50 each.
Estimate the total revenue by rounding the numbers to **1 significant figure**.

**Ex 10:** Estimate the value of the following calculations by rounding each number to **1 significant figure**.

$$\frac{4.12 \times 19.8}{0.49} \approx$$

## B ERROR FORMULAS

### B.1 CALCULATING ABSOLUTE AND PERCENTAGE ERRORS

**Ex 11:** The exact value of $\pi$ is approximately 3.14159. An ancient approximation uses the fraction $\frac{22}{7}$.

1. Calculate the value of $\frac{22}{7}$ to 5 decimal places.

2. Find the absolute error when using $\frac{22}{7}$ as an approximation for $\pi$ (use $\pi \approx 3.14159$).

3. Calculate the percentage error (to 2 decimal places).

%

**Ex 12:** A student measures the length of a piece of wire to be 15.4 cm. The manufacturer states the exact length is 15.0 cm.

1. Calculate the error.

2. Calculate the percentage error (to 2 decimal places).

%

**Ex 13:** The population of a town is exactly 31,467 people. A newspaper reports the population as 31,500 (rounded to 3 significant figures).

1. Find the absolute error.

2. Calculate the percentage error (to 2 decimal places).

%

**Ex 14:** A carpenter measures a board to be 78 cm long. The actual length is 77.5 cm.

1. Find the absolute error.

2. Calculate the percentage error (to 2 decimal places).

%

# C MEASUREMENT ACCURACY

## C.1 DETERMINING ACCURACY AND RANGES

**Ex 15:** State the accuracy (the error interval ±) of the following measuring devices:

1. A tape measure marked in cm.

   ± ☐ cm

2. A measuring cylinder with 1 mL graduations.

   ± ☐ mL

3. A set of scales with marks every 500 g.

   ± ☐ g

4. A thermometer with marks every 0.1°C.

   ± ☐ °C

**Ex 16:** Tom's digital thermometer indicates a temperature of 36.4°C.

1. What is the smallest division of the thermometer based on this reading?

   ☐ °C

2. Determine the range of values in which Tom's actual temperature $T$ lies.

   ☐ $\leq T <$ ☐

**Ex 17:** Joanne's exercise watch displays the distance she has run to **3 significant figures**.Find the **least** distance Joanne could have run for each display:

1. Display: 1.06 km.

   ☐ km

2. Display: 10.1 km.

   ☐ km

**Ex 18:** Hasan has measured the length of several ropes to be 2.4 m each.

1. What is the accuracy of his measurement?

   ± ☐ m

2. If Hasan places 10 of these ropes end to end, what is the maximum possible total length?

   ☐ m

**Ex 19:** In a race, the times recorded for Jiao and Liang were 128 s and 133 s respectively, measured to the nearest second.Find the range of possible values for the time difference $d$ by which Jiao beat Liang (i.e., Liang's time minus Jiao's time).

☐ $< d <$ ☐

# D BOUNDS

## D.1 DETERMINING LOWER AND UPPER BOUNDS

**Ex 20:** For each of the following measurements, determine the lower bound and upper bound.

1. $x = 5$ cm (nearest cm).

   Lower Bound: ☐ Upper Bound: ☐

2. $y = 8.4$ kg (nearest 0.1 kg).

   Lower Bound: ☐ Upper Bound: ☐

3. $z = 120$ m (nearest 10 m).

   Lower Bound: ☐ Upper Bound: ☐

4. $t = 3.45$ s (2 decimal places).

   Lower Bound: ☐ Upper Bound: ☐

**Ex 21:** A distance is measured as $D = 400$ km correct to 1 significant figure.

1. Determine the lower bound.

   ☐

2. Determine the upper bound.

   ☐

## D.2 CALCULATING WITH BOUNDS

**Ex 22:** A rectangle has length $L = 8.5$ cm and width $W = 4.2$ cm, both measured to 1 decimal place.

1. Determine the lower and upper bounds for the length $L$.

   Lower: ☐ Upper: ☐

2. Determine the lower and upper bounds for the width $W$.

   Lower: ☐ Upper: ☐

3. Calculate the maximum possible perimeter of the rectangle.

   ☐

4. Calculate the minimum possible area of the rectangle.

   ☐

**Ex 23:** The distance traveled by a car is $d = 200$ km (nearest 10 km) and the time taken is $t = 4.0$ hours (nearest 0.1 h).

1. Write down the upper bound for the distance.

   ☐

2. Write down the lower bound for the time.

3. Calculate the maximum possible average speed (in km/h).

**Ex 24:** Two lengths are given as $A = 15$ cm and $B = 12$ cm, both to the nearest cm.
Calculate the bounds for the difference $A - B$.

Lower Bound: Upper Bound:

**Ex 25:**  A box has a height $h = 10$ cm and a square base with side $s = 5$ cm. Both measurements are correct to the nearest cm.
Calculate the maximum possible volume of the box.