DISCRETE RANDOM VARIABLES

A RANDOM VARIABLES

A.1 DEFINITIONS

A.1.1 FINDING THE VALUE OF A RANDOM VARIABLE

Ex 1: Let X represent the number of heads obtained when tossing two fair coins: a red coin and a blue coin. Determine the value of X for each of the following outcomes:

- $X(T,T) = \square$,
- $X(H,T) = \square$,
- $X(T,H) = \square$,
- $X(H,H) = \square$.

Ex 2: Let X represent the number of heads obtained when tossing three fair coins: a red coin, a blue coin, and a green coin. Determine the value of X for each of the following outcomes:

- $X(T,T,T) = \square$,
- $X(H,T,T) = \square$,
- $X(T,H,T) = \square$,
- $X(T,T,H) = \square$,
- $X(H,H,T) = \square$,
- $X(H,T,H) = \square$,
- $X(T,H,H) = \square$,
- $X(H,H,H) = \square$.

Ex 3: Let X represent the sum of the numbers obtained when rolling two fair six-sided dice: a red die and a blue die. Determine the value of X for each of the following outcomes:

- $X(1,1) = \square$,
- $X(2,3) = \square$,
- $X(3,2) = \square$,
- $X(4,5) = \square$,
- $X(5,4) = \square$,
- $X(6,6) = \square$.

Ex 4: Let X represent the product of the numbers obtained when rolling two fair six-sided dice: a red die and a blue die. Determine the value of X for each of the following outcomes:

- $X(1,1) = \square$,
- $X(2,3) = \square$,
- $X(3,2) = \square$,
- $X(4,5) = \square$,
- $X(5,4) = \square$,
- $X(6,6) = \square$.

Ex 5: Let X represent the net gain (in dollars) from a game where you pay \$3 to play and roll one fair six-sided die. You receive a payout equal to the number of dollars shown on the die. Determine the value of X for each of the following outcomes:

- $X(1) = \square$,
- $X(2) = \square$,
- $X(3) = \square$,
- $X(4) = \square$,
- $X(5) = \square$,
- $X(6) = \square$.

Ex 6: Let X represent the net gain (in dollars) from a game where you pay \$2 to play and toss two fair coins: a red coin and a blue coin. You receive a payout of \$1 if no heads appear, \$3 if one head appears, and \$5 if two heads appear. Determine the value of X for each of the following outcomes:

- $X(T,T) = \square$,
- $X(H,T) = \square$,
- $X(T,H) = \square$,
- $X(H,H) = \square$.

A.1.2 IDENTIFYING THE POSSIBLE VALUES

MCQ 7: Let X represent the number of heads obtained when tossing two fair coins: a red coin and a blue coin. Identify the possible values of X.

Choose the one correct answer:

- ☐ $\{0\}$,
- ☐ $\{0,1\}$,
- ☐ $\{0,1,2\}$,
- ☐ $\{0,1,2,3\}$.

MCQ 8: Let X represent the number of heads obtained when tossing three fair coins: a red coin, a blue coin, and a green coin. Identify the possible values of X.

Choose the one correct answer:

- ☐ $\{0\}$,

- ☐ $\{0, 1\}$,
- ☐ $\{0, 1, 2\}$,
- ☐ $\{0, 1, 2, 3\}$.

MCQ 9: Let X represent the net gain (in dollars) from a game where you pay \$4 to play and roll one fair six-sided die. You receive a payout equal to the number of dollars shown on the die. Identify the possible values of X.

Choose the one correct answer:

- ☐ $\{-3, -2, -1, 0, 1, 2\}$,
- ☐ $\{-4\}$,
- ☐ $\{0, 1, 2, 3, 4, 5, 6\}$,
- ☐ $\{-4, -3, -2, -1, 0, 1\}$.

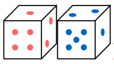
MCQ 10: Let S represent the sum of the numbers obtained when rolling two fair six-sided dice: a red die and a blue die. Identify the possible values of S.

Choose the one correct answer:

- ☐ $\{1, 2\}$,
- ☐ $\{1, 2, 3, 4, 5, 6\}$,
- ☐ $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$,
- ☐ $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$.

