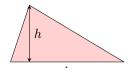
# A DEFINITIONS

## A.1 FINDING THE SUBJECTS



MCQ 1: For a triangle

we have the

formula

$$A = \frac{b \times h}{2}.$$

Find the subject of the formula.

 $\boxtimes$  A: the area

 $\Box$  b: the base

 $\square$  h: the height

Answer: The subject of the formula is the area A.

MCQ 2: In an electrical circuit, we have the formula

$$U = RI$$

.Find the subject of the formula.

 $\boxtimes U$ : the voltage

 $\square$  R: the resistance

 $\square$  I: the current

Answer: The subject of the formula is the voltage U.

MCQ 3: In physics, we have the formula

$$v = \frac{d}{t}$$

.Find the subject of the formula.

 $\boxtimes v$ : the velocity

 $\Box$  d: the distance

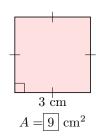
 $\Box$  t: the time

Answer: The subject of the formula is the velocity v.

## **B PROBLEM SOLVING**

#### **B.1 CALCULATING MEASURES IN GEOMETRY**

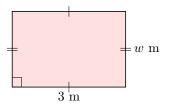
Ex 4: Find the area of the figure:



Answer:

$$A = s^2$$
 (identifying the formula)  
=  $3^2$  (substituting  $s = 3$ )  
=  $9 \text{ cm}^2$  (solving)

**Ex 5:** The area of the rectangle is  $6 \text{ m}^2$  and the length of one side is 3 m.



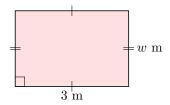
Find the width of the rectangle.

$$w = \boxed{2} \text{ m}$$

Answer:

$$A = L \times w \quad \text{(identifying the formula)}$$
 
$$6 = 3 \times w \quad \text{(substituting } A = 6 \text{ and } L = 3\text{)}$$
 
$$w = \frac{6}{3} \quad \text{(dividing by 3)}$$
 
$$w = 2 \, \text{m}$$

Ex 6: The perimeter of the rectangle is 10 m.



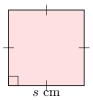
Find the width of the rectangle.

$$w = \boxed{2} \text{ m}$$

Answer:

$$P=2L+2w$$
 (identifying the formula)  
 $10=2\times 3+2w$  (substituting  $P=10$  and  $L=3$ )  
 $10=6+2w$   
 $2w=10-6$  (subtracting 6)  
 $2w=4$   
 $w=\frac{4}{2}=2\,\mathrm{m}$ 

Ex 7: The area of a square is  $10 \,\mathrm{cm}^2$ .

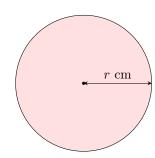


Find the length of the side of the square (round to 2 decimal places).

$$s = 3.16$$
 cm

$$A=s^2$$
 (identifying the formula) 
$$10=s^2$$
 (substituting  $A=10$ ) 
$$s=\sqrt{10}$$
 (taking the square root) 
$$s\approx 3.16\,\mathrm{cm}$$
 (rounding to 2 decimal places)

**Ex 8:** The area of a circle is  $10 \,\mathrm{cm}^2$ . Recall the formula:  $A = \pi r^2$ .



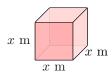
Find the radius of the circle (round your answer to 2 decimal places).

$$r = \boxed{1.78} \text{ cm}$$

Answer:

$$A = \pi r^2 \qquad \text{(recall the formula)}$$
 
$$10 = \pi r^2 \qquad \text{(substitute } A = 10\text{)}$$
 
$$r^2 = \frac{10}{\pi} \qquad \text{(divide both sides by } \pi\text{)}$$
 
$$r = \sqrt{\frac{10}{\pi}} \qquad \text{(take the square root)}$$
 
$$r \approx 1.78 \, \text{cm} \qquad \text{(rounded to 2 decimal places)}$$

**Ex 9:** The volume of the cube is  $10 \,\mathrm{m}^3$ .



Find the length of the side of the cube (round to 2 decimal places and  $\sqrt[3]{10} = 2.1544...$ ).

$$x = 2.15 \text{ m}$$

Answer:

$$V=x^3$$
 (identifying the formula)  
 $10=x^3$  (substituting  $V=10$ )  
 $x=\sqrt[3]{10}$  (taking the cube root of both sides)  
 $x\approx 2.15\,\mathrm{m}$  (rounding to 2 decimal places)

#### **B.2 CALCULATING MEASURES IN PHYSICS**

Ex 10: A car travels at a constant speed of 120 km/h for 2 hours.

