EXPONENT

A DEFINITIONS

A.1 WRITING REPEATED MULTIPLICATION IN EXPONENT FORM

Ex 1: Write in exponent form:

$$2 \times 2 \times 2 = \boxed{2^3}$$

Answer: $\overbrace{2 \times 2 \times 2}^{3 \text{ factors}} = 2^3$

Ex 2: Write in exponent form:

$$3 \times 3 \times 3 \times 3 = \boxed{3^4}$$

Answer: $\overbrace{3 \times 3 \times 3 \times 3}^{4 \text{ factors}} = 3^4$

Ex 3: Write in exponent form:

$$5 \times 5 = \boxed{5^2}$$

Answer: $\overbrace{5 \times 5}^{2 \text{ factors}} = 5^2$

Ex 4: Write in exponent form:

$$7 \times 7 \times 7 = \boxed{7^3}$$

Answer: $\overbrace{7 \times 7 \times 7}^{3 \text{ factors}} = 7^3$

Ex 5: Write in exponent form:

$$10 \times 10 \times 10 \times 10 \times 10 = \boxed{10^5}$$

Answer: $\overbrace{10 \times 10 \times 10 \times 10 \times 10}^{5 \text{ factors}} = 10^5$

A.2 WRITING IN EXPONENT FORM FROM VERBAL EXPRESSIONS

Ex 6: Write in exponent form:

$$2 \text{ raised to the power } 3 = \boxed{2^3}$$

Answer: 2 raised to the power of $3 = 2^3$

Ex 7: Write in exponent form:

$$5 \text{ raised to the power } 2 = \boxed{5^2}$$

Answer: 5 raised to the power of $2 = 5^2$

Ex 8: Write in exponent form:

$$7 \text{ raised to the power } 4 = \boxed{7^4}$$

Answer: 7 raised to the power of $4 = 7^4$

Ex 9: Write in exponent form:

$$10 \text{ raised to the power } 5 = \boxed{10^5}$$

Answer: 10 raised to the power of $5 = 10^5$

A.3 CALCULATING POWERS

Ex 10: Evaluate the power:

$$2^3 = \boxed{8}$$

Answer:

$$\begin{aligned} 2^3 &= 2 \times 2 \times 2 \\ &= 8 \end{aligned}$$

Ex 11: Evaluate the power:

$$5^2 = \boxed{25}$$

Answer:

$$\begin{aligned} 5^2 &= 5 \times 5 \\ &= 25 \end{aligned}$$

Ex 12: Evaluate the power:

$$3^4 = \boxed{81}$$

Answer:

$$\begin{aligned} 3^4 &= 3 \times 3 \times 3 \times 3 \\ &= 81 \end{aligned}$$

Ex 13: Evaluate the power:

$$10^3 = \boxed{1000}$$

Answer:

$$\begin{aligned} 10^3 &= 10 \times 10 \times 10 \\ &= 1000 \end{aligned}$$

A.4 EXPRESSING NUMBERS IN EXPONENT FORM

Ex 14: Write in exponent form:

$$8 = \boxed{2^3}$$

Answer:

$$\begin{aligned} 8 &= 2 \times 2 \times 2 \\ &= 2^3 \end{aligned}$$

Ex 15: Write in exponent form:

$$27 = \boxed{3^3}$$

Answer:

$$\begin{aligned} 27 &= 3 \times 3 \times 3 \\ &= 3^3 \end{aligned}$$

Ex 16: Write in exponent form:

$$16 = \boxed{2^4}$$

Answer:

$$\begin{aligned} 16 &= 2 \times 2 \times 2 \times 2 \\ &= 2^4 \end{aligned}$$

Ex 17: Write in exponent form:

$$100 = \boxed{10^2}$$

Answer:

$$\begin{aligned} 100 &= 10 \times 10 \\ &= 10^2 \end{aligned}$$

A.5 INTERPRETING POWERS

MCQ 18: Determine if the following statement is True or False:

$$2^3 = 2 + 2 + 2$$

☐ True

☒ False

Answer:

- The expression 2^3 represents $2 \times 2 \times 2$, not $2 + 2 + 2$.
- Therefore, the statement $2^3 = 2 + 2 + 2$ is **False**.