A.1.3 DEFINING A RANDOM VARIABLE

MCQ 11: Two fair six-sided dice (one red and one blue) are rolled, and the sum of the numbers on their top faces is recorded, as shown in this example:

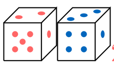
 $1 + 2 = 3$

Define the random variable S to model this situation.

Choose the one correct answer:

- ☐ The random variable S represents the difference between the numbers on the two dice: $S(i, j) = |i - j|$,
- ☐ The random variable S represents the product of the numbers on the two dice: $S(i, j) = i \times j$,
- ☐ The random variable S represents the sum of the numbers on the two dice: $S(i, j) = i + j$.

MCQ 12: Two fair six-sided dice (one red and one blue) are rolled, and the product of the numbers on their top faces is recorded, as shown in this example:

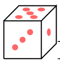
 $2 \times 3 = 6$

Define the random variable S to model this situation.

Choose the one correct answer:

- ☐ The random variable S represents the difference between the numbers on the two dice: $S(i, j) = |i - j|$,
- ☐ The random variable S represents the sum of the numbers on the two dice: $S(i, j) = i + j$,
- ☐ The random variable S represents the product of the numbers on the two dice: $S(i, j) = i \times j$.

MCQ 13: A fair six-sided die is rolled in a game where you pay \$3 to play and receive a payout equal to the number of dollars shown on the die, as shown in this example:

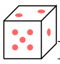
 Payout: \$5, Net Gain: $5 - 3 = 2$

The random variable X represents the net gain from the game.

Choose the one correct answer:

- ☐ $X(i) = 3 - i$,
- ☐ $X(i) = i - 3$,
- ☐ $X(i) = i + 3$.

MCQ 14: A fair six-sided die is rolled in a game where you pay \$3 to play and receive a payout (in dollars) equal to 2 times the number shown on the die, as shown in this example:

 Payout: $2 \times 2 = 4$ dollars, Net Gain: $4 - 3 = 1$

The random variable X represents the net gain from the game.

Choose the one correct answer:

- ☐ $X(i) = 2 \times i - 3$,
- ☐ $X(i) = 3 - 2 \times i$,
- ☐ $X(i) = 2 \times i$.

A.1.4 FINDING EVENTS INVOLVING A RANDOM VARIABLE

MCQ 15: Let X be the number of heads when tossing 2 coins: and . List the outcomes for $(X = 1)$.

Choose the one correct answer:

- ☐ $\{(T, T)\}$,
- ☐ $\{(T, H)\}$,
- ☐ $\{(T, H), (H, T)\}$,
- ☐ $\{(H, H)\}$.

MCQ 16: Let X be the number of heads when tossing 3 fair coins: a red coin, a blue coin, and a green coin. List the outcomes for $(X = 2)$.

Choose the one correct answer:

- ☐ $\{(H, H, H), (T, T, T)\}$ (all heads or all tails),
- ☐ $\{(H, H, T), (H, T, H), (T, H, H)\}$ (exactly two heads),
- ☐ $\{(T, T, H), (H, H, T)\}$ (green coin decides),
- ☐ $\{(H, H, H)\}$ (maximum heads).

MCQ 17: Let S be the sum of the numbers when rolling two fair six-sided dice: a red die and a blue die. List the outcomes for $(S = 4)$.

Choose the one correct answer:

- ☐ $\{(1,3),(2,2),(3,1)\}$,
- ☐ $\{(1,3),(2,2)\}$,
- ☐ $\{(1,3),(2,2),(3,1),(4,0)\}$,
- ☐ $\{(4,4)\}$.

MCQ 18: Let S be the sum of the numbers when rolling two fair six-sided dice: a red die and a blue die. List the outcomes for $(S \geq 10)$.

Choose the one correct answer:

- ☐ $\{(4,6),(5,5),(6,4)\}$,
- ☐ $\{(4,6),(5,5)\}$,
- ☐ $\{(4,6),(5,5),(5,6),(6,4),(6,5),(6,6)\}$,
- ☐ $\{(6,6)\}$.

A.1.5 DEFINING A RANDOM VARIABLE TO MODEL A SITUATION

MCQ 19: In a quality control study, a manufacturing plant produces batches of 100 products each, and we count the number of defective products in each batch to assess the plant's reliability. Define the random variable X to model this situation.

