# A DISTRIBUTIVE LAW 1

### A.1 EXPANDING WITH ADDITION: LEVEL 1

**Ex 1:** Expand and simplify:

$$5(x+3) = 5x+15$$

Answer:

$$5(x+3)=5 \times x + 5 \times 3$$
$$= 5x + 15$$

Ex 2: Expand and simplify:

$$2(3+x) = 6+2x$$

Answer:

$$2(3+x)=2\times 3+2\times x$$
  
=  $6+2x$ 

Ex 3: Expand and simplify:

$$3(2x+2) = 6x+6$$

Answer:

$$3(2x+2)=3 \times 2x + 3 \times 2$$
  
=  $6x + 6$ 

Ex 4: Expand and simplify:

$$2(5+3x) = 10+6x$$

Answer:

$$2(5+3x)=2 \times 5 + 2 \times 3x$$
  
=  $10 + 6x$ 

## A.2 EXPANDING WITH ADDITION: LEVEL 2

**Ex 5:** Expand and simplify:

$$x(x+1) = x^2 + x$$

Answer:

$$x(x+1) = x \times x + x \times 1$$

$$= x^2 + x$$

Ex 6: Expand and simplify:

$$x(2x+3) = 2x^2 + 3x$$

Answer:

$$\widehat{x(2x+3)} = x \times 2x + x \times 3$$

$$= 2x^2 + 3x$$

Ex 7: Expand and simplify:

$$2x(x+2) = 2x^2 + 4x$$

Answer:

$$2x(x+2) = 2x \times x + 2x \times 2$$
$$= 2x^2 + 4x$$

Ex 8: Expand and simplify:

$$3x(2x+5) = 6x^2 + 15x$$

Answer:

$$3x(2x+5) = 3x \times 2x + 3x \times 5$$
  
=  $6x^2 + 15x$ 

### A.3 EXPANDING WITH ADDITION: LEVEL 3

Ex 9: Expand and simplify:

$$2(x+1) + x = 3x + 2$$

Answer:

$$2(x+1) + x = 2 \times x + 2 \times 1 + x \quad \text{(expanding)}$$

$$= 2x + 2 + x$$

$$= (2+1)x + 2 \quad \text{(combining)}$$

$$= 3x + 2 \quad \text{(simplifying)}$$

Ex 10: Expand and simplify:

$$2(2x+3) - 3x = x+6$$

Answer:

$$2(2x+3) - 3x = 2 \times 2x + 2 \times 3 - 3x \quad \text{(expanding)}$$

$$= 4x + 6 - 3x$$

$$= (4-3)x + 6 \quad \text{(combining)}$$

$$= x + 6 \quad \text{(simplifying)}$$

Ex 11: Expand and simplify:

$$x(x+2) - x^2 = 2x$$

Answer:

$$x(x+2) - x^2 = x \times x + x \times 2 - x^2 \quad \text{(expanding)}$$
$$= x^2 + 2x - x^2$$
$$= 2x \quad \text{(combining)}$$

Ex 12: Expand and simplify:

$$2x(3x+2) - 8x = 6x^2 - 4x$$

Answer:

$$2x(3x+2) - 8x = 2x \times 3x + 2x \times 2 - 8x \quad \text{(expanding)}$$
$$= 6x^2 + 4x - 8x$$
$$= 6x^2 + (4-8)x \quad \text{(combining)}$$
$$= 6x^2 - 4x \quad \text{(simplifying)}$$

