EXPANSION

A DISTRIBUTIVE LAW 1

A.1 EXPANDING WITH ADDITION: LEVEL 1

Ex 1: Expand and simplify:

$$5(x+3) = \boxed{5x+15}$$

Answer:

$$\begin{aligned} 5\ (x+3) &= 5 \times x + 5 \times 3 \\ &= 5x + 15 \end{aligned}$$

Ex 2: Expand and simplify:

$$2(3+x) = \boxed{6+2x}$$

Answer:

$$\begin{aligned} 2\ (3+x) &= 2 \times 3 + 2 \times x \\ &= 6 + 2x \end{aligned}$$

Ex 3: Expand and simplify:

$$3(2x+2) = \boxed{6x+6}$$

Answer:

$$\begin{aligned} 3\ (2x+2) &= 3 \times 2x + 3 \times 2 \\ &= 6x + 6 \end{aligned}$$

Ex 4: Expand and simplify:

$$2(5+3x) = \boxed{10+6x}$$

Answer:

$$\begin{aligned} 2\ (5+3x) &= 2 \times 5 + 2 \times 3x \\ &= 10 + 6x \end{aligned}$$

A.2 EXPANDING WITH ADDITION: LEVEL 2

Ex 5: Expand and simplify:

$$x(x+1) = \boxed{x^2+x}$$

Answer:

$$\begin{aligned} x\ (x+1) &= x \times x + x \times 1 \\ &= x^2 + x \end{aligned}$$

Ex 6: Expand and simplify:

$$x(2x+3) = \boxed{2x^2+3x}$$

Answer:

$$\begin{aligned} x\ (2x+3) &= x \times 2x + x \times 3 \\ &= 2x^2 + 3x \end{aligned}$$

Ex 7: Expand and simplify:

$$2x(x+2) = \boxed{2x^2+4x}$$

Answer:

$$\begin{aligned} 2x\ (x+2) &= 2x \times x + 2x \times 2 \\ &= 2x^2 + 4x \end{aligned}$$

Ex 8: Expand and simplify:

$$3x(2x+5) = \boxed{6x^2+15x}$$

Answer:

$$\begin{aligned} 3x\ (2x+5) &= 3x \times 2x + 3x \times 5 \\ &= 6x^2 + 15x \end{aligned}$$

A.3 EXPANDING WITH ADDITION: LEVEL 3

Ex 9: Expand and simplify:

$$2(x+1) + x = \boxed{3x+2}$$

Answer:

$$\begin{aligned} 2(x+1) + x &= 2 \times x + 2 \times 1 + x && \text{(expanding)} \\ &= 2x + 2 + x \\ &= (2+1)x + 2 && \text{(combining)} \\ &= 3x + 2 && \text{(simplifying)} \end{aligned}$$

Ex 10: Expand and simplify:

$$2(2x+3) - 3x = \boxed{x+6}$$

Answer:

$$\begin{aligned} 2(2x+3) - 3x &= 2 \times 2x + 2 \times 3 - 3x && \text{(expanding)} \\ &= 4x + 6 - 3x \\ &= (4-3)x + 6 && \text{(combining)} \\ &= x + 6 && \text{(simplifying)} \end{aligned}$$

Ex 11: Expand and simplify:

$$x(x+2) - x^2 = \boxed{2x}$$

Answer:

$$\begin{aligned} x(x+2) - x^2 &= x \times x + x \times 2 - x^2 && \text{(expanding)} \\ &= x^2 + 2x - x^2 \\ &= 2x && \text{(combining)} \end{aligned}$$

Ex 12: Expand and simplify:

$$2x(3x+2) - 8x = \boxed{6x^2-4x}$$

Answer:

$$\begin{aligned} 2x(3x+2) - 8x &= 2x \times 3x + 2x \times 2 - 8x && \text{(expanding)} \\ &= 6x^2 + 4x - 8x \\ &= 6x^2 + (4-8)x && \text{(combining)} \\ &= 6x^2 - 4x && \text{(simplifying)} \end{aligned}$$

A.4 EXPANDING WITH SUBTRACTION: LEVEL 1

Ex 13: Expand and simplify:

$$2(x-2) = \boxed{2x-4}$$

Answer:

$$\begin{aligned} 2\,(x-2) &= 2\times x - 2\times 2 \\ &= 2x-4 \end{aligned}$$

Ex 14: Expand and simplify:

$$3(5x-6) = \boxed{15x-18}$$

Answer:

$$\begin{aligned} 3\,(5x-6) &= 3\times 5x - 3\times 6 \\ &= 15x-18 \end{aligned}$$