Find the distance traveled by the car. Recall the formula:  $v = \frac{d}{t}$ .

$$d = 240 \text{ km}$$

Answer:

$$v=rac{d}{t}$$
 (identifying the formula)  $20=rac{d}{2}$  (substituting  $v=120$  and  $t=2$ )  $d=120\times 2$  (multiplying both sides by 2)  $d=240\,\mathrm{km}$ 

Ex 11: A circuit has a resistance of 5  $\Omega$  and a voltage of 20 V

Find the current flowing through the circuit. Recall the formula: U=RI.

$$I = \boxed{4}$$
 A

Answer:

$$U=RI$$
 (identifying the formula)  
 $20=5I$  (substituting  $U=20$  and  $R=5$ )  
 $I=\frac{20}{5}$  (dividing both sides by 5)  
 $I=4$  A

**Ex 12:** The formula  $F = \frac{9}{5}C + 32$  converts a temperature from Celsius (C) to Fahrenheit (F).

Given a temperature of 68°F, find the temperature in Celsius.

$$C = 20$$
°C

Answer:

$$F = \frac{9}{5}C + 32 \quad \text{(identifying the formula)}$$

$$68 = \frac{9}{5}C + 32 \quad \text{(substituting } F = 68\text{)}$$

$$68 - 32 = \frac{9}{5}C \quad \text{(subtracting 32 from both sides)}$$

$$36 = \frac{9}{5}C$$

$$C = 36 \times \frac{5}{9} \quad \text{(multiplying both sides by } \frac{5}{9}\text{)}$$

Ex 13: The formula  $E_p = mgh$  calculates the gravitational potential energy  $(E_p)$  of an object, where m is the mass in kilograms, g is the acceleration due to gravity (9.8 m/s<sup>2</sup>), and h is the height in meters.

Given an object with a mass of 10 kg and a gravitational potential energy of 490 J, find the height at which the object is located.

$$h = \boxed{5}$$
 m

$$\begin{split} E_p &= mgh & \text{(identifying the formula)} \\ 490 &= 10 \times 9.8 \times h & \text{(substituting } E_p = 490, \ m = 10, \ g = 9.8) \\ 490 &= 98h & \text{(calculating } 10 \times 9.8) \\ h &= \frac{490}{98} & \text{(dividing both sides by } 98) \\ h &= 5 \text{ m} \end{split}$$

## C REARRANGING FORMULAE

#### **C.1 REARRANGING LINEAR EQUATIONS**

**Ex 14:** Rearrange the equation 3x + 4y = 13 to make y the subject.

$$y = \boxed{\frac{13 - 3x}{4}}$$

Answer:

$$3x + 4y = 13$$
 
$$4y = 13 - 3x (subtracting 3x from both sides)$$
 
$$y = \frac{13 - 3x}{4} (dividing both sides by 4)$$

**Ex 15:** Rearrange the equation 5x - 2y = 10 to make y the subject.

$$y = \boxed{\frac{5x - 10}{2}}$$

Answer:

$$5x - 2y = 10$$

$$-2y = 10 - 5x (subtracting 5x from both sides)$$

$$-2y = -5x + 10 (reordering)$$

$$y = \frac{-5x + 10}{-2} (dividing both sides by -2)$$

$$y = \frac{5x - 10}{2} (simplifying)$$

**Ex 16:** Rearrange the equation 3y + 2x = -x + 3 to make y the subject.

$$y = \boxed{1 - x}$$

Answer:

$$3y + 2x = -x + 3$$

$$3y = -x + 3 - 2x$$
 (subtracting 2x from both sides)
$$3y = 3 - 3x$$

$$y = \frac{3 - 3x}{3}$$
 (dividing both sides by 3)
$$y = 1 - x$$
 (simplifying)

**Ex 17:** Rearrange the equation 7y - 5x = 2x + 14 to make y the subject.

$$y = \boxed{2+x}$$

Answer:

$$7y - 5x = 2x + 14$$

$$7y = 2x + 14 + 5x \quad \text{(adding } 5x \text{ to both sides)}$$

$$7y = 7x + 14$$

$$y = \frac{7x + 14}{7} \quad \text{(dividing both sides by } 7\text{)}$$

$$y = x + 2 \quad \text{(simplifying)}$$

#### C.2 REARRANGING GEOMETRIC FORMULAE

MCQ 18: The formula for the circumference (perimeter) of a circle is  $C=2\pi r$ .