Choose the one correct answer:

- ☐ The random variable X represents the total number of products in each batch, which is fixed at 100.
- ☐ The random variable X represents the plant's reliability, recorded as a binary outcome where 1 indicates a "reliable" batch (no defects) and 0 indicates an "unreliable" batch (at least one defect).
- ☐ The random variable X represents the number of defective products in a batch of 100.

MCQ 20: In a study of a new training program's impact, each employee's performance is assessed before and after, with improvement classified as 'Improved' or 'Not Improved'. Define the random variable Z_i, where i is the i-th employee, to model this study.

Choose the one correct answer:

- ☐ The random variable Z_i represents the number of employees in the training program.
- ☐ The random variable Z_i represents the improvement status of the i-th employee, where 1 = 'Improved' and 0 = 'Not Improved'.
- ☐ The random variable Z_i represents the duration of the training program.
- ☐ The random variable Z_i represents the performance score of the i-th employee.

MCQ 21: In a study of public transportation, a researcher records the time (in minutes) a passenger waits at a bus stop until the next bus arrives. Define the random variable T to model this situation.

Choose the one correct answer:

- ☐ The random variable T represents the waiting time (in minutes) until the next bus arrives.
- ☐ The random variable T represents the number of buses arriving per hour, which is fixed by the schedule.
- ☐ The random variable T represents whether a bus is late, recorded as 1 for "late" and 0 for "on time."

MCQ 22: During a soccer tournament analysis, a statistician tracks the number of goals scored by a team in a single match. Define the random variable G to model this situation.

Choose the one correct answer:

- ☐ The random variable G represents the duration of the match, fixed at 90 minutes.
- ☐ The random variable G represents the number of goals scored by the team in a match.
- ☐ The random variable G represents the match outcome, recorded as 1 for "win" and 0 for "loss or draw."

A.2 PROBABILITY DISTRIBUTION

A.2.1 FINDING THE PROBABILITY DISTRIBUTION

Ex 23: Let X represent the number obtained when rolling one fair four-sided die with faces numbered 1, 2, 3, and 4. Complete the table with the probabilities as decimal numbers.

x	1	2	3	4
$P(X = x)$				

Ex 24: Let X represent the number of girls in a family with two children, where each child is equally likely to be a boy or a girl. Complete the table with the probabilities as decimal numbers.

x	0	1	2
$P(X = x)$			

Ex 25: Let X represent the number of heads obtained when tossing three fair coins: a red coin, a blue coin, and a green coin. Complete the table with the probabilities as decimal numbers.

x	0	1	2	3
$P(X = x)$				

MCQ 26: Let S represent the sum of the numbers obtained when rolling two fair four-sided dice, each numbered 1, 2, 3, and 4: . Identify the probability distribution of S.

Choose the one correct answer:

- [ ]

s	2	3	4	5	6	7	8
$P(S=s)$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

- [ ]

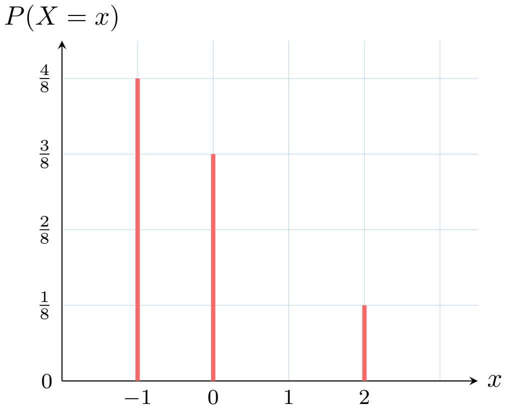
s	2	3	4	5	6	7	8
$P(S=s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

- [ ]

s	2	3	4	5	6	7	8
$P(S=s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{5}{16}$	$\frac{6}{16}$	$\frac{7}{16}$

A.2.2 READING PROBABILITY DISTRIBUTIONS FROM GRAPHS

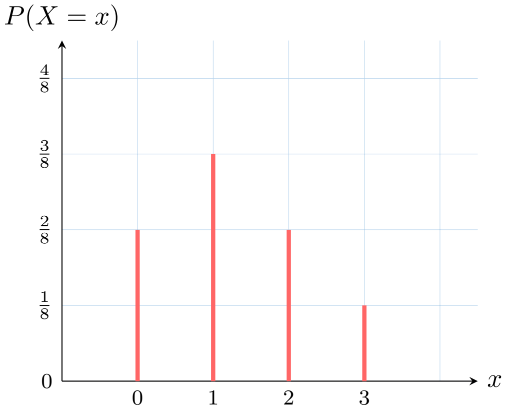
Ex 27: Consider the following probability distribution for a random variable X, represented by the graph below:



