

EXPANSION

In mathematics, we often want to transform the form of an expression to make it easier to work with, to simplify calculations, or to solve equations. **Expanding** an expression means rewriting a multiplication as a sum of terms.

A EXPANSION

Definition Expansion

Expansion is the process of writing an expression as a sum of terms by distributing multiplication over addition.

Ex 1: Expand and simplify $5(x + 3)$.

Answer:

$$5(x+3) = 5 \times x + 5 \times 3 = 5x + 15$$

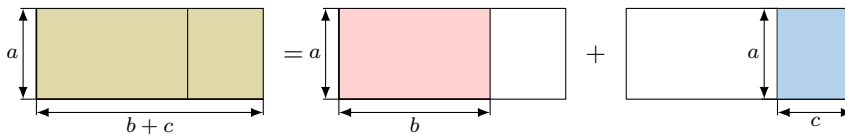
B DISTRIBUTIVE LAW 1

Proposition Distributive Law 1

Multiplication is distributive over addition and subtraction:

- Addition:**

$$a(b+c) = ab + ac$$



- Subtraction:**

$$a(b-c) = ab - ac$$

Ex: Expand and simplify $2(l + L)$.

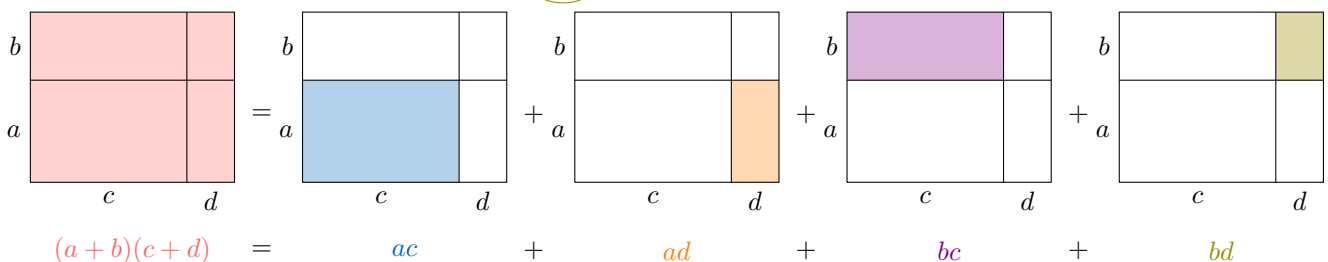
Answer:

$$2(l+L) = 2 \times l + 2 \times L = 2l + 2L$$

C DISTRIBUTIVE LAW 2

Proposition Distributive law 2

$$(a+b)(c+d) = ac + ad + bc + bd$$



Ex: Expand and simplify $(x + 4)(2x + 2)$

Answer:

$$\begin{aligned}
 (x+4) \cdot (2x+2) &= x \times 2x + x \times 2 + 4 \times 2x + 4 \times 2 \\
 &= 2x^2 + 2x + 8x + 8 \\
 &= 2x^2 + 10x + 8
 \end{aligned}$$

D DIFFERENCE OF TWO SQUARES

Proposition Difference of Two Squares

$$(a - b)(a + b) = a^2 - b^2.$$

Proof

$$\begin{aligned}
 (a - b)(b + a) &= ab + aa - bb - ba \quad (\text{expanding}) \\
 &= \cancel{ab} + a^2 - b^2 - \cancel{ba} \\
 &= a^2 - b^2
 \end{aligned}$$

Ex: Expand and simplify: $(x - 3)(x + 3)$.

Answer:

$$\begin{aligned}
 (x - 3)(x + 3) &= x^2 - 3^2 \\
 &= x^2 - 9
 \end{aligned}$$

E PERFECT SQUARES EXPANSION

Proposition Perfect Squares Expansion

$$\begin{aligned}
 (a + b)^2 &= a^2 + 2ab + b^2
 \end{aligned}$$

and

$$(a - b)^2 = a^2 - 2ab + b^2.$$

Proof

$$\begin{aligned}
 (a + b)^2 &= (a + b)(a + b) \quad (\text{square definition}) \\
 &= aa + ab + ba + bb \quad (\text{expanding}) \\
 &= a^2 + 2ab + b^2 \quad (\text{combining})
 \end{aligned}$$

and

$$\begin{aligned}
 (a - b)^2 &= (a - b)(a - b) \quad (\text{square definition}) \\
 &= aa - ab - ba + bb \quad (\text{expanding}) \\
 &= a^2 - 2ab + b^2 \quad (\text{combining})
 \end{aligned}$$

Ex: Expand and simplify $(x + 2)^2$

Answer: In the perfect squares expansion, we substitute $a = x$ and $b = 2$:

$$\begin{aligned}
 (x + 2)^2 &= x^2 + 2 \times x \times 2 + 2^2 \\
 &= x^2 + 4x + 4
 \end{aligned}$$