APPROXIMATIONS AND ERRORS

A ROUNDING

A.1 ROUNDING

Ex 1: Round 12.3456 to 2 decimal places (2 dp).

$$\boxed{12.35}$$

Answer: To round 12.3456 to 2 decimal places, look at the third decimal digit, which is 5.
Since the digit is 5 or greater, round up the second decimal digit from 4 to 5.
Thus, 12.3456 rounded to 2 dp is 12.35.

Ex 2: Round 0.004567 to 2 significant figures (2 sf).

$$\boxed{0.0046}$$

Answer: The first two significant figures are 4 and 5 (starting from the first non-zero digit).
The next digit is 6, which is 5 or greater, so round up the 5 to 6.
Thus, 0.004567 rounded to 2 sf is 0.0046.

Ex 3: Round 98765 to 3 significant figures (3 sf).

$$\boxed{98800}$$

Answer: The first three significant figures are 9, 8 and 7.
The next digit is 6, which is 5 or greater, so round up the 7 to 8.
Thus, 98765 rounded to 3 sf is 98800 (or 9.88×10^4).

Ex 4: Round 3.14159 to 3 decimal places (3 dp).

$$\boxed{3.142}$$

Answer: Look at the fourth decimal digit, which is 5.
Since it is 5 or greater, round up the third decimal digit from 1 to 2.
Thus, 3.14159 rounded to 3 dp is 3.142.

Ex 5: Round the number $N = 459.982$ to:

Rounded to	Answer
nearest whole number	$\boxed{460}$
1 decimal place	$\boxed{460.0}$
2 significant figures	$\boxed{460}$

Answer:

1. Nearest whole number: look at the first decimal digit ($9 \geq 5$), round up $\rightarrow$ 460.
2. 1 decimal place: look at the second decimal digit ($8 \geq 5$), round up the 9 (which carries over) $\rightarrow$ 460.0.
3. 2 significant figures: first two digits 4 and 5, next digit $9 \geq 5$, round up $\rightarrow$ 460 (or 4.6×10^2).

Ex 6: Complete the table by rounding each number as indicated.

Number	to 3 s.f.	to 2 d.p.
34.052	$\boxed{34.1}$	$\boxed{34.05}$
0.08961	$\boxed{0.0896}$	$\boxed{0.09}$
109.99	$\boxed{110}$	$\boxed{109.99}$

Answer:

- 34.052: to 3 s.f. $\rightarrow$ next digit 5, round up $34.0 \rightarrow 34.1$; to 2 d.p. $\rightarrow$ third digit $2 < 5 \rightarrow 34.05$.
- 0.08961: to 3 s.f. $\rightarrow$ fourth significant digit $6 \geq 5$, round up $\rightarrow$ 0.0896; to 2 d.p. $\rightarrow$ third digit $9 \geq 5$, round up $\rightarrow$ 0.09.
- 109.99: to 3 s.f. $\rightarrow$ next digit $9 \geq 5$, round up $\rightarrow$ 110; to 2 d.p. $\rightarrow$ remains 109.99 (already at 2 d.p.).

A.2 ESTIMATING VALUES

Ex 7: A group of 5 friends go to a restaurant. The price of a meal is \$ 9.99 per person.
Estimate the total bill by rounding the price to **1 significant figure**.

$$\$ \boxed{50}$$

Answer: Rounding to 1 s.f.:

- Number of persons: 5 (already 1 s.f.)
- Price: $9.99 \approx 10$

Calculation:

$$\text{Estimate} = 5 \times 10 = 50.$$

Ex 8: A person has a monthly revenue of \$ 1957.
Estimate the annual revenue by rounding the monthly amount to **1 significant figure**.

$$\$\boxed{24000}$$

Answer:

- Rounding monthly revenue to 1 s.f.: $1957 \approx 2000$.
- There are exactly 12 months in a year.

Calculation:

$$\text{Estimate} = 2000 \times 12 = 24\,000.$$

(Exact value: 23 484).