Determine the probability:

$$P(X=2) = \boxed{}$$

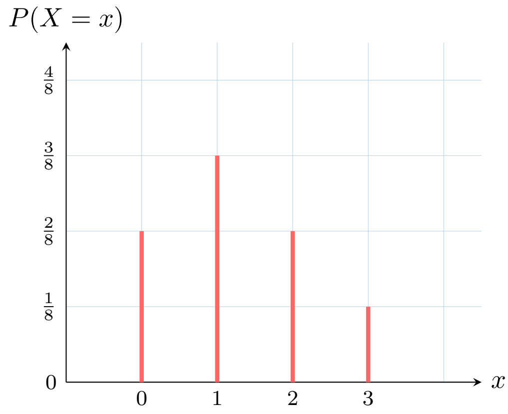
Ex 28: Consider the following probability distribution for a random variable X, represented by the graph below:



Determine the probability:

$$P(X=1) = \boxed{}$$

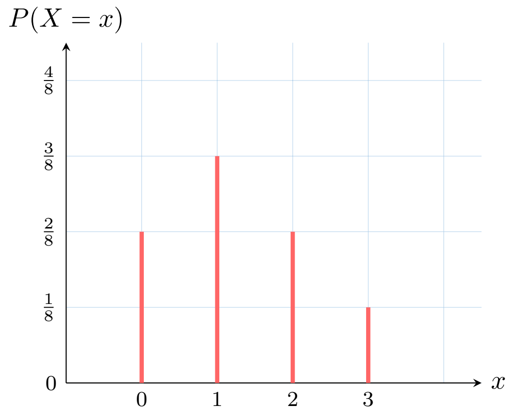
Ex 29: Consider the following probability distribution for a random variable X, represented by the graph below:



Determine the probability:

$$P(X \geq 1) = \boxed{}$$

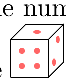
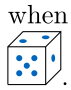
Ex 30: Consider the following probability distribution for a random variable X, represented by the graph below:

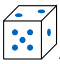


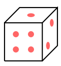
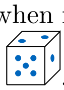
Determine the probability:

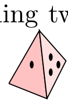
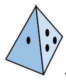
$$P(X \leq 1) = \boxed{}$$

A.2.3 CONSTRUCTING A PROBABILITY DISTRIBUTION

Ex 31: Let S represent the sum of the numbers obtained when rolling two fair six-sided dice: a red die  and a blue die . Construct the probability distribution table for S.

Ex 32: Let M represent the maximum of the numbers obtained when rolling two fair six-sided dice: a red die  and a blue die . Construct the probability distribution table for M.

Ex 33: Let D represent the absolute difference of the numbers obtained when rolling two fair six-sided dice: a red die  and a blue die . Construct the probability distribution table for D.

Ex 34: Let P represent the product of the numbers obtained when rolling two fair four-sided dice, each numbered 1, 2, 3, 4: a red die  and a blue die . Construct the probability distribution table for P.

A.3 EXISTENCE OF A RANDOM VARIABLE WITH A GIVEN PROBABILITY DISTRIBUTION

A.3.1 VERIFYING PROBABILITY DISTRIBUTION VALIDITY

MCQ 35: Determine whether the following table represents a valid probability distribution for a random variable X:

x	1	2	3
$P(X = x)$	0.3	0.4	0.3

Select the one correct answer:

☐ Valid probability distribution

☐ Not a valid probability distribution

MCQ 36: Determine whether the following table represents a valid probability distribution for a random variable X:

x	1	2	3
$P(X = x)$	0.2	0.5	0.4

Select the one correct answer:

☐ Valid probability distribution

☐ Not a valid probability distribution

MCQ 37: Determine whether the following table represents a valid probability distribution for a random variable X:

x	1	2	3
$P(X = x)$	0.5	-0.1	0.6

Select the one correct answer:

☐ Valid probability distribution

☐ Not a valid probability distribution

MCQ 38: Determine whether the following table represents a valid probability distribution for a random variable X:

x	0	1	2	3
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{4}$

Select the one correct answer:

☐ Valid probability distribution

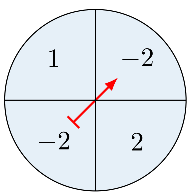
☐ Not a valid probability distribution

A.3.2 DEFINING A RANDOM VARIABLE AND ITS PROBABILITY DISTRIBUTION

Ex 39: We survey a class of 30 students about their siblings and obtain these results: 6 students have 0 siblings, 15 have 1 sibling, 6 have 2 siblings, and 3 have 3 siblings. Let X represent the number of siblings of a student chosen at random from this class. Complete the table with the probabilities as decimal numbers.

x	0	1	2	3
$P(X = x)$				

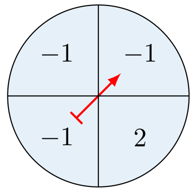
Ex 40: In a game of chance, a player spins a fair spinner divided into four equal sections, each labeled with a gain (in dollars) as shown below:



Let X represent the player's gain (in dollars) from the game. Complete the table with the probabilities as decimal numbers.

x	-2	1	2
$P(X = x)$			

Ex 41: In a game of chance, a player spins a fair spinner divided into four equal sections, with gains (in dollars) labeled as shown below:



Let X represent the player's gain (in dollars) from the game. Complete the table with the probabilities as decimal numbers.

x	-1	2
$P(X = x)$		

Ex 42: A bag contains 20 marbles: 4 red, 10 blue, 4 green, and 2 yellow. A marble is drawn at random, and points are awarded based on its color: 0 points for red, 1 point for blue, 2 points for green, and 3 points for yellow. Let X represent the number of points earned. Complete the table with the probabilities as decimal numbers.

x	0	1	2	3
$P(X = x)$				

A.3.3 FINDING PROBABILITIES IN EVERYDAY SCENARIOS

Ex 43: The random variable X represents the number of goals scored by a soccer player in a match, with the probability distribution given below:

x	0	1	2	3
$P(X = x)$	0.35	0.3	0.25	0.1

Find the probability that the player scores exactly 2 goals in a match.

The probability is ☐.

Ex 44: The random variable X represents the number of times a student visits the nurse's office in a day, with the probability distribution given below:

x	0	1	2	3
$P(X = x)$	0.35	0.3	0.25	0.1

Find the probability that a student visits the nurse's office at least twice in a day.

The probability is ☐.

Ex 45: The random variable X represents the number of rainy days in a week, with the probability distribution given below:

x	0	1	2	3
$P(X = x)$	0.35	0.3	0.25	0.1

Find the probability that there is at least one rainy day in a week.

The probability is ☐.

Ex 46: The random variable X represents the number of defective items in a batch of 7, with the probability distribution given below:

x	0	1	2	3	4	5	6	7
$P(X=x)$	0.4	0.25	0.15	0.1	0.05	0.03	0.02	0.01

Find the probability that there is at least one defective item in the batch.

The probability is $\boxed{}$.

B EXPECTATION

B.1 DEFINITION

B.1.1 CALCULATING EXPECTATIONS

Ex 47: The random variable X represents the number of goals scored by a soccer player in a match, with the probability distribution given below:

x	0	1	2	3
$P(X=x)$	0.1	0.3	0.5	0.1

Calculate the expected value $E(X)$, the average number of goals scored per match.

$$E(X) = \boxed{}$$

Ex 48: The random variable X represents the number of hours a student spends studying on a weekend, with the probability distribution given below:

x	0	1	2	3	4
$P(X=x)$	0.2	0.25	0.3	0.15	0.1

Calculate the expected value $E(X)$, the average number of hours spent studying per weekend.

$$E(X) = \boxed{}$$

Ex 49: The random variable X represents the number of customers served by a cashier in an hour, with the probability distribution given below:

x	0	1	2	3	4
$P(X=x)$	0.15	0.2	0.35	0.25	0.05

Calculate the expected value $E(X)$, the average number of customers served per hour.

$$E(X) = \boxed{}$$

Ex 50: The random variable X represents the number of emails received by an employee in an hour, with the probability distribution given below:

x	0	1	2	3	4
$P(X=x)$	0.3	0.25	0.2	0.15	0.1

Calculate the expected value $E(X)$, the average number of emails received per hour.

$$E(X) = \boxed{}$$

B.1.2 EXPLORING EXPECTED VALUES

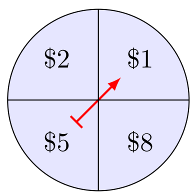
Ex 51: In a game of chance, a player rolls a standard six-sided die. The number rolled is the outcome of interest.