Ex 15: Expand and simplify:

$$2(3-x) = \boxed{6-2x}$$

Answer:

$$\begin{aligned} 2\,(3-x) &= 2\times 3 - 2\times x \\ &= 6-2x \end{aligned}$$

Ex 16: Expand and simplify:

$$4(3-5x) = \boxed{12-20x}$$

Answer:

$$\begin{aligned} 4\,(3-5x) &= 4\times 3 - 4\times 5x \\ &= 12-20x \end{aligned}$$

A.5 EXPANDING WITH SUBTRACTION: LEVEL 2

Ex 17: Expand and simplify:

$$x(x-1) = \boxed{x^2-x}$$

Answer:

$$\begin{aligned} x\,(x-1) &= x\times x - x\times 1 \\ &= x^2-x \end{aligned}$$

Ex 18: Expand and simplify:

$$x(2x-3) = \boxed{2x^2-3x}$$

Answer:

$$\begin{aligned} x\,(2x-3) &= x\times 2x - x\times 3 \\ &= 2x^2-3x \end{aligned}$$

Ex 19: Expand and simplify:

$$2x(x-2) = \boxed{2x^2-4x}$$

Answer:

$$\begin{aligned} 2x\,(x-2) &= 2x\times x - 2x\times 2 \\ &= 2x^2-4x \end{aligned}$$

Ex 20: Expand and simplify:

$$3x(2x-5) = \boxed{6x^2-15x}$$

Answer:

$$\begin{aligned} 3x\,(2x-5) &= 3x\times 2x - 3x\times 5 \\ &= 6x^2-15x \end{aligned}$$

A.6 EXPANDING WITH SUBTRACTION: LEVEL 3

Ex 21: Expand and simplify

$$2(x-2)+4 = \boxed{2x}$$

Answer:

$$\begin{aligned} 2(x-2)+4 &= 2\times x - 2\times 2 + 4 && \text{(expanding)} \\ &= 2x-4+4 \\ &= 2x+0 && \text{(simplifying)} \\ &= 2x \end{aligned}$$

Ex 22: Expand and simplify

$$4(x-3)-5x = \boxed{-x-12}$$

Answer:

$$\begin{aligned} 4(x-3)-5x &= 4\times x - 4\times 3 - 5x && \text{(expanding)} \\ &= 4x-12-5x \\ &= (4-5)x-12 && \text{(combining like terms)} \\ &= -x-12 && \text{(simplifying)} \end{aligned}$$

Ex 23: Expand and simplify

$$x(x-2)+6 = \boxed{x^2-2x+6}$$

Answer:

$$\begin{aligned} x(x-2)+6 &= x\times x - x\times 2 + 6 && \text{(expanding)} \\ &= x^2-2x+6 \end{aligned}$$

Ex 24: Expand and simplify

$$2(x-2)+3x-10 = \boxed{5x-14}$$

Answer:

$$\begin{aligned} 2(x-2)+3x-10 &= 2\times x - 2\times 2 + 3x - 10 && \text{(expanding)} \\ &= 2x-4+3x-10 \\ &= (2+3)x-4-10 && \text{(combining like terms)} \\ &= 5x-14 && \text{(simplifying)} \end{aligned}$$

B DISTRIBUTIVE LAW 2

B.1 EXPANDING WITH ADDITION

Ex 25: Expand and simplify

$$(x+4)(2x+2) = \boxed{2x^2+10x+8}$$

Answer:

$$\begin{aligned}(x+4)\cdot(2x+2) &= x\times 2x + x\times 2 + 4\times 2x + 4\times 2\\ &= 2x^2+2x+8x+8\\ &= 2x^2+10x+8\end{aligned}$$

Ex 26: Expand and simplify

$$(x+2)(x+1) = \boxed{x^2+3x+2}$$

Answer:

$$\begin{aligned}(x+2)\cdot(x+1) &= x\times x + x\times 1 + 2\times x + 2\times 1\\ &= x^2+x+2x+2\\ &= x^2+3x+2\end{aligned}$$

Ex 27: Expand and simplify

$$(x+3)(x+4) = \boxed{x^2+7x+12}$$

Answer:

$$\begin{aligned}(x+3)\cdot(x+4) &= x\times x + x\times 4 + 3\times x + 3\times 4\\ &= x^2+4x+3x+12\\ &= x^2+7x+12\end{aligned}$$