Ex 9: A theatre sold 495 tickets at a price of \$19.50 each.
Estimate the total revenue by rounding the numbers to **1 significant figure**.

$$\boxed{10000}$$

Answer: Rounding to 1 s.f.:

- $495 \approx 500$
- $19.50 \approx 20$

Calculation:

$$\text{Estimate} = 500 \times 20 = 10\,000.$$

Ex 10: Estimate the value of the following calculations by rounding each number to **1 significant figure**.

$$\frac{4.12 \times 19.8}{0.49} \approx \boxed{160}$$

Answer: Rounding each number to 1 s.f.: $4.12 \approx 4$, $19.8 \approx 20$, $0.49 \approx 0.5$.

$$\text{Estimate} = \frac{4 \times 20}{0.5} = \frac{80}{0.5} = 160.$$

B ERROR FORMULAS

B.1 CALCULATING ABSOLUTE AND PERCENTAGE ERRORS

Ex 11: The exact value of π is approximately 3.14159. An ancient approximation uses the fraction $\frac{22}{7}$.

1. Calculate the value of $\frac{22}{7}$ to 5 decimal places.

$$\boxed{3.14286}$$

2. Find the absolute error when using $\frac{22}{7}$ as an approximation for π (use $\pi \approx 3.14159$).

$$\boxed{0.00127}$$

3. Calculate the percentage error (to 2 decimal places).

$$\boxed{0.04}\%$$

Answer:

1. Value calculation: $\frac{22}{7} = 3.142857... \approx 3.14286$.
2. Absolute Error: $|V_{approx} - V_{exact}| = |3.14286 - 3.14159| = 0.00127$.
3. Percentage Error: $\frac{0.00127}{3.14159} \times 100 = 0.0404... \approx 0.04\%$.

Ex 12: A student measures the length of a piece of wire to be 15.4 cm. The manufacturer states the exact length is 15.0 cm.

1. Calculate the error.

$$\boxed{0.4}$$

2. Calculate the percentage error (to 2 decimal places).

$$\boxed{2.67}\%$$

Answer:

1. Error: $V_{approx} - V_{exact} = 15.4 - 15.0 = 0.4$ cm.
2. Percentage Error: $\left|\frac{0.4}{15.0}\right| \times 100 = 2.666... \approx 2.67\%$.

Ex 13: The population of a town is exactly 31,467 people. A newspaper reports the population as 31,500 (rounded to 3 significant figures).

1. Find the absolute error.

$$\boxed{33}$$

2. Calculate the percentage error (to 2 decimal places).

$$\boxed{0.10}\%$$

Answer:

1. Absolute Error: $|V_{approx} - V_{exact}| = |31500 - 31467| = 33$.
2. Percentage Error: $\frac{33}{31467} \times 100 \approx 0.1048... \approx 0.10\%$.

Ex 14: A carpenter measures a board to be 78 cm long. The actual length is 77.5 cm.

1. Find the absolute error.

$$\boxed{0.5}$$

2. Calculate the percentage error (to 2 decimal places).

$$\boxed{0.65}\%$$

Answer:

1. Absolute Error: $|78 - 77.5| = 0.5$ cm.
2. Percentage Error: $\frac{0.5}{77.5} \times 100 \approx 0.645... \approx 0.65\%$.

C MEASUREMENT ACCURACY

C.1 DETERMINING ACCURACY AND RANGES

Ex 15: State the accuracy (the error interval $\pm$) of the following measuring devices:

1. A tape measure marked in cm.

$$\pm \boxed{0.5} \text{ cm}$$

2. A measuring cylinder with 1 mL graduations.

$$\pm \boxed{0.5} \text{ mL}$$

3. A set of scales with marks every 500 g.

$$\pm \boxed{250} \text{ g}$$

4. A thermometer with marks every 0.1°C.

$$\pm \boxed{0.05} \text{ °C}$$

Answer: The accuracy is half of the smallest division (graduation).