- Calculate the expected value $E(X)$ of the roll.
- Interpret the result in terms of the player's average outcome per roll.

Ex 52: An insurance company offers you a contract for 150 dollars. There's a 5

- Calculate the expected profit $E(X)$ of the insurance company.
- Explain what the result means in terms of the average expected profit for the company.

Ex 53: In a game of chance, a player spins a spinner:



The player wins the amount of money indicated by the arrow, but it costs \$5 to play each game. In gambling, the gain is defined as the payout minus the cost to play.

- Calculate the expected gain $E(X)$ of the player.
- Interpret the result in terms of the player's average outcome per game.

Ex 54: In a game of chance, a player bets \$1 on a single number in a classical roulette wheel numbered from 0 to 36. If the chosen number comes up, the player wins 35 times their bet plus their bet back, receiving a total payout of \$36; otherwise, they lose their bet. In gambling, the gain is defined as the payout minus the cost to play.

- Calculate the expected gain $E(X)$ of the player.
- Interpret the result in terms of the player's average outcome per game.

C VARIANCE AND STANDARD DEVIATION

C.1 DEFINITIONS

C.1.1 CALCULATING STANDARD DEVIATION

Ex 55: A soccer player's number of goals scored in a match is represented by the random variable X. The probability distribution for X is shown below:

x (goals)	0	1	2	3
$P(X=x)$	0.6	0.1	0.1	0.3

Calculate the standard deviation $\sigma(X)$, which shows how much the number of goals typically varies from the average per match (round at two decimal places).

$$\sigma(X) = \boxed{}$$

Ex 56: The random variable X represents the number of cups of coffee a teacher drinks in a day. The probability distribution for X is shown below:

x (cups)	0	1	2	3
$P(X=x)$	0.1	0.3	0.4	0.2

Calculate the standard deviation $\sigma(X)$, which shows how much the number of cups typically varies from the average per day (round to two decimal places).

$$\sigma(X) = \boxed{}$$

Ex 57: The random variable X represents the number of siblings a student in a class has. The probability distribution for X is shown below:

x (siblings)	0	1	2	3
$P(X=x)$	0.3	0.4	0.2	0.1

Calculate the standard deviation $\sigma(X)$, which shows how much the number of siblings typically varies from the average per student (round to two decimal places).

$$\sigma(X) = \boxed{}$$

Ex 58: The random variable X represents the number of car accidents a driver has in a year. The probability distribution for X is shown below:

x (accidents)	0	1	2	3
$P(X=x)$	0.7	0.2	0.08	0.02

Calculate the standard deviation $\sigma(X)$, which shows how much the number of car accidents typically varies from the average per year (round to two decimal places).

$$\sigma(X) = \boxed{}$$

C.1.2 CHOOSING BASED ON EXPECTED VALUES AND RISK

Ex 59: A customer is choosing between two financial options for a \$1000 investment over one year. The random variables X and Y represent the gain (in dollars) for Option A and Option B, respectively, with:

- Expected gain: $E(X) = 67.5$ for Option A and $E(Y) = 120$ for Option B
- Standard deviation: $\sigma(X) = 16.01$ for Option A and $\sigma(Y) = 95.39$ for Option B

The customer wants to minimize risk (safe placement). Which option should they choose, and why? Justify your answer based on the given expected gains and standard deviations.

Ex 60: A customer is choosing between two financial options for a \$1000 investment over one year. The random variables X and Y represent the gain (in dollars) for Option A and Option B, respectively, with:

- Expected gain: $E(X) = 67.5$ for Option A and $E(Y) = 120$ for Option B
- Standard deviation: $\sigma(X) = 16.01$ for Option A and $\sigma(Y) = 95.39$ for Option B

The customer wants to maximize profit regardless of the risk. Which option should they choose, and why? Justify your answer based on the given expected gains and standard deviations.