Rearrange the formula to make r the subject.

Choose the correct answer:

$$\Box r = 2\pi C$$

$$\Box 2\pi r = C$$

$$\boxtimes \ r = \frac{C}{2\pi}$$

Answer:

$$C=2\pi r$$
 (given formula) 
$$r=\frac{C}{2\pi}$$
 (dividing both sides by  $2\pi$ )

MCQ 19: The formula for the volume of a cube is  $V = s^3$ . Rearrange the formula to make s the subject.

Choose the correct answer:

$$\square \ s = V^3$$

$$\square s = \frac{V}{3}$$

$$\boxtimes s = \sqrt[3]{V}$$

Answer:

$$V=s^3$$
 (given formula) 
$$s=\sqrt[3]{V}$$
 (taking the cube root of both sides)

MCQ 20: The formula for the area of a triangle is  $A = \frac{1}{2}bh$ . Rearrange the formula to make h the subject.

Choose the correct answer:

$$\Box h = \frac{b}{2A}$$

$$\Box h = \frac{A}{2h}$$

$$\boxtimes h = \frac{2A}{h}$$

Answer:

$$A = \frac{1}{2}bh \quad \text{(given formula)}$$

$$2A = bh \quad \text{(multiplying both sides by 2)}$$

$$h = \frac{2A}{b} \quad \text{(dividing both sides by } b\text{)}$$

### C.3 REARRANGING PHYSICS FORMULAE

MCQ 21: The formula  $F = \frac{9}{5}C + 32$  converts a temperature from Celsius (C) to Fahrenheit (F).

Rearrange the formula to make C the subject.

Choose the correct answer:

$$\boxtimes C = \frac{5}{9}(F - 32)$$

$$\Box C = \frac{9}{5}(F - 32)$$

$$\Box \ C = \frac{5}{9}F + 32$$

Answer:

$$F = \frac{9}{5}C + 32 \qquad \text{(given formula)}$$
 
$$F - 32 = \frac{9}{5}C \qquad \text{(subtracting 32 from both sides)}$$
 
$$C = \frac{5}{9}(F - 32) \quad \text{(multiplying both sides by } \frac{5}{9}\text{)}$$

**MCQ 22:** The formula  $v = \frac{d}{t}$  relates speed (v), distance (d), and time (t).

Rearrange the formula to make t the subject.

Choose the correct answer:

$$\boxtimes t = \frac{d}{v}$$

$$\Box t = \frac{v}{d}$$

$$\Box t = dv$$

Answer:

$$v = \frac{d}{t}$$
 (given formula)

vt = d (multiplying both sides by t)

$$t = \frac{d}{v}$$
 (dividing both sides by  $v$ )

**MCQ 23:** The formula U = RI relates voltage (U), resistance (R), and current (I).

Rearrange the formula to make I the subject.

Choose the correct answer:

$$\boxtimes I = \frac{U}{R}$$

$$\square I = UR$$

$$\square I = \frac{R}{U}$$

Answer:

$$U = RI$$
 (given formula)

$$I = \frac{U}{R}$$
 (dividing both sides by  $R$ )

MCQ 24: The formula  $E = \frac{1}{2}mv^2$  relates kinetic energy (E), mass (m), and speed (v).

Rearrange the formula to make v the subject.