1. $1 \text{ cm} \div 2 = 0.5$ cm.
2. $1 \text{ mL} \div 2 = 0.5$ mL.
3. $500 \text{ g} \div 2 = 250$ g.
4. $0.1°\text{C} \div 2 = 0.05°\text{C}$.

Ex 16: Tom's digital thermometer indicates a temperature of 36.4°C.

1. What is the smallest division of the thermometer based on this reading?

$$\boxed{0.1}\ {}^\circ\mathrm{C}$$

2. Determine the range of values in which Tom's actual temperature T lies.

$$\boxed{36.35} \leq T < \boxed{36.45}$$

Answer:

1. The value is given to 1 decimal place, so the division is 0.1.
2. Accuracy is $\pm\frac{0.1}{2} = \pm 0.05$.

$$36.4 - 0.05 \leq T < 36.4 + 0.05 \implies 36.35 \leq T < 36.45.$$

Ex 17: Joanne's exercise watch displays the distance she has run to **3 significant figures**.Find the **least** distance Joanne could have run for each display:

1. Display: 1.06 km.

$$\boxed{1.055}\ \text{km}$$

2. Display: 10.1 km.

$$\boxed{10.05}\ \text{km}$$

Answer: We look for the lower bound.

1. 1.06 (2 dp): Unit is 0.01. Error ± 0.005. Lower bound $= 1.06 - 0.005 = 1.055$.
2. 10.1 (1 dp): Unit is 0.1. Error ± 0.05. Lower bound $= 10.1 - 0.05 = 10.05$.

Ex 18: Hasan has measured the length of several ropes to be 2.4 m each.

1. What is the accuracy of his measurement?

$$\pm\ \boxed{0.05}\ \text{m}$$

2. If Hasan places 10 of these ropes end to end, what is the maximum possible total length?

$$\boxed{24.5}\ \text{m}$$

Answer:

1. Measurement 2.4 (1 dp) $\rightarrow$ unit 0.1. Accuracy ± 0.05.
2. Upper bound for one rope $= 2.45$. For 10 ropes: $10 \times 2.45 = 24.5$ m.

Ex 19: In a race, the times recorded for Jiao and Liang were 128 s and 133 s respectively, measured to the nearest second.Find the range of possible values for the time difference d by which Jiao beat Liang (i.e., Liang's time minus Jiao's time).

$$\boxed{4} < d < \boxed{6}$$

Answer: Bounds (nearest second $\rightarrow \pm 0.5$):

- Jiao (J): $127.5 \leq J < 128.5$.
- Liang (L): $132.5 \leq L < 133.5$.

Difference $D = L - J$:

- $D_{\text{min}} = L_{\text{min}} - J_{\text{max}} = 132.5 - 128.5 = 4$.
- $D_{\text{max}} = L_{\text{max}} - J_{\text{min}} = 133.5 - 127.5 = 6$.

D BOUNDS

D.1 DETERMINING LOWER AND UPPER BOUNDS

Ex 20: For each of the following measurements, determine the lower bound and upper bound.

1. $x = 5$ cm (nearest cm).

Lower Bound: $\boxed{4.5}$ Upper Bound: $\boxed{5.5}$

2. $y = 8.4$ kg (nearest 0.1 kg).

Lower Bound: $\boxed{8.35}$ Upper Bound: $\boxed{8.45}$

3. $z = 120$ m (nearest 10 m).

Lower Bound: $\boxed{115}$ Upper Bound: $\boxed{125}$

4. $t = 3.45$ s (2 decimal places).

Lower Bound: $\boxed{3.445}$ Upper Bound: $\boxed{3.455}$

Answer: The bounds are found by adding and subtracting half the degree of accuracy ($\frac{\text{unit}}{2}$).

1. Unit $=$ 1 cm. Half-unit $=$ 0.5. Bounds: $5 \pm 0.5 \implies [4.5, 5.5)$.
2. Unit $=$ 0.1 kg. Half-unit $=$ 0.05. Bounds: $8.4 \pm 0.05 \implies [8.35, 8.45)$.
3. Unit $=$ 10 m. Half-unit $=$ 5. Bounds: $120 \pm 5 \implies [115, 125)$.
4. Unit $=$ 0.01 s. Half-unit $=$ 0.005. Bounds: $3.45 \pm 0.005 \implies [3.445, 3.455)$.