MCQ 19: Determine if the following statement is True or False:

$$3^2 = 2 \times 2 \times 2$$

☐ True

☒ False

Answer:

- The expression 3^2 represents 3×3, not $2 \times 2 \times 2$.
- Therefore, the statement $3^2 = 2 \times 2 \times 2$ is **False**.

MCQ 20: Determine if the following statement is True or False:

$$4^3 = 4 \times 4 \times 4$$

☒ True

☐ False

Answer:

- The expression 4^3 represents $4 \times 4 \times 4$.
- Therefore, the statement $4^3 = 4 \times 4 \times 4$ is **True**.

MCQ 21: Determine if the following statement is True or False:

$$3 \times 4 = 4 + 4 + 4$$

☒ True

☐ False

Answer:

- The expression 3×4 represents 3 groups of 4, which is $4 + 4 + 4$.
- Therefore, the statement $3 \times 4 = 4 + 4 + 4$ is **True**.

MCQ 22: Determine if the following statement is True or False:

$$3^2 = 2 \times 2 \times 2$$

☐ True

☒ False

Answer:

- The expression 3^2 means 3×3, which equals 9.
- The expression $2 \times 2 \times 2$ means multiplying 2 three times, which equals 8.
- Therefore, the statement $3^2 = 2 \times 2 \times 2$ is **False**.

A.6 EVALUATING EXPRESSIONS WITH POWERS

Ex 23: Evaluate the expression:

$$2^3 \times 3^2 = \boxed{72}$$

Answer:

$$\begin{aligned} 2^3 \times 3^2 &= (2 \times 2 \times 2) \times (3 \times 3) \\ &= 8 \times 9 \\ &= 72 \end{aligned}$$

Ex 24: Evaluate the expression:

$$3^2 \times 10^2 = \boxed{900}$$

Answer:

$$\begin{aligned} 3^2 \times 10^2 &= (3 \times 3) \times (10 \times 10) \\ &= 9 \times 100 \\ &= 900 \end{aligned}$$

Ex 25: Evaluate the expression:

$$6 \times 10^3 = \boxed{6000}$$

Answer:

$$\begin{aligned} 6 \times 10^3 &= 6 \times (10 \times 10 \times 10) \\ &= 6 \times 1000 \\ &= 6\,000 \end{aligned}$$

Ex 26: Evaluate the expression:

$$2.5 \times 10^2 = \boxed{250}$$

Answer:

$$\begin{aligned} 2.5 \times 10^2 &= 2.5 \times (10 \times 10) \\ &= 2.5 \times 100 \\ &= 250 \end{aligned}$$

A.7 CHECKING EQUALITY BETWEEN PRODUCTS AND POWERS

MCQ 27: Determine if the following statement is True or False:

$$2 \times 2 \times 3 \times 3 = 2^4$$

☐ True

☒ False

Answer:

- The expression $2 \times 2 \times 3 \times 3$ is equal to $2^2 \times 3^2 = 4 \times 9 = 36$.
- The expression $2^4 = 2 \times 2 \times 2 \times 2 = 16$.
- Therefore, the statement $2 \times 2 \times 3 \times 3 = 2^4$ is **False**.

MCQ 28: Determine if the following statement is True or False:

$$2 \times 2 \times 2 = 3^2$$

☐ True

☒ False

Answer:

- $2 \times 2 \times 2 = 8$
- $3^2 = 3 \times 3 = 9$
- Therefore, the statement $2 \times 2 \times 2 = 3^2$ is **False**.

MCQ 29: Determine if the following statement is True or False:

$$2 \times 3 \times 2 \times 3 = 2^2 \times 3^2$$

☒ True

☐ False

Answer:

$$\begin{aligned} 2 \times 3 \times 2 \times 3 &= (2 \times 2) \times (3 \times 3) \\ &= 2^2 \times 3^2 \end{aligned}$$

Therefore, the statement $2 \times 3 \times 2 \times 3 = 2^2 \times 3^2$ is **True**.

MCQ 30: Determine if the following statement is True or False:

$$5 \times 5 \times 5 \times 4 = 5^3 \times 2^2$$

☒ True

☐ False

Answer:

$$\begin{aligned} 5 \times 5 \times 5 \times 4 &= (5 \times 5 \times 5) \times 4 \\ &= 5^3 \times 4 \\ &= 5^3 \times (2 \times 2) \\ &= 5^3 \times 2^2 \end{aligned}$$

Therefore, the statement $5 \times 5 \times 5 \times 4 = 5^3 \times 2^2$ is **True**.