### A.4 EXPANDING WITH SUBTRACTION: LEVEL 1

Ex 13: Expand and simplify:

$$2(x-2) = 2x-4$$

Answer:

$$2(x-2)=2 \times x-2 \times 2$$

$$= 2x-4$$

Ex 14: Expand and simplify:

$$3(5x - 6) = \boxed{15x - 18}$$

Answer:

$$3(5x-6)=3 \times 5x-3 \times 6$$
  
=  $15x-18$ 

**Ex 15:** Expand and simplify:

$$2(3-x) = 6-2x$$

Answer:

$$2(3-x)=2 \times 3-2 \times x$$

$$= 6-2x$$

Ex 16: Expand and simplify:

$$4(3-5x) = 12 - 20x$$

Answer:

$$4(3-5x)=4 \times 3-4 \times 5x$$
  
=  $12-20x$ 

### A.5 EXPANDING WITH SUBTRACTION: LEVEL 2

Ex 17: Expand and simplify:

$$x(x-1) = x^2 - x$$

Answer:

$$x(x-1)=x \times x - x \times 1$$

$$= x^2 - x$$

Ex 18: Expand and simplify:

$$x(2x-3) = 2x^2 - 3x$$

Answer:

$$\widehat{x(2x-3)} = x \times 2x - x \times 3 \\
= 2x^2 - 3x$$

Ex 19: Expand and simplify:

$$2x(x-2) = 2x^2 - 4x$$

Answer:

$$2x(x-2)=2x \times x - 2x \times 2$$

$$= 2x^2 - 4x$$

Ex 20: Expand and simplify:

$$3x(2x-5) = 6x^2 - 15x$$

Answer:

$$3x (2x-5) = 3x \times 2x - 3x \times 5 = 6x^2 - 15x$$

### A.6 EXPANDING WITH SUBTRACTION: LEVEL 3

Ex 21: Expand and simplify

$$2(x-2)+4=2x$$

Answer:

$$2(x-2) + 4 = 2 \times x - 2 \times 2 + 4 \quad \text{(expanding)}$$

$$= 2x - 4 + 4$$

$$= 2x + 0 \quad \text{(simplifying)}$$

$$= 2x$$

Ex 22: Expand and simplify

$$4(x-3) - 5x = -x - 12$$

Answer:

$$4(x-3) - 5x = 4 \times x - 4 \times 3 - 5x \quad \text{(expanding)}$$

$$= 4x - 12 - 5x$$

$$= (4-5)x - 12 \quad \text{(combining like terms)}$$

$$= -x - 12 \quad \text{(simplifying)}$$

Ex 23: Expand and simplify

$$x(x-2) + 6 = x^2 - 2x + 6$$

Answer:

$$x(x-2) + 6 = x \times x - x \times 2 + 6 \quad \text{(expanding)}$$
$$= x^2 - 2x + 6$$

Ex 24: Expand and simplify

$$2(x-2) + 3x - 10 = 5x - 14$$

Answer:

$$2(x-2) + 3x - 10 = 2 \times x - 2 \times 2 + 3x - 10$$
 (expanding)  
=  $2x - 4 + 3x - 10$   
=  $(2+3)x - 4 - 10$  (combining like term  
=  $5x - 14$  (simplifying)

### A.7 EXPANDING: LEVEL 4

Ex 25: Expand and simplify:

$$x(x^2 - x + 1) = x^3 - x^2 + x$$

Answer:

$$\mathbf{x}(x^2 - x + 1) = \mathbf{x} \times x^2 + \mathbf{x} \times (-x) + \mathbf{x} \times 1$$
$$= x^3 - x^2 + x$$

Ex 26: Expand and simplify:

$$2x(-x^3 + 2x - 2) = \boxed{-2x^4 + 4x^2 - 4x}$$

Answer:

$$2x(-x^3 + 2x - 2) = 2x \times (-x^3) + 2x \times 2x + 2x \times (-2)$$
$$= -2x^4 + 4x^2 - 4x$$

Ex 27: Expand and simplify:

$$-2(x^2 - 2x - 3) = \boxed{-2x^2 + 4x + 6}$$

Answer:

$$-2(x^{2} - 2x - 3) = (-2) \times x^{2} + (-2) \times (-2x) + (-2) \times (-3)$$
$$= -2x^{2} + 4x + 6$$