Ex 21: A distance is measured as $D = 400$ km correct to 1 significant figure.

1. Determine the lower bound.

$$\boxed{350}$$

2. Determine the upper bound.

$$\boxed{450}$$

Answer: The first significant figure is the '4', which is in the **hundreds** place.

- The accuracy unit is 100.
- The tolerance is half the unit: $\frac{100}{2} = 50$.
- Lower Bound: $400 - 50 = 350$.
- Upper Bound: $400 + 50 = 450$.
- The range is $350 \leq D < 450$.

D.2 CALCULATING WITH BOUNDS

Ex 22: A rectangle has length $L = 8.5$ cm and width $W = 4.2$ cm, both measured to 1 decimal place.

1. Determine the lower and upper bounds for the length L.

   Lower: $\boxed{8.45}$ Upper: $\boxed{8.55}$

2. Determine the lower and upper bounds for the width W.

   Lower: $\boxed{4.15}$ Upper: $\boxed{4.25}$

3. Calculate the maximum possible perimeter of the rectangle.

   $\boxed{25.6}$

4. Calculate the minimum possible area of the rectangle.

   $\boxed{35.0675}$

Answer: The unit of accuracy is 0.1, so the error is ± 0.05.

1. Bounds for L: $[8.45, 8.55)$.
2. Bounds for W: $[4.15, 4.25)$.
3. Max Perimeter $= 2 \times (L_{max} + W_{max}) = 2 \times (8.55 + 4.25) = 2 \times 12.8 = 25.6$ cm.
4. Min Area $= L_{min} \times W_{min} = 8.45 \times 4.15 = 35.0675$ cm^2.

Ex 23: The distance traveled by a car is $d = 200$ km (nearest 10 km) and the time taken is $t = 4.0$ hours (nearest 0.1 h).

1. Write down the upper bound for the distance.

   $\boxed{205}$

2. Write down the lower bound for the time.

   $\boxed{3.95}$

3. Calculate the maximum possible average speed (in km/h).

   $\boxed{51.90}$

Answer:

1. Distance (nearest 10): error is ± 5. $d \in [195, 205)$. Upper bound $d_{max} = 205$.
2. Time (nearest 0.1): error is ± 0.05. $t \in [3.95, 4.05)$. Lower bound $t_{min} = 3.95$.
3. To maximize a fraction, divide the largest numerator by the smallest denominator:

$$\text{Max Speed} = \frac{d_{max}}{t_{min}} = \frac{205}{3.95} \approx 51.898... \approx 51.90 \text{ km/h}.$$

Ex 24: Two lengths are given as $A = 15$ cm and $B = 12$ cm, both to the nearest cm.
Calculate the bounds for the difference $A - B$.

Lower Bound: $\boxed{2}$ Upper Bound: $\boxed{4}$

Answer: Bounds for A: $[14.5, 15.5)$. Bounds for B: $[11.5, 12.5)$.
To find the difference bounds:

- Max Difference: $A_{max} - B_{min} = 15.5 - 11.5 = 4$.
- Min Difference: $A_{min} - B_{max} = 14.5 - 12.5 = 2$.

So $2 < A - B < 4$.

Ex 25: A box has a height $h = 10$ cm and a square base with side $s = 5$ cm. Both measurements are correct to the nearest cm.
Calculate the maximum possible volume of the box.

$\boxed{317.625}$

Answer: Volume $V = s^2 \times h$.

- Bounds for s: $4.5 \leq s < 5.5$. Upper bound $s_{max} = 5.5$.
- Bounds for h: $9.5 \leq h < 10.5$. Upper bound $h_{max} = 10.5$.
- Max Volume $= (5.5)^2 \times 10.5 = 30.25 \times 10.5 = 317.625$ cm^3.