Ex 28: Expand and simplify

$$(2x+1)(3x+2) = \boxed{6x^2+7x+2}$$

Answer:

$$\begin{aligned}(2x+1)\cdot(3x+2) &= 2x\times 3x + 2x\times 2 + 1\times 3x + 1\times 2\\ &= 6x^2+4x+3x+2\\ &= 6x^2+7x+2\end{aligned}$$

Ex 29: Expand and simplify

$$(2x+1)(3+x^2) = \boxed{2x^3+x^2+6x+3}$$

Answer:

$$\begin{aligned}(2x+1)\cdot(3+x^2) &= 2x\times 3 + 2x\times x^2 + 1\times 3 + 1\times x^2\\ &= 6x+2x^3+3+x^2\\ &= 2x^3+x^2+6x+3\end{aligned}$$

Ex 30: Expand and simplify

$$(x+1)^2 = \boxed{x^2+2x+1}$$

Answer:

$$\begin{aligned}(x+1)^2 &= (x+1)(x+1) && \text{(square definition)}\\ &= x\times x + x\times 1 + 1\times x + 1\times 1 && \text{(expanding)}\\ &= x^2+2x+1 && \text{(combining)}\end{aligned}$$

B.2 EXPANDING WITH SUBTRACTION

Ex 31: Expand and simplify

$$(x+2)(x-1) = \boxed{x^2+x-2}$$

Answer:

$$\begin{aligned}(x+2)\cdot(x-1) &= x\times x + x\times(-1) + 2\times x + 2\times(-1)\\ &= x^2-x+2x-2\\ &= x^2+x-2\end{aligned}$$

Ex 32: Expand and simplify

$$(x-1)(x-2) = \boxed{x^2-3x+2}$$

Answer:

$$\begin{aligned}(x-1)\cdot(x-2) &= x\times x + x\times(-2) + (-1)\times x + (-1)\times(-2)\\ &= x^2-2x-x+2\\ &= x^2-3x+2\end{aligned}$$

Ex 33: Expand and simplify

$$(x+3)(x-2) = \boxed{x^2+x-6}$$

Answer:

$$\begin{aligned}(x+3)\cdot(x-2) &= x\times x + x\times(-2) + 3\times x + 3\times(-2)\\ &= x^2-2x+3x-6\\ &= x^2+x-6\end{aligned}$$

Ex 34: Expand and simplify

$$(2x+1)(1-2x) = \boxed{1-4x^2}$$

Answer:

$$\begin{aligned}(2x+1)\cdot(1-2x) &= 2x\times 1 + 2x\times(-2x) + 1\times 1 + 1\times(-2x)\\ &= 2x-4x^2+1-2x\\ &= 1-4x^2\end{aligned}$$

Ex 35: Expand and simplify

$$(-1+2x)(1-x) = \boxed{-2x^2+3x-1}$$

Answer:

$$\begin{aligned}(-1+2x)\cdot(1-x) &= (-1)\times 1 + (-1)\times(-x) + 2x\times 1 + 2x\times(-x)\\ &= -1+x+2x-2x^2\\ &= -2x^2+3x-1\end{aligned}$$

C DIFFERENCE OF TWO SQUARES

C.1 EXPANDING: LEVEL 1

Ex 36: Expand and simplify

$$(x+1)(x-1) = \boxed{x^2-1}$$

Answer:

$$\begin{aligned}(x+1)(x-1) &= x^2 - 1^2\\ &= x^2 - 1\end{aligned}$$

Ex 37: Expand and simplify

$$(x-3)(x+3) = \boxed{x^2-9}$$

Answer:

$$\begin{aligned}(x-3)(x+3) &= x^2 - 3^2\\ &= x^2 - 9\end{aligned}$$

Ex 38: Expand and simplify

$$(4-x)(4+x) = \boxed{16-x^2}$$

Answer:

$$\begin{aligned}(4-x)(4+x) &= 4^2 - x^2\\ &= 16 - x^2\end{aligned}$$

Ex 39: Expand and simplify

$$(5+x)(5-x) = \boxed{25-x^2}$$

Answer:

$$\begin{aligned}(5+x)(5-x) &= 5^2 - x^2\\ &= 25 - x^2\end{aligned}$$

C.2 EXPANDING: LEVEL 2

Ex 40: Expand and simplify

$$(2x-4)(2x+4) = \boxed{4x^2-16}$$

Answer:

$$\begin{aligned}(2x-4)(2x+4) &= (2x)^2 - 4^2\\ &= 2^2x^2 - 4^2\\ &= 4x^2 - 16\end{aligned}$$