Ex 28: Expand and simplify:

$$-2x(-x^2 - 3x + 2) = 2x^3 + 6x^2 - 4x$$

$$-2x(-x^2 - 3x + 2) = (-2x) \times (-x^2) + (-2x) \times (-3x) + (-2x) \times 2$$
=  $2x^3 + 6x^2 - 4x$  Expand and simplify

## A.8 EXPANDING: LEVEL 4

Ex 29: Expand and simplify:

$$2(x+2) - x(x+2) = -x^2 + 4$$

Answer

$$2(x+2)-x(x+2) = 2 \times x + 2 \times 2 + (-x) \times x + (-x) \times 2$$
$$= 2x + 4 - x^2 - 2x$$
$$= -x^2 + 4$$

Ex 30: Expand and simplify:

$$-2x(x-4) + x(x-3) = \boxed{-x^2 + 5x}$$

$$-2x(x-4)+x(x-3) = (-2x) \times x + (-2x) \times (-4) + x \times x + x \times (-3)$$

$$= -2x^2 + 8x + x^2 - 3x$$

$$= -x^2 + 5x$$
(2x+1)(3x+3)

$$2x - 2(4 - x + x^2) = \boxed{-2x^2 + 4x - 8}$$

$$2x - 2(4 - x + x^{2}) = 2x + (-2) \times 4 + (-2) \times x + +(-2) \times x^{2}$$
$$= 2x - 8 + 2x - 2x^{2}$$
$$= -2x^{2} + 4x - 8$$

Ex 32: Expand and simplify:

$$x - x(x^2 + x + 1) = -x^3 - x^2$$

$$x - x(x^{2} + x + 1) = x - x \times x^{2} - x \times x - x \times 1$$

$$= x - x^{3} - x^{2} - x$$

$$= -x^{3} - x^{2}$$

## B DISTRIBUTIVE LAW 2

## **B.1 EXPANDING WITH ADDITION**

Ex 33: Expand and simplify

$$(x+2)(x+1) = x^2 + 3x + 2$$

Answer:

$$(x+2)\cdot(x+1)=x \times x + x \times 1 + 2 \times x + 2 \times 1$$
  
=  $x^2 + x + 2x + 2$   
=  $x^2 + 3x + 2$ 

$$(x+3)(x+4) = x^2 + 7x + 12$$

Answer:

$$(x+3)\cdot(x+4) = x \times x + x \times 4 + 3 \times x + \frac{3}{3} \times 4$$

$$= x^2 + 4x + 3x + 12$$

$$= x^2 + 7x + 12$$

Ex 35: Expand and simplify

$$(x+4)(2x+2) = 2x^2 + 10x + 8$$

Answer:

Answer:

$$(x+4)\cdot(2x+2)=x \times 2x + x \times 2 + 4 \times 2x + 4 \times 2$$
  
=  $2x^2 + 2x + 8x + 8$   
=  $2x^2 + 10x + 8$ 

$$(2x+1)(3x+2) = \boxed{6x^2 + 7x + 2}$$

**Ex 31:** Expand and simplify:

$$(2x+1)\cdot(3x+2) = 2x \times 3x + 2x \times 2 + 1 \times 3x + 1 \times 2$$

$$= 6x^{2} + 4x + 3x + 2$$

$$= 6x^{2} + 7x + 2$$

Ex 37: Expand and simplify

$$(x+1)^2 = \boxed{x^2 + 2x + 1}$$

Answer:

$$(x+1)^2 = (x+1)(x+1)$$
 (square definition)  
=  $x \times x + x \times 1 + 1 \times x + 1 \times 1$  (expanding)  
=  $x^2 + 2x + 1$  (combining)

