EXPONENTS

Exponents are an efficient way of expressing repeated multiplication, and they help us work with large numbers more easily.

A POSITIVE EXPONENTS

Definition Exponentiation

Exponentiation is repeated multiplication of a number by itself:

$$\underset{\text{base}\,\rightarrow}{} \overset{\text{exponent}}{2^{4}} = \overbrace{2 \times 2 \times 2 \times 2}^{4 \text{ factors}}$$

Ex: Write using exponent notation: $5 \times 5 \times 5$

Answer: $5 \times 5 \times 5 = 5^3$

Definition Vocabulary

Value	Expanded form	Exponent notation	Spoken form
2	2	2^1	2 or 2 raised to the power 1
4	2×2	2^2	2 squared or 2 raised to the power 2
8	$2 \times 2 \times 2$	2^3	2 cubed or 2 raised to the power 3
16	$2 \times 2 \times 2 \times 2$	2^4	2 raised to the power 4
32	$2 \times 2 \times 2 \times 2 \times 2$	2^5	2 raised to the power 5

Ex: Find the value for 2^3.

Answer:

$$\begin{aligned} 2^3 &= 2 \times 2 \times 2 \\ &= 8 \end{aligned}$$

B NEGATIVE EXPONENTS

Definition Exponentiation for a negative exponent

For a negative exponent, exponentiation is repeated division:

$$\begin{aligned} a^{-n} &= \frac{1}{\underbrace{a \times a \times \cdots \times a}_{n \text{ factors}}} \qquad \text{and} \qquad a^0 = 1 \\ &= \frac{1}{a^n} \end{aligned}$$

In particular, $a^{-1} = \frac{1}{a}$.

Ex: Write 3^{-2} as a fraction.

Answer:

$$\begin{aligned} 3^{-2} &= \frac{1}{3 \times 3} \\ &= \frac{1}{9} \end{aligned}$$

C EXPONENT LAW 1

Proposition Exponent law 1

$$a^m \times a^n = a^{m+n}$$

Ex: Simplify $5^2 \times 5^4$.

Answer:

$$\begin{aligned} 5^2 \times 5^4 &= 5^{2+4} \\ &= 5^6 \end{aligned}$$

D EXPONENT LAW 2

Proposition Exponent Law 2

$$\frac{a^m}{a^n} = a^{m-n}$$

Ex: Simplify $\dfrac{5^7}{5^3}$.

Answer:

$$\begin{aligned}\frac{5^7}{5^3} &= 5^{7-3}\\ &= 5^4\end{aligned}$$

E EXPONENT LAW 3

Proposition Exponent Law 3

$$(a^m)^n = a^{m\times n}$$

Ex: Simplify $\left(5^2\right)^5$.

Answer:

$$\begin{aligned}\left(5^2\right)^5 &= 5^{2\times 5}\\ &= 5^{10}\end{aligned}$$

F EXPONENT LAW 4

Proposition Exponent Law 4

$$(ab)^n = a^n\, b^n$$

Ex: Simplify $(2\times 5)^3$.

Answer:

$$(2\times 5)^3 = 2^3\, 5^3$$

G EXPONENT LAW 5

Proposition Exponent Law 5

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Ex: Calculate $\left(\dfrac{5}{3}\right)^2$.

Answer:

$$\begin{aligned}\left(\frac{5}{3}\right)^2 &= \frac{5^2}{3^2}\\ &= \frac{25}{9}\end{aligned}$$

H ORDER OF OPERATIONS

The order of operations is a set of guidelines that help us solve mathematical expressions in a consistent manner.

Definition **Order of Operations**

To solve mathematical expressions accurately, we follow the **order of operations**, which is commonly remembered using the acronym **PEMDAS**:

1. P: Parentheses
2. E: Exponents
3. M: Multiplication
4. D: Division
5. A: Addition
6. S: Subtraction

The order of operations proceeds from top to bottom, meaning we start with parentheses, then exponents, and so on. However, multiplication and division, as well as addition and subtraction, have the same level of priority. In these cases, we work from left to right.

Ex: Evaluate $(1+2)\times 2^3+4$

Answer:

$$\begin{aligned}
(1+2)\times 2^3+4 &= (1+2)\times(2^3+4 && \text{(parentheses: } (1+2)=3\text{)}\\
&= 3\times 2^3+4 && \text{(exponent: } 2^3=8\text{)}\\
&= 3\times 8+4 && \text{(multiplication: } 3\times 8=24\text{)}\\
&= 24+4 && \text{(addition: } 24+4=28\\
&= 28\text{)}
\end{aligned}$$