Choose the correct answer:

$$\boxtimes v = \sqrt{\frac{2E}{m}}$$

$$\square \ v = \frac{E}{m}$$

$$\square \ v = \frac{2E}{m}$$

Answer:

$$E = \frac{1}{2}mv^2$$
 (given formula)

$$2E = mv^2$$
 (multiplying both sides by 2)

$$v^2 = \frac{2E}{m}$$
 (dividing both sides by  $m$ )

$$v = \sqrt{\frac{2E}{m}}$$
 (taking the square root of both sides)

### C.4 REARRANGING RATIO EQUATIONS

**Ex 25:** Rearrange the equation  $\frac{2}{y} = \frac{x}{6}$  to make y the subject:

$$y = \boxed{\frac{12}{x}}$$

Answer:

$$\frac{2}{y}$$
  $\frac{x}{6}$ 

 $y \times x = 2 \times 6$  (cross multiplication)

$$y = \frac{12}{x}$$
 (dividing both sides by  $x$ )

**Ex 26:** Rearrange the equation  $\frac{1}{x} = \frac{2}{y}$  to make y the subject:

$$y = \boxed{2x}$$

Answer:



 $1 \times y = x \times 2$  (cross multiplication)

$$y = 2x$$

**Ex 27:** Rearrange the equation  $\frac{x}{2} = \frac{4}{u}$  to make y the subject:

$$y = \boxed{\frac{8}{x}}$$

Answer:

$$\frac{x}{2}$$
  $\frac{4}{y}$ 

 $x \times y = 2 \times 4$  (cross multiplication)

$$y = \frac{8}{x}$$
 (dividing by  $x$ )

**Ex 28:** Rearrange the equation  $\frac{y}{x} = \frac{4}{3}$  to make y the subject:

$$y = \boxed{\frac{4x}{3}}$$

Answer:



 $y \times 3 = x \times 4$  (cross multiplication)

$$y = \frac{4x}{3}$$

# **D CONSTRUCTING FORMULAE**

# D.1 MODELING LINEAR RELATIONSHIPS WITH ALGEBRA

Ex 29: A mechanic charges a \$40 call-out fee and \$30 per hour thereafter.

Find the mechanic's fee M for a job which takes x hours.

$$M = 40 + 30x$$

Answer: The total fee is \$40 plus \$30 for each of x hours.

- For x = 1,  $M = 40 + 30 \times 1$
- For x = 2,  $M = 40 + 30 \times 2$
- •
- For x,  $M = 40 + 30 \times x$

**Ex 30:** A car rental company charges a fixed distance of 50 km included in the rental and 15 km for each extra hour of rental. Find the total distance D the car can travel in terms of the rental time x (in hours).

$$D = 50 + 15x$$

Answer: The total distance is 50 km plus 15 km for each extra hour.

- For x = 1,  $D = 50 + 15 \times 1$
- For x = 2,  $D = 50 + 15 \times 2$
- •
- For x,  $D = 50 + 15 \times x$

Ex 31: A gym membership includes a one-time joining fee of \$25 and a monthly fee of \$40.

Find the total cost C after x months.

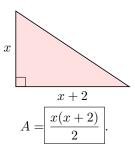
$$C = 25 + 40x$$

Answer: The total cost is the joining fee of \$25 plus \$40 for each of x months.

- For x = 1,  $C = 25 + 40 \times 1$
- For x = 2,  $C = 25 + 40 \times 2$
- . :
- For x,  $C = 25 + 40 \times x$

# D.2 MODELING AREAS AND VOLUMES WITH ALGEBRA: LEVEL 1

**Ex 32:** A right-angled triangle has a base length of x + 2 units and a height of x units. Find the area A of the triangle.



Answer: The area of a triangle is half the product of its base and height.

$$A = \frac{\text{base} \times \text{height}}{2}$$
$$= \frac{x(x+2)}{2}$$

**Ex 33:** A garden is in the shape of a semi-circle with diameter x meters.



Find the area A of the garden in terms of the diameter x of the semi-circle.

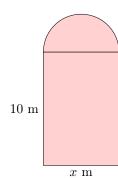
$$A = \boxed{\frac{\pi x^2}{8}} \quad \text{m}^2.$$

Answer:

- The radius r is half of the diameter:  $r = \frac{x}{2}$ .
- The area of a circle is  $\pi r^2$ , where r is the radius.

Area of semi-circle 
$$=\frac{1}{2}$$
Area of circle  $=\frac{1}{2}\pi r^2$   $=\frac{1}{2}\pi\left(\frac{x}{2}\right)^2$   $=\frac{1}{2}\pi\frac{x^2}{4}$   $=\frac{\pi x^2}{8}$ 

**Ex 34:** The door is composed of a rectangle with height 10 meters and width x meters, topped with a semi-circle of diameter x meters.