A.8 WRITING REPEATED MULTIPLICATION OF AN ALGEBRAIC EXPRESSION IN EXPONENT FORM

Ex 31: Write in exponent form:

$$x \times x \times x = \boxed{x^3}$$

Answer: $\overbrace{x \times x \times x}^{3 \text{ factors}} = x^3$

Ex 32: Write in exponent form:

$$x \times x = \boxed{x^2}$$

Answer: $\overbrace{x \times x}^{2 \text{ factors}} = x^2$

MCQ 33: Which expressions are equal to x?

Choose all answers that apply:

☐ x^2

☒ x^1

☐ 1

Answer:

- x^2 means $x \times x$, which is not equal to x (unless $x = 1$ or $x = 0$).
- $x^1 = x$: **(correct)**
- 1 is only equal to x if $x = 1$, but in general, $x \neq 1$.

Ex 34: Write in exponent form:

$$x \times x \times x \times x = \boxed{x^4}$$

Answer: $\overbrace{x \times x \times x \times x}^{4 \text{ factors}} = x^4$

A.9 WRITING ALGEBRAIC EXPRESSIONS IN EXPONENT FORM FROM VERBAL DESCRIPTIONS

Ex 35: Write in exponent form:

$$x \text{ squared} = \boxed{x^2}$$

Answer: x squared $= x^2$

Ex 36: Write in exponent form:

$$x \text{ to the power of } 4 = \boxed{x^4}$$

Answer: x to the power of $4 = x^4$

Ex 37: Write in exponent form:

$$x \text{ cubed} = \boxed{x^3}$$

Answer: x cubed $= x^3$

Ex 38: Write in exponent form:

$$x \text{ to the power of } 5 = \boxed{x^5}$$

Answer: x to the power of $5 = x^5$

B EXPONENT LAW

B.1 SIMPLIFYING PRODUCTS OF POWERS

Ex 39: Simplify:

$$7^3 \times 7^2 = \boxed{7^5}$$

Answer:

$$\begin{aligned} 7^3 \times 7^2 &= \overbrace{7 \times 7 \times 7}^{3 \text{ factors}} \times \overbrace{7 \times 7}^{2 \text{ factors}} \\ &= \overbrace{7 \times 7 \times 7 \times 7 \times 7}^{3+2 \text{ factors}} \\ &= 7^{3+2} \\ &= 7^5 \end{aligned}$$

Ex 40: Simplify:

$$2^4 \times 2^3 = \boxed{2^7}$$

Answer:

$$\begin{aligned} 2^4 \times 2^3 &= \overbrace{2 \times 2 \times 2 \times 2}^{4 \text{ factors}} \times \overbrace{2 \times 2 \times 2}^{3 \text{ factors}} \\ &= \overbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}^{4+3 \text{ factors}} \\ &= 2^{4+3} \\ &= 2^7 \end{aligned}$$

Ex 41: Simplify:

$$3^5 \times 3^2 = \boxed{3^7}$$

Answer:

$$\begin{aligned} 3^5 \times 3^2 &= \overbrace{3 \times 3 \times 3 \times 3 \times 3}^{5 \text{ factors}} \times \overbrace{3 \times 3}^{2 \text{ factors}} \\ &= \overbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}^{5+2 \text{ factors}} \\ &= 3^{5+2} \\ &= 3^7 \end{aligned}$$

Ex 42: Simplify:

$$10^6 \times 10^2 = \boxed{10^8}$$

Answer:

$$\begin{aligned} 10^6 \times 10^2 &= \overbrace{10 \times \cdots \times 10}^{6 \text{ factors}} \times \overbrace{10 \times 10}^{2 \text{ factors}} \\ &= \overbrace{10 \times \cdots \times 10}^{6+2 \text{ factors}} \\ &= 10^{6+2} \\ &= 10^8 \end{aligned}$$