Ex 41: Expand and simplify

$$(x+\sqrt{2})(x-\sqrt{2}) = \boxed{x^2-2}$$

Answer:

$$\begin{aligned}(x+\sqrt{2})(x-\sqrt{2}) &= x^2 - (\sqrt{2})^2\\ &= x^2 - 2\end{aligned}$$

Ex 42: Expand and simplify

$$\left(\frac{1}{2}-x\right)\left(\frac{1}{2}+x\right) = \boxed{\frac{1}{4}-x^2}$$

Answer:

$$\begin{aligned}\left(\frac{1}{2}-x\right)\left(\frac{1}{2}+x\right) &= \left(\frac{1}{2}\right)^2 - x^2\\ &= \frac{1^2}{2^2} - x^2\\ &= \frac{1}{4} - x^2\end{aligned}$$

Ex 43: Expand and simplify

$$\left(\tfrac{x}{2}-1\right)\left(\tfrac{x}{2}+1\right) = \boxed{\frac{x^2}{4}-1}$$

Answer:

$$\begin{aligned}\left(\frac{x}{2}-1\right)\left(\frac{x}{2}+1\right) &= \left(\frac{x}{2}\right)^2 - 1^2\\ &= \frac{x^2}{2^2} - 1^2\\ &= \frac{x^2}{4} - 1\end{aligned}$$

D PERFECT SQUARES EXPANSION

D.1 EXPANDING WITH ADDITION

Ex 44: Expand and simplify

$$(x+2)^2 = \boxed{x^2+4x+4}$$

Answer: In the perfect squares expansion, we substitute $a = x$ and $b = 2$:

$$\begin{aligned}(x+2)^2 &= x^2 + 2 \times x \times 2 + 2^2\\ &= x^2 + 4x + 4\end{aligned}$$

Ex 45: Expand and simplify

$$(3+x)^2 = \boxed{9+6x+x^2}$$

Answer: In the perfect squares expansion, we substitute $a = 3$ and $b = x$:

$$\begin{aligned}(3+x)^2 &= 3^2 + 2 \times 3 \times x + x^2\\ &= 9 + 6x + x^2\end{aligned}$$

Ex 46: Expand and simplify

$$(2x+1)^2 = \boxed{4x^2+4x+1}$$

Answer: In the perfect squares expansion, we substitute $a = 2x$ and $b = 1$:

$$\begin{aligned}(2x+1)^2 &= (2x)^2 + 2 \times 2x \times 1 + 1^2\\ &= 2^2x^2 + 4x + 1\\ &= 4x^2 + 4x + 1\end{aligned}$$

Ex 47: Expand and simplify

$$(2+3x)^2 = \boxed{4+12x+9x^2}$$

Answer: In the perfect squares expansion, we substitute $a = 2$ and $b = 3x$:

$$\begin{aligned}(2+3x)^2 &= 2^2 + 2 \times 2 \times 3x + (3x)^2\\ &= 4 + 12x + 3^2x^2\\ &= 4 + 12x + 9x^2\end{aligned}$$

D.2 EXPANDING WITH SUBTRACTION

Ex 48: Expand and simplify

$$(x-2)^2 = \boxed{x^2-4x+4}$$

Answer: In the perfect squares expansion, we substitute $a = x$ and $b = 2$:

$$\begin{aligned}(x-2)^2 &= x^2 - 2 \times x \times 2 + 2^2 \\ &= x^2 - 4x + 4\end{aligned}$$

Ex 49: Expand and simplify

$$(3-x)^2 = \boxed{9-6x+x^2}$$

Answer: In the perfect squares expansion, we substitute $a = 3$ and $b = x$:

$$\begin{aligned}(3-x)^2 &= 3^2 - 2 \times 3 \times x + x^2 \\ &= 9 - 6x + x^2\end{aligned}$$

Ex 50: Expand and simplify

$$(2x-1)^2 = \boxed{4x^2-4x+1}$$

Answer: In the perfect squares expansion, we substitute $a = 2x$ and $b = 1$:

$$\begin{aligned}(2x-1)^2 &= (2x)^2 - 2 \times 2x \times 1 + 1^2 \\ &= 4x^2 - 4x + 1\end{aligned}$$

Ex 51: Expand and simplify

$$(2-3x)^2 = \boxed{4-12x+9x^2}$$

Answer: In the perfect squares expansion, we substitute $a = 2$ and $b = 3x$:

$$\begin{aligned}(2-3x)^2 &= 2^2 - 2 \times 2 \times 3x + (3x)^2 \\ &= 4 - 12x + 9x^2\end{aligned}$$