Ex 61: A person is choosing between two countries to live in based on the average annual temperature (in degrees Celsius). The random variables X and Y represent the temperature for Country A and Country B, respectively, with:

- Expected temperature: $E(X) = 18.5$ for Country A and $E(Y) = 25.0$ for Country B
- Standard deviation: $\sigma(X) = 3.2$ for Country A and $\sigma(Y) = 8.5$ for Country B

The person wants to maximize warmth regardless of temperature variability. Which country should they choose, and why? Justify your answer based on the given expected temperatures and standard deviations.

Ex 62: A student is choosing between two part-time jobs based on weekly earnings (in dollars). The random variables X and Y represent the earnings for Job A and Job B, respectively, with:

- Expected earnings: $E(X) = 150$ for Job A and $E(Y) = 175$ for Job B
- Standard deviation: $\sigma(X) = 10.5$ for Job A and $\sigma(Y) = 25.8$ for Job B

The student wants to minimize variability in their earnings (stable income). Which job should they choose, and why? Justify your answer based on the given expected earnings and standard deviations.

D CLASSICAL DISTRIBUTIONS

D.1 UNIFORM DISTRIBUTION

D.1.1 IDENTIFYING UNIFORM DISTRIBUTIONS

MCQ 63: When rolling a fair six-sided die, the random variable X represents the number that appears on the top face.
Does the random variable X follow a uniform distribution?
Select the one correct answer:

- ☐ True
- ☐ False

MCQ 64: A standard deck of 52 playing cards is divided into 4 suits: hearts, diamonds, clubs, and spades, with each suit containing 13 cards. A card is drawn at random from the deck, and the random variable Z represents the suit of the card, assigned as follows: 1 for hearts, 2 for diamonds, 3 for clubs, and 4 for spades.
Does the random variable Z follow a uniform distribution?
Select the one correct answer:

- ☐ True
- ☐ False

MCQ 65: A bag contains 5 red, 4 blue, and 3 green marbles, making a total of 12 marbles. A marble is drawn at random, and the random variable Y represents the color of the marble, assigned as follows: 1 for red, 2 for blue, and 3 for green.
Does the random variable Y follow a uniform distribution?
Select the one correct answer:

- ☐ True
- ☐ False

MCQ 66: A telephone number is selected at random, and the random variable X represents its last digit, which can be any integer from 0 to 9.
Does the random variable X follow a uniform distribution?
Select the one correct answer:

- ☐ True
- ☐ False

D.2 BERNOULLI DISTRIBUTION

D.2.1 IDENTIFYING BERNOULLI DISTRIBUTIONS

MCQ 67: A fair coin is tossed once, and the random variable X represents the outcome, with values assigned as follows: 0 for tails and 1 for heads.
Does the random variable X follow a Bernoulli distribution?
Select the one correct answer:

- ☐ True
- ☐ False

MCQ 68: Two fair coins are tossed, and the random variable X represents the number of heads obtained, with possible values 0, 1, or 2.
Does the random variable X follow a Bernoulli distribution?
Select the one correct answer:

- ☐ True
- ☐ False

MCQ 69: Mia asks a boy, "Do you like me?" and he answers with either "Yes" or "No." The random variable X represents his response, with values assigned as follows: 1 for "Yes" and 0 for "No." Assume the boy's answer is a random guess with equal chances for each response.
Does the random variable X follow a Bernoulli distribution?
Select the one correct answer:

- ☐ True

☐ False

MCQ 70: A student is selected at random from a class containing 5 girls and 10 boys, making a total of 15 students. The random variable X represents the gender of the selected student, with values assigned as follows: 0 for boy and 1 for girl. Does the random variable X follow a Bernoulli distribution?