Find the area A of the door in terms of x.

$$A = \boxed{10x + \frac{\pi x^2}{8}} \quad \text{m}^2.$$

Answer:

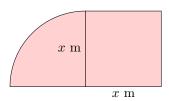
- The area of the rectangle is width times height:  $x \times 10 = 10x$ .
- The radius r of the semi-circle is half of the diameter:  $r = \frac{x}{2}$ .
- The area of a full circle is  $\pi r^2$ , so the area of the semi-circle is half of that.

Area of semi-circle 
$$= \frac{1}{2}\pi r^2$$
 
$$= \frac{1}{2}\pi \left(\frac{x}{2}\right)^2$$
 
$$= \frac{1}{2}\pi \frac{x^2}{4}$$
 
$$= \frac{\pi x^2}{8}$$

• The total area of the door is the sum of the rectangle and semi-circle areas:

$$A = 10x + \frac{\pi x^2}{8}$$

**Ex 35:** The door consists of a square with side length x meters and a quarter-circle with radius x meters.



Find the area A of the door in terms of x.

$$A = x^2 + \frac{\pi x^2}{4} \quad m^2.$$

Answer:

- The area of the square is side times side:  $x \times x = x^2$ .
- The radius r of the quarter-circle is r = x.
- The area of a full circle is  $\pi r^2$ , so the area of the quarter-circle is one-fourth of that.

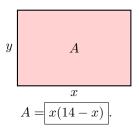
Area of quarter-circle = 
$$\frac{1}{4}\pi r^2$$
  
=  $\frac{1}{4}\pi x^2$ 

• The total area of the door is the sum of the square and quarter-circle areas:

$$A = x^2 + \frac{\pi x^2}{4}$$

# D.3 MODELING AREAS AND VOLUMES WITH ALGEBRA: LEVEL 2

**Ex 36:** You have 28 meters of fencing to enclose a rectangular vegetable garden. Let x be the length of the rectangle and y be the width. Find the area A of the garden in terms of x.



Answer:

• The perimeter of the rectangle is 28 m, so:

$$2x + 2y = 28$$

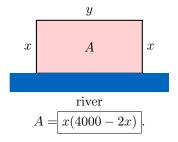
• Solve for y in terms of x:

$$2x + 2y = 28$$
$$2y = 28 - 2x$$
$$y = 14 - x$$

• The area of the rectangle is  $A = x \times y$ . Substituting y = 14 - x:

$$A = x(14 - x)$$

Ex 37: A farmer has 4000 meters of fencing to enclose a rectangular field along a river. Because one side is along the river, fencing is required on only three sides. Let x be the length perpendicular to the river and y the length parallel to the river. Find the area A of the field in terms of x.



Answer:

• Since the fence is on three sides only, the total fencing is:

$$x + x + y = 4000$$
 or  $2x + y = 4000$ 

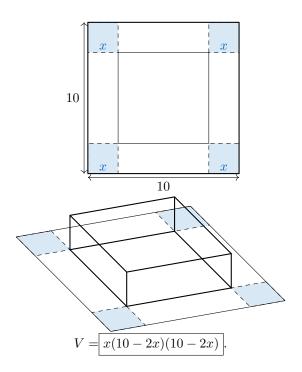
• Solve for y in terms of x:

$$2x + y = 4000$$
$$y = 4000 - 2x$$

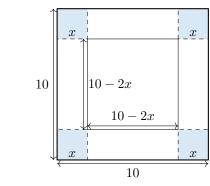
• The area of the rectangle is  $A = x \times y$ . Substituting y = 4000 - 2x:

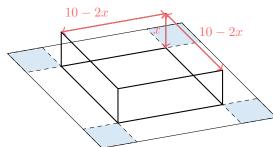
$$A = x(4000 - 2x)$$

**Ex 38:** A sheet of paper  $10 \text{ cm} \times 10 \text{ cm}$  is made into an open box by cutting x-cm squares out of each corner and folding up the sides. Find the volume V of the box in terms of x.