Ex 43: Simplify:

$$2^3 \times 2 = \boxed{2^4}$$

Answer:

$$\begin{aligned} 2^3 \times 2 &= 2^3 \times 2^1 \\ &= \overbrace{2 \times 2 \times 2}^{3 \text{ factors}} \times \overbrace{2}^{1 \text{ factor}} \\ &= \overbrace{2 \times 2 \times 2 \times 2}^{3+1 \text{ factors}} \\ &= 2^{3+1} \\ &= 2^4 \end{aligned}$$

Ex 44: Simplify:

$$3 \times 3^4 = \boxed{3^5}$$

Answer:

$$\begin{aligned} 3 \times 3^4 &= 3^1 \times 3^4 \\ &= \overbrace{3}^{1 \text{ factor}} \times \overbrace{3 \times 3 \times 3 \times 3}^{4 \text{ factors}} \\ &= \overbrace{3 \times 3 \times 3 \times 3 \times 3}^{1+4 \text{ factors}} \\ &= 3^{1+4} \\ &= 3^5 \end{aligned}$$

B.2 SIMPLIFYING PRODUCTS OF ALGEBRAIC POWERS

Ex 45: Simplify:

$$x^2 \times x^3 = \boxed{x^5}$$

Answer:

$$\begin{aligned} x^2 \times x^3 &= \overbrace{x \times x}^{2 \text{ factors}} \times \overbrace{x \times x \times x}^{3 \text{ factors}} \\ &= \overbrace{x \times x \times x \times x \times x}^{2+3 \text{ factors}} \\ &= x^{2+3} \\ &= x^5 \end{aligned}$$

Ex 46: Simplify:

$$x \times x^2 = \boxed{x^3}$$

Answer:

$$\begin{aligned} x \times x^2 &= x^1 \times x^2 \\ &= \overbrace{x}^{1 \text{ factor}} \times \overbrace{x \times x}^{2 \text{ factors}} \\ &= \overbrace{x \times x \times x}^{1+2 \text{ factors}} \\ &= x^{1+2} \\ &= x^3 \end{aligned}$$

Ex 47: Simplify:

$$x^2 \times x^2 = \boxed{x^4}$$

Answer:

$$\begin{aligned} x^2 \times x^2 &= \overbrace{x \times x}^{2 \text{ factors}} \times \overbrace{x \times x}^{2 \text{ factors}} \\ &= \overbrace{x \times x \times x \times x}^{2+2 \text{ factors}} \\ &= x^{2+2} \\ &= x^4 \end{aligned}$$

Ex 48: Simplify:

$$x^3 \times x = \boxed{x^4}$$

Answer:

$$\begin{aligned} x^3 \times x &= x^3 \times x^1 \\ &= \overbrace{x \times x \times x}^{3 \text{ factors}} \times \overbrace{x}^{1 \text{ factor}} \\ &= \overbrace{x \times x \times x \times x}^{3+1 \text{ factors}} \\ &= x^{3+1} \\ &= x^4 \end{aligned}$$

B.3 IDENTIFYING CORRECT EXPONENTIAL EXPRESSIONS

MCQ 49: Which expressions are equal to $2^2 + 2^1$?

Choose all answers that apply:

- ☐ 6
- ☐ 2^3
- ☒ 4^3

Answer:

- $2^3 = 8$ which is not equal to $2^2 + 2^1 = 6$.

- $4^3 = 64$ which is not equal to $2^2 + 2^1 = 6$.
- $2^2 + 2^1 = 4 + 2 = 6$:**(correct)**

MCQ 50: Which expressions are equal to $5^2 \times 5^1$?
Choose all answers that apply:

☐ 25

☒ 125

☒ 5^3

Answer:

- $5^2 \times 5^1 = 5^{2+1} = 5^3 = 125$: **(correct)**
- 25 is just 5^2, not $5^2 \times 5^1$.
- 5^3 is the same as $5^2 \times 5^1$ by the law of exponents : **(correct)**

MCQ 51: Which expressions are equal to $3^2 + 3^1$?
Choose all answers that apply:

☒ 12

☐ 3^3

☐ 9^3

Answer:

- $3^2 + 3^1 = 9 + 3 = 12$: **(correct)**
- $3^3 = 27$ which is not equal to $3^2 + 3^1 = 12$.
- $9^3 = 729$ which is not equal to $3^2 + 3^1 = 12$.
- "None of the above" is incorrect because 12 is a correct answer.