Select the one correct answer:

☐ True

☐ False

D.2.2 FINDING A PROBABILITY OF SUCCESS

Ex 71: A fair coin is tossed once, and the random variable X represents the outcome, with values assigned as follows: 0 for tails and 1 for heads.
Find the probability of success for the random variable X.

$$p = \boxed{}$$

Ex 72: A student is selected at random from a class containing 5 girls and 10 boys, making a total of 15 students. The random variable X represents the gender of the selected student, with values assigned as follows: 0 for boy and 1 for girl.
Find the probability of success for the random variable X.

$$p = \boxed{}$$

Ex 73: A basketball player has made 300 successful shots out of 400 attempts in practice. For a single shot, the random variable X represents the outcome, with values assigned as follows: 0 for a miss and 1 for a success.
Find the probability of success for the random variable X.

$$p = \boxed{}$$

Ex 74: A student, Leo, guesses the answer to a true/false quiz question. He has a tendency to guess "True" 60% of the time and "False" 40% of the time. The random variable X represents the outcome of his guess, with values assigned as follows: 0 for "False" and 1 for "True."
Find the probability of success for the random variable X.

$$p = \boxed{}$$

D.3 BINOMIAL DISTRIBUTION

D.3.1 IDENTIFYING BINOMIAL DISTRIBUTIONS

MCQ 75: A fair coin is tossed 100 times independently, and the random variable X represents the number of heads obtained, with possible values ranging from 0 to 100.
Does the random variable X follow a binomial distribution?

Select the one correct answer:

☐ True

☐ False

MCQ 76: An urn contains 3 red balls and 7 blue balls. A ball is drawn at random and replaced 50 times independently, and the random variable X represents the number of red balls drawn, with possible values ranging from 0 to 50.
Does the random variable X follow a binomial distribution?

Select the one correct answer:

☐ True

☐ False

MCQ 77: An urn contains 3 red balls and 7 blue balls, making a total of 10 balls. A ball is drawn at random without replacement 5 times, and the random variable X represents the number of red balls drawn, with possible values ranging from 0 to 3.
Does the random variable X follow a binomial distribution?

Select the one correct answer:

☐ True

☐ False

MCQ 78: In a group of 10 friends, a student invites 5 friends one by one to a party. Initially, each friend has a 40% chance of accepting, but for each friend who says "Yes," the next friend's probability of accepting increases by 5% due to excitement about the party. The random variable X represents the number of friends who accept, with possible values ranging from 0 to 5.
Does the random variable X follow a binomial distribution?

Select the one correct answer:

☐ True

☐ False

D.3.2 FINDING PROBABILITIES IN EVERYDAY SCENARIOS

Ex 79: Of all electric light bulbs produced, 5% are defective at manufacture. Six bulbs are randomly selected and tested. Find the probability that exactly two are defective (round to three decimal places).

$$P(\text{"2 defectives"}) \approx \boxed{}$$

Ex 80: Weather forecasts indicate that on any given day in June, there is a 30% chance of rain. Over a week (7 days), the random variable X represents the number of rainy days. Find the probability that exactly 4 days are rainy (round to three decimal places).

$$P(\text{"4 rainy days"}) \approx \boxed{}$$

Ex 81: A student guesses on a 5-question true/false quiz, with a 50% chance of guessing each question correctly. The random variable X represents the number of correct answers. Find the probability that the student gets 1 or fewer questions correct (round to three decimal places).

$P(\text{"1 or fewer correct"}) \approx \boxed{}$

Ex 82: A student sends party invitations to 4 friends via text message, and each friend has a 70% chance of accepting, independently of the others. The random variable X represents the number of friends who accept. Find the probability that all 4 friends accept (round to three decimal places).

$P(\text{"only successes"}) \approx \boxed{}$

D.3.3 CALCULATING AN EXPECTED VALUE

Ex 83: A student guesses randomly on a 10-question multiple-choice quiz, where each question has 4 options and only one is correct. Find the average number of correct guesses the student makes.

Average correct guesses $= \boxed{}$

Ex 84: Weather forecasts indicate that on any given day in June, there is a 30% chance of rain. Find the average number of rainy days in a week (7 days).

Average rainy days $= \boxed{}$

Ex 85: A delivery service has a 60% chance of delivering a package on time each day. Over 8 days, the random variable X represents the number of on-time deliveries. Find the average number of on-time deliveries over these 8 days. .

Average on-time deliveries $= \boxed{}$

Ex 86: A basketball player is practicing 3-point shots. The probability that he successfully scores each shot is $\frac{4}{5}$, and each successful shot is worth 3 points. He takes 100 shots independently. Find the average number of points he scores after taking all 100 shots .

Average points $= \boxed{}$