EXPANSION

In mathematics, we often want to transform the form of an expression to make it easier to work with, to simplify calculations, or to solve equations. **Expand** is the process of writing an expression as a sum of terms by distributing multiplication over addition.

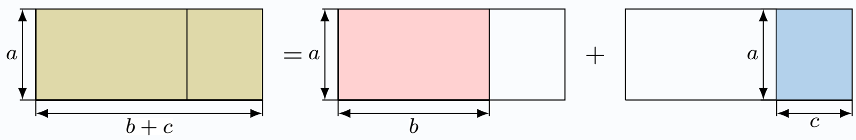
A DISTRIBUTIVE LAW 1

Proposition Distributive Law 1

Multiplication is distributive over addition and subtraction:

- **Addition:**

$$a\,(\,b+c\,) = ab + ac$$



- **Subtraction:**

$$a\,(\,b-c\,) = ab - ac$$

Ex: Expand and simplify $2(l+L)$.

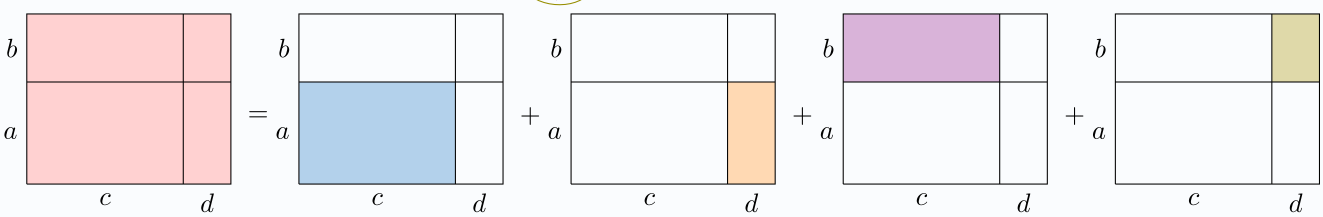
Answer:

$$\begin{aligned} 2\,(\,l+L\,) &= 2\times l + 2\times L \\ &= 2l + 2L \end{aligned}$$

B DISTRIBUTIVE LAW 2

Proposition Distributive law 2

$$(a+b)\cdot(c+d) = ac + ad + bc + bd$$



$$(a+b)(c+d) \quad = \quad ac \quad + \quad ad \quad + \quad bc \quad + \quad bd$$

Ex: Expand and simplify $(x+4)(2x+2)$

Answer:

$$\begin{aligned} (x+4)\cdot(2x+2) &= x\times 2x + x\times 2 + 4\times 2x + 4\times 2 \\ &= 2x^2 + 2x + 8x + 8 \\ &= 2x^2 + 10x + 8 \end{aligned}$$

C DIFFERENCE OF TWO SQUARES

Proposition Difference of Two Squares

$$(a-b)(a+b) = a^2 - b^2$$

Proof

$$\begin{aligned}(a-b)(b+a) &= ab + aa - bb - ba \quad \text{(expanding)}\\ &= \cancel{ab} + a^2 - b^2 - \cancel{ab}\\ &= a^2 - b^2\end{aligned}$$

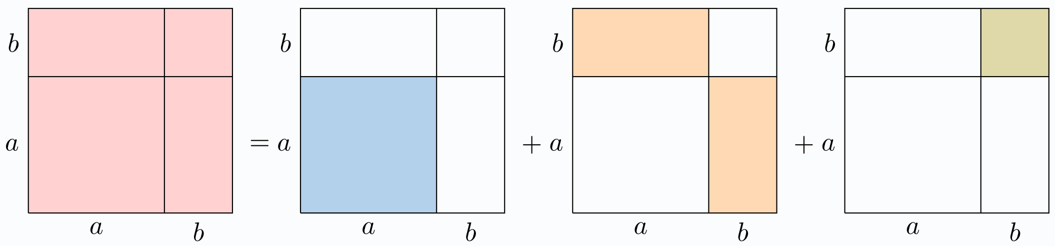
Ex: Expand and simplify: $(x-3)(x+3)$.

Answer:

$$\begin{aligned}(x-3)(x+3) &= x^2 - 3^2\\ &= x^2 - 9\end{aligned}$$

D BINOMIAL EXPANSION

Proposition Perfect Squares Expansion



$$(a+b)^2 = a^2 + 2ab + b^2$$

and

$$(a-b)^2 = a^2 - 2ab + b^2.$$

Proof

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b) && \text{(square definition)}\\ &= aa + ab + ba + bb && \text{(expanding)}\\ &= a^2 + 2ab + b^2 && \text{(combining)}\end{aligned}$$

and

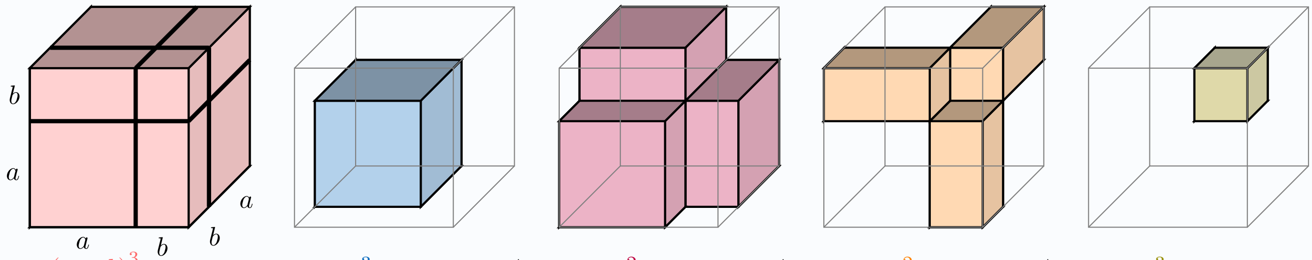
$$\begin{aligned}(a-b)^2 &= (a-b)(a-b) && \text{(square definition)}\\ &= aa - ab - ba + bb && \text{(expanding)}\\ &= a^2 - 2ab + b^2 && \text{(combining)}\end{aligned}$$

Ex: Expand and simplify $(x+2)^2$

Answer: In the perfect squares expansion, we substitute $a = x$ and $b = 2$:

$$\begin{aligned}(x+2)^2 &= x^2 + 2 \times x \times 2 + 2^2\\ &= x^2 + 4x + 4\end{aligned}$$

Proposition **Perfect Cube Expansion**



$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + a^3$$

Proof

$$\begin{aligned}
(a+b)^3 &= (a+b)(a+b)^2 && \text{(cube definition)}\\
&= (a+b)(a^2+2ab+b^2) && \text{(using square expansion)}\\
&= a(a^2+2ab+b^2) + b(a^2+2ab+b^2) && \text{(distribute)}\\
&= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3 && \text{(expand)}\\
&= a^3 + 3a^2b + 3ab^2 + b^3 && \text{(combine like terms)}
\end{aligned}$$

Ex: Expand and simplify $(x+2)^3$

Answer: In the perfect cube expansion, we substitute $a = x$ and $b = 2$:

$$\begin{aligned}
(x+2)^3 &= x^3 + 3 \times x^2 \times 2 + 3 \times x \times 2^2 + 2^3\\
&= x^3 + 6x^2 + 12x + 8
\end{aligned}$$