MCQ 52: Which expressions are equal to $4^3 \times 4^2$?
Choose all answers that apply:

☒ 4^5

☐ 64

☒ 1024

Answer:

- $4^3 \times 4^2 = 4^{3+2} = 4^5$: **(correct)**
- 64 is 4^3, not $4^3 \times 4^2$.
- 1024 is 4^5, and thus equal to $4^3 \times 4^2$: **(correct)**

C ORDER OF OPERATIONS

C.1 EVALUATING EXPRESSIONS WITH EXPONENTS IN 2 STEPS

Ex 53: Evaluate this expression:

$$2 \times 5^2 = \boxed{50}$$

Answer:

$$\begin{aligned} 2 \times 5^2 &= 2 \times 5^2 && \text{(exponent: } 5^2 = 25\text{)} \\ &= 2 \times 25 && \text{(multiplication: } 2 \times 25 = 50\text{)} \\ &= 50 \end{aligned}$$

Ex 54: Evaluate this expression:

$$2^3 - 1 = \boxed{7}$$

Answer:

$$\begin{aligned} 2^3 - 1 &= 2^3 - 1 && \text{(exponent: } 2^3 = 8\text{)} \\ &= 8 - 1 && \text{(subtraction: } 8 - 1 = 7\text{)} \\ &= 7 \end{aligned}$$

Ex 55: Evaluate this expression:

$$(2 + 1)^2 = \boxed{9}$$

Answer:

$$\begin{aligned} (2 + 1)^2 &= (2 + 1)^2 && \text{(parentheses: } 2 + 1 = 3\text{)} \\ &= 3^2 && \text{(exponent: } 3^2 = 9\text{)} \\ &= 9 \end{aligned}$$

Ex 56: Evaluate this expression:

$$2^3 \div 4 = \boxed{2}$$

Answer:

$$\begin{aligned} 2^3 \div 4 &= 2^3 \div 4 && \text{(exponent: } 2^3 = 8\text{)} \\ &= 8 \div 4 && \text{(division: } 8 \div 4 = 2\text{)} \\ &= 2 \end{aligned}$$

Ex 57: Evaluate this expression:

$$(5 - 2)^2 = \boxed{9}$$

Answer:

$$\begin{aligned} (5 - 2)^2 &= (5 - 2)^2 && \text{(parentheses: } 5 - 2 = 3\text{)} \\ &= 3^2 && \text{(exponent: } 3^2 = 9\text{)} \\ &= 9 \end{aligned}$$

C.2 EVALUATING EXPRESSIONS WITH EXPONENTS IN 3 STEPS

Ex 58: Evaluate this expression:

$$2^3 \times (8-6) = \boxed{16}$$

Answer:

$$\begin{aligned} 2^3 \times (8-6) &= 2^3 \times (8-6) && \text{(parentheses: } 8-6=2\text{)} \\ &= 2^3 \times 2 && \text{(exponent: } 2^3=8\text{)} \\ &= 8 \times 2 && \text{(multiplication: } 8\times 2=16\text{)} \\ &= 16 \end{aligned}$$

Ex 59: Evaluate this expression:

$$(2+1)^2 - 1 = \boxed{8}$$

Answer:

$$\begin{aligned} (2+1)^2 - 1 &= (2+1)^2 - 1 && \text{(parentheses: } 2+1=3\text{)} \\ &= 3^2 - 1 && \text{(exponent: } 3^2=9\text{)} \\ &= 9 - 1 && \text{(subtraction: } 9-1=8\text{)} \\ &= 8 \end{aligned}$$

Ex 60: Evaluate this expression:

$$(3^2 - 1) \times 4 = \boxed{32}$$

Answer:

$$\begin{aligned} (3^2 - 1) \times 4 &= (3^2 - 1) \times 4 && \text{(eval parenthesis: } 3^2=9\text{)} \\ &= (9 - 1) \times 4 && \text{(eval parenthesis: } 9-1=8\text{)} \\ &= 8 \times 4 && \text{(multiplication: } 8\times 4=32\text{)} \\ &= 32 \end{aligned}$$