## **B.2 EXPANDING WITH SUBTRACTION**

Ex 38: Expand and simplify

$$(x+2)(x-1) = x^2 + x - 2$$

Answer:

$$(x+2)\cdot(x-1)=x \times x + x \times (-1) + 2 \times x + 2 \times (-1)$$

$$= x^2 - x + 2x - 2$$

$$= x^2 + x - 2$$

Ex 39: Expand and simplify

$$(x-1)(x-2) = x^2 - 3x + 2$$

Answer:

$$(x-1)\cdot(x-2)=x \times x + x \times (-2) + (-1) \times x + (-1) \times (-2)$$

$$= x^2 - 2x - x + 2$$

$$= x^2 - 3x + 2$$

Ex 40: Expand and simplify

$$(x+3)(x-2) = x^2 + x - 6$$

Answer:

$$(x+3)\cdot(x-2)=x \times x + x \times (-2) + 3 \times x + 3 \times (-2)$$

$$= x^2 - 2x + 3x - 6$$

$$= x^2 + x - 6$$

Ex 41: Expand and simplify

$$(2x+1)(1-2x) = 1 - 4x^2$$

Answer:

$$(2x+1)\cdot(1-2x) = 2x \times 1 + 2x \times (-2x) + 1 \times 1 + 1 \times (-2x)$$

$$= 2x - 4x^{2} + 1 - 2x$$

$$= 1 - 4x^{2}$$

Ex 42: Expand and simplify

$$(-1+2x)(1-x) = \boxed{-2x^2 + 3x - 1}$$

Answer:

$$(-1+2x)\cdot(1-x) = (-1) \times 1 + (-1) \times (-x) + 2x \times 1 + 2x \times (-x)$$

$$= -1 + x + 2x - 2x^{2}$$

$$= -2x^{2} + 3x - 1$$

### **B.3 EXPANDING**

Ex 43: Expand and simplify:

$$(x-5)(2x+7) = 2x^2 - 3x - 35$$

Answer:

$$(x-5)(2x+7) = x \times 2x + x \times 7 + (-5) \times 2x + (-5) \times 7$$
$$= 2x^2 + 7x - 10x - 35$$
$$= 2x^2 - 3x - 35$$

Ex 44: Expand and simplify:

$$(4x+1)(x-6) = 4x^2 - 23x - 6$$

Answer.

$$(4x+1)(x-6) = 4x \times x + 4x \times (-6) + 1 \times x + 1 \times (-6)$$

$$= 4x^2 - 24x + x - 6$$

$$= 4x^2 - 23x - 6$$

**Ex 45:** Expand and simplify:

$$(2x-3)(-x+4) = -2x^2 + 11x - 12$$

Answer.

$$(2x-3)(-x+4) = 2x \times (-x) + 2x \times 4 + (-3) \times (-x) + (-3) \times 4$$
$$= -2x^2 + 8x + 3x - 12$$
$$= -2x^2 + 11x - 12$$

Ex 46: Expand and simplify:

$$(-x+2)(3x-4) = 3x^2 + 10x - 8$$

Answer:

$$(-x+2)(3x-4) = (-x) \times 3x + (-x) \times (-4) + 2 \times 3x + 2 \times (-4)$$
$$= -3x^2 + 4x + 6x - 8$$
$$= -3x^2 + 10x - 8$$

## C DIFFERENCE OF TWO SQUARES

## C.1 EXPANDING: LEVEL 1

Ex 47: Expand and simplify

$$(x+1)(x-1) = x^2 - 1$$

Answer:

$$(x+1)(x-1) = x^2 - 1^2$$
  
=  $x^2 - 1$ 

Ex 48: Expand and simplify

$$(x-3)(x+3) = x^2 - 9$$

Answer:

$$(x-3)(x+3) = x^2 - 3^2$$
$$= x^2 - 9$$

Ex 49: Expand and simplify



$$(4-x)(4+x) = \boxed{16-x^2}$$

Answer:

$$(4 - x)(4 + x) = 4^{2} - x^{2}$$
$$= 16 - x^{2}$$

Ex 50: Expand and simplify

$$(5+x)(5-x) = 25 - x^2$$

Answer:

$$(5+x)(5-x) = 5^2 - x^2$$
$$= 25 - x^2$$

### C.2 EXPANDING: LEVEL 2

Ex 51: Expand and simplify

$$(2x-4)(2x+4) = 4x^2 - 16$$

Answer:

$$(2x-4)(2x+4) = (2x)^2 - (4)^2$$
$$= 2^2x^2 - 4^2$$
$$= 4x^2 - 16$$

Ex 52: Expand and simplify

$$(x+\sqrt{2})(x-\sqrt{2}) = x^2-2$$

Answer:

$$(x + \sqrt{2})(x - \sqrt{2}) = x^2 - (\sqrt{2})^2$$
$$- x^2 - 2$$

Ex 53: Expand and simplify

$$\left(\frac{1}{2} - x\right)\left(\frac{1}{2} + x\right) = \boxed{\frac{1}{4} - x^2}$$

Answer:

$$\left(\frac{1}{2} - x\right) \left(\frac{1}{2} + x\right) = \left(\frac{1}{2}\right)^2 - x^2$$
$$= \frac{1^2}{2^2} - x^2$$
$$= \frac{1}{4} - x^2$$

Ex 54: Expand and simplify

$$\left(\frac{x}{2} - 1\right)\left(\frac{x}{2} + 1\right) = \boxed{\frac{x^2}{4} - 1}$$

Answer:

$$\left(\frac{x}{2} - 1\right) \left(\frac{x}{2} + 1\right) = \left(\frac{x}{2}\right)^2 - (1)^2$$
$$= \frac{x^2}{2^2} - 1^2$$
$$= \frac{x^2}{4} - 1$$

## **D BINOMIAL EXPANSION**

# D.1 EXPANDING PERFECT SQUARE BINOMIALS (ADDITION)

Ex 55: Expand and simplify

$$(x+2)^2 = x^2 + 4x + 4$$

Answer: In the perfect squares expansion, we substitute a=x and b=2:

$$(x + 2)^{2} = x^{2} + 2 \times x \times 2 + 2^{2}$$

$$= x^{2} + 4x + 4$$

Ex 56: Expand and simplify

$$(3+x)^2 = 9 + 6x + x^2$$

Answer: In the perfect squares expansion, we substitute a=3 and b=x:

$$(3+x)^2 = 3^2 + 2 \times 3 \times x + x^2$$
$$= 9 + 6x + x^2$$

Ex 57: Expand and simplify

$$(2x+1)^2 = \boxed{4x^2 + 4x + 1}$$

Answer: In the perfect squares expansion, we substitute a=2x and b=1:

$$(2x+1)^2 = (2x)^2 + 2 \times 2x \times 1 + (1)^2$$
$$= 2^2x^2 + 4x + 1$$
$$= 4x^2 + 4x + 1$$

Ex 58: Expand and simplify

$$(2+3x)^2 = \boxed{4+12x+9x^2}$$

Answer: In the perfect squares expansion, we substitute a=2 and b=3x:

$$(2+3x)^2 = 2^2 + 2 \times 2 \times 3x + (3x)^2$$
$$= 4 + 12x + 3^2x^2$$
$$= 4 + 12x + 9x^2$$

# D.2 EXPANDING PERFECT SQUARE BINOMIALS (SUBTRACTION)

Ex 59: Expand and simplify

$$(x-2)^2 = x^2 - 4x + 4$$

Answer: In the perfect squares expansion, we substitute a=x and b=2:

$$(x-2)^2 = x^2 - 2 \times x \times 2 + 2^2$$
$$= x^2 - 4x + 4$$

Ex 60: Expand and simplify

$$(3-x)^2 = \boxed{9 - 6x + x^2}$$

Answer: In the perfect squares expansion, we substitute a=3 and b=x:

$$(3-x)^2 = 3^2 - 2 \times 3 \times x + x^2$$
$$= 9 - 6x + x^2$$

Ex 61: Expand and simplify

$$(2x-1)^2 = \boxed{4x^2 - 4x + 1}$$

Answer: In the perfect squares expansion, we substitute a=2x and b=1:

$$(2x-1)^2 = (2x)^2 - 2 \times 2x \times 1 + (1)^2$$
$$= 4x^2 - 4x + 1$$

Ex 62: Expand and simplify

$$(2-3x)^2 = \boxed{4-12x+9x^2}$$

Answer: In the perfect squares expansion, we substitute a=2 and b=3x:

$$(2 - 3x)^2 = 2^2 - 2 \times 2 \times 3x + (3x)^2$$
$$= 4 - 12x + 9x^2$$

# D.3 EXPANDING PERFECT CUBE BINOMIALS (ADDITION)

Ex 63: Expand and simplify

$$(x+1)^3 = x^3 + 3x^2 + 3x + 1$$

Answer: In the perfect cubes expansion, we substitute a=x and b=1:

$$(x+1)^3 = x^3 + 3 \times x^2 \times 1 + 3 \times x \times 1^2 + 1^3$$
  
=  $x^3 + 3x^2 + 3x + 1$ 

Ex 64: Expand and simplify

$$(x+3)^3 = x^3 + 9x^2 + 27x + 27$$

Answer: In the perfect cubes expansion, we substitute a=x and b=3:

$$(x+3)^3 = x^3 + 3 \times x^2 \times 3 + 3 \times x \times 3^2 + 3^3$$
$$= x^3 + 9x^2 + 27x + 27$$

Ex 65: Expand and simplify

$$(2x+2)^3 = 8x^3 + 24x^2 + 24x + 8$$

Answer: In the perfect cubes expansion, we substitute a = 2x and b = 2:

$$(2x + 2)^3 = (2x)^3 + 3 \times (2x)^2 \times 2 + 3 \times 2x \times (2)^2 + (2)^3$$

$$= 8x^3 + 24x^2 + 24x + 8$$

Ex 66: Expand and simplify

$$(2x+3)^3 = 8x^3 + 36x^2 + 54x + 27$$

Answer: In the perfect cubes expansion, we substitute a = 2x and b = 2.

$$(2x+3)^3 = (2x)^3 + 3 \times (2x)^2 \times 3 + 3 \times 2x \times (3)^2 + (3)^3$$

$$= 8x^3 + 36x^2 + 54x + 27$$

# D.4 EXPANDING PERFECT CUBE BINOMIALS (SUBTRACTION)

Ex 67: Expand and simplify

$$(x-1)^3 = \boxed{x^3 - 3x^2 + 3x - 1}$$

Answer: In the perfect cubes expansion, we substitute a=x and b=(-1):

$$(x + (-1))^3 = x^3 + 3 \times x^2 \times (-1) + 3 \times x \times ((-1))^2 + ((-1))^3$$
$$= x^3 - 3x^2 + 3x - 1$$

Ex 68: Expand and simplify

$$(x-2)^3 = x^3 - 6x^2 + 12x - 8$$

Answer: In the perfect cubes expansion, we substitute a = x and b = (-2):

$$(x + (-2))^3 = x^3 + 3 \times x^2 \times (-2) + 3 \times x \times (-2)^2 + (-2)^3$$
  
=  $x^3 - 6x^2 + 12x - 8$ 

Ex 69: Expand and simplify

$$(3-x)^3 = 27 - 27x + 9x^2 - x^3$$

Answer: In the perfect cubes expansion, we substitute a=3 and b=(-x):

$$(3 + (-x))^3 = 3^3 + 3 \times 3^2 \times (-x) + 3 \times 3 \times ((-x))^2 + ((-x))^3$$
$$= 27 - 27x + 9x^2 - x^3$$

Ex 70: Expand and simplify

$$(2x-1)^3 = 8x^3 - 12x^2 + 6x - 1$$

Answer: In the perfect cubes expansion, we substitute a = 2x and b = (-1):

$$(2x + (-1))^3 = (2x)^3 + 3 \times (2x)^2 \times (-1) + 3 \times 2x \times ((-1))^2 + ((-1)^2 + (-1)^2$$