• After cutting out x-cm squares, the height of the box is x. The new length is 10 - 2x and the new width is 10 - 2x.





• The volume of a box is

 $V = \text{height} \times \text{length} \times \text{width}.$ 

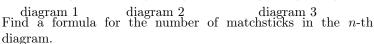
• Substituting the dimensions:

$$V = x(10 - 2x)(10 - 2x).$$

### D.4 FINDING PATTERNS AND WRITING FORMULAE

Ex 39: Look at the following matchstick pattern:





Number of matchsticks = 3n + 1

Answer: Counting the matchsticks in each diagram:

- Diagram 1:  $4 = 3 \times 1 + 1$
- Diagram 2:  $7 = 3 \times 2 + 1$
- Diagram 3:  $10 = 3 \times 3 + 1$
- :
- Diagram  $n: 3 \times n + 1$

Ex 40: Look at the following triangular matchstick pattern:







diagram 1 diagram 2 diagram 3 Find a formula for the number of matchsticks in the n-th diagram.

Number of matchsticks = 2n+1

Answer: Counting the matchsticks in each diagram:

- Diagram 1:  $3 = 2 \times 1 + 1$
- Diagram 2:  $5 = 2 \times 2 + 1$
- Diagram 3:  $7 = 2 \times 3 + 1$
- •
- Diagram  $n: 2 \times n + 1$

Ex 41: Find the *n*-th term of the sequence  $5, 10, 15, 20, 25, \ldots$ 

$$n$$
-th term =  $5n$ .

Answer:

- $1^{\text{st}}$  term:  $5 = 5 \times 1$
- $2^{nd}$  term:  $10 = 5 \times 2$
- $3^{\text{rd}}$  term:  $15 = 5 \times 3$
- •
- $n^{\text{th}}$  term:  $5 \times n$

**Ex 42:** Find the n-th term of the sequence 6, 12, 18, 24, 30, 36, ...

$$n$$
-th term =  $6n$ 

Answer:

- $1^{st}$  term:  $6 = 6 \times 1$
- $2^{nd}$  term:  $12 = 6 \times 2$
- $3^{\rm rd}$  term:  $18 = 6 \times 3$
- . :
- $n^{\text{th}}$  term:  $6 \times n$

Ex 43: Find the *n*-th term of the sequence  $1, 3, 5, 7, 9, 11, \ldots$ 

$$n$$
-th term =  $2n-1$ .

•  $1^{st}$  term:  $1 = 2 \times 1 - 1$ 

•  $2^{nd}$  term:  $3 = 2 \times 2 - 1$ 

•  $3^{\text{rd}}$  term:  $5 = 2 \times 3 - 1$ 

• :

•  $n^{\text{th}}$  term:  $2 \times n - 1$ 

**Ex 44:** Find the *n*-th term of the sequence  $2, 4, 8, 16, 32, 64, \ldots$ 

$$n$$
-th term =  $2^n$ .

Answer:

•  $1^{st}$  term:  $2 = 2^1$ 

•  $2^{\text{nd}}$  term:  $4 = 2^2$ 

•  $3^{\text{rd}}$  term:  $8 = 2^3$ 

• :

•  $n^{\text{th}}$  term:  $2^n$ 

**Ex 45:** Find the *n*-th term of the sequence  $1, 4, 9, 16, 25, 36, \ldots$ 

$$n$$
-th term =  $n^2$ .

Answer:

•  $1^{st}$  term:  $1 = 1^2$ 

•  $2^{\text{nd}}$  term:  $4 = 2^2$ 

•  $3^{rd}$  term:  $9 = 3^2$ 

• :

•  $n^{\text{th}}$  term:  $n^2$ 

**Ex 46:** Find the *n*-th term of the sequence  $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \dots$ 

$$n$$
-th term =  $\left\lceil \frac{1}{2n} \right\rceil$ .

Answer:

• 1<sup>st</sup> term:  $\frac{1}{2} = \frac{1}{2 \times 1}$ 

•  $2^{\text{nd}}$  term:  $\frac{1}{4} = \frac{1}{2 \times 2}$ 

•  $3^{\text{rd}}$  term:  $\frac{1}{6} = \frac{1}{2 \times 3}$ 

•  $n^{\text{th}}$  term:  $\frac{1}{2n}$