Ex 61: Evaluate this expression:

$$\frac{3^2 - 1}{2} = \boxed{4}$$

Answer:

$$\begin{aligned} \frac{3^2 - 1}{2} &= \frac{3^2 - 1}{2} && \text{(eval numerator: } 3^2=9\text{)} \\ &= \frac{9 - 1}{2} && \text{(eval numerator: } 9-1=8\text{)} \\ &= \frac{8}{2} && \text{(division: } 8\div 2=4\text{)} \\ &= 4 \end{aligned}$$

C.3 FINDING THE OPERATORS

Ex 62:

$$3^3 \ \boxed{-}\ 2^2 = 23$$

Answer:

- $3^3 + 2^2 = 27 + 4 = 31$, so it's not true.
- $3^3 - 2^2 = 27 - 4 = 23$, so it's true.
- $3^3 \times 2^2 = 27 \times 4 = 108$, so it's not true.
- $3^3 \div 2^2 = 27 \div 4 = 6.75$, so it's not true.

Ex 63:

$$2^4 \ \boxed{\times}\ 3^2 = 144$$

Answer:

- $2^4 + 3^2 = 16 + 9 = 25$, so it's not true.
- $2^4 - 3^2 = 16 - 9 = 7$, so it's not true.
- $2^4 \times 3^2 = 16 \times 9 = 144$, so it's true.
- $2^4 \div 3^2 = 16 \div 9 \approx 1.78$, so it's not true.

Ex 64:

$$2^3 \ \boxed{\div}\ 4 = 2$$

Answer:

- $2^3 + 4 = 8 + 4 = 12$, so it's not true.
- $2^3 - 4 = 8 - 4 = 4$, so it's not true.
- $2^3 \times 4 = 8 \times 4 = 32$, so it's not true.
- $2^3 \div 4 = 8 \div 4 = 2$, so it's true.

Ex 65:

$$(2+1)^2 \ \boxed{+}\ 1 = 10$$

Answer:

- $(2+1)^2 + 1 = 9 + 1 = 10$, so it's true.
- $(2+1)^2 - 1 = 9 - 1 = 8$, so it's not true.
- $(2+1)^2 \times 1 = 9 \times 1 = 9$, so it's not true.
- $(2+1)^2 \div 1 = 9 \div 1 = 9$, so it's not true.

C.4 SIMPLIFYING ALGEBRAIC EXPRESSIONS

Ex 66: Simplify the expression:

$$2x^2 + 3x^2 = \boxed{5x^2}$$

Answer:

$$\begin{aligned} 2x^2 + 3x^2 &= (2+3)x^2 && \text{(combine like terms)} \\ &= 5x^2 \end{aligned}$$

Ex 67: Simplify the expression:

$$3x^2 - x^2 = \boxed{2x^2}$$

Answer:

$$\begin{aligned} 3x^2 - x^2 &= (3-1)x^2 \quad \text{(combine like terms)} \\ &= 2x^2 \end{aligned}$$

Ex 68: Simplify the expression:

$$2x^2 + 3x + x = \boxed{2x^2 + 4x}$$

Answer:

$$\begin{aligned} 2x^2 + 3x + x &= 2x^2 + (3+1)x \quad \text{(combine like terms)} \\ &= 2x^2 + 4x \end{aligned}$$

Ex 69: Simplify the expression:

$$x^2 + 2x + x^2 + 5x + 1 = \boxed{2x^2 + 7x + 1}$$

Answer:

$$\begin{aligned} x^2 + 2x + x^2 + 5x + 1 &= (x^2 + x^2) + (2x + 5x) + 1 \\ &= 2x^2 + 7x + 1 \end{aligned}$$

Ex 70: Simplify the expression:

$$3x^2 + 4 + 2x + x^2 + 6x + 1 = \boxed{4x^2 + 8x + 5}$$

Answer:

$$\begin{aligned} 3x^2 + 4 + 2x + x^2 + 6x + 1 &= (3x^2 + x^2) + (2x + 6x) + (4 + 1) \\ &= 4x^2 + 8x + 5 \end{aligned}$$

Ex 71: Simplify the expression:

$$(2x - x)^2 = \boxed{x^2}$$

Answer:

$$\begin{aligned} (2x - x)^2 &= (x)^2 \quad \text{(combine like terms in the parentheses)} \\ &= x^2 \end{aligned}$$