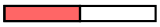
FRACTIONS

A DEFINITIONS

Discover: Hugo is very hungry after playing soccer. His dad baked two identical cakes.
Hugo eats one whole cake:

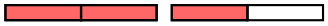
Then, Hugo is still hungry, so he eats half of the second cake:



How much cake does Hugo eat in total? Write your answer as a fraction.

Answer:

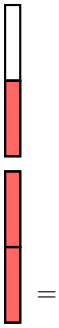
- Hugo eats one whole cake and half of another cake.



- The numerator (top number) shows how many parts Hugo eats: 3.
- The denominator (bottom number) shows how many equal parts make one cake: 2.
- So Hugo eats $\frac{3}{2}$ cakes in total.

Definition **Fraction**

A **fraction** includes two numbers: the **numerator** and the **denominator**, separated by a bar.



$= \frac{3}{2}$

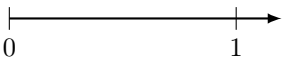
numerator: number of equal parts considered

denominator: number of equal parts the unit is divided

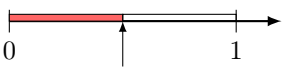
B ON THE NUMBER LINE

Discover:

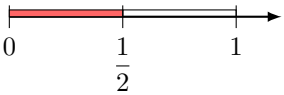
- Hugo is walking along a path.



- He stops and asks himself, "Where am I?"



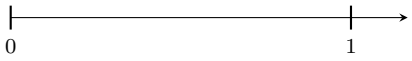
- His father says, "You are at half of the way that is $\frac{1}{2}$."



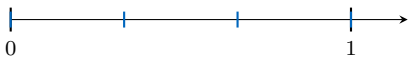
Method **Representing a Fraction on the Number Line**

To represent the fraction $\frac{2}{3}$ on a number line.

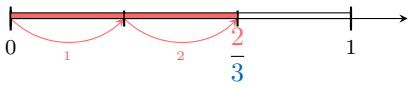
1. Draw a straight line and mark the points 0 and 1.



2. Divide the line between 0 and 1 into 3 equal parts.



3. Count 2 parts from 0 and mark the point.



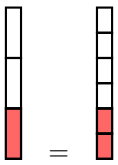
C EQUIVALENT FRACTIONS

Discover: Mr. Tariel has a cake that he cuts into **3 equal parts**. He plans to give **1 part** to his son, Louis.

Louis says, "I want **2 pieces**!"
His dad replies, "Alright," and cuts each of the **3 parts** in half, making **6 smaller equal parts**. He then gives Louis **2 of these smaller pieces**.

Louis looks at his plate and feels disappointed.
Why is Louis still not happy?

Answer: Even though Louis got **2 pieces** instead of 1, the total amount of cake he received is the same as before. His dad just cut the cake into smaller pieces.

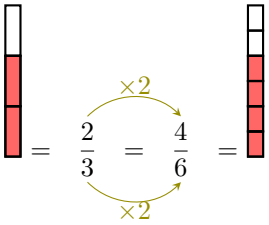


In fractions:

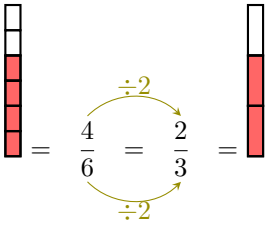
$$\frac{1}{3} = \frac{2}{6}$$

Definition **Equivalent Fractions**

- When you multiply the numerator and the denominator by the same number, the fractions are equals.

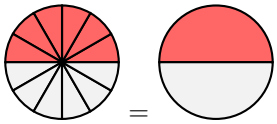


- When you divide the numerator and the denominator by the same number, the fractions are equals.



D SIMPLIFICATION

Discover: Pizza Time! Louis eats $\frac{6}{12}$ of a pizza. Hugo says, "Hey, $\frac{6}{12}$ is the same as $\frac{1}{2}$. It's easier to understand if you simplify the fraction!".

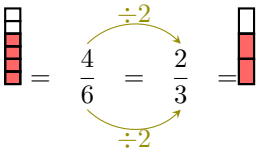


- **Louis:** "How is $\frac{1}{2}$ easier?"
- **Hugo:** "Because $\frac{1}{2}$ is the simplified form of $\frac{6}{12}$. It means you ate 1 out of 2 slices instead of 6 out of 12 slices. It's the same amount of pizza, but it's simpler to understand!"

Method **Simplifying a fraction**

To simplify a fraction, we find an equivalent fraction with the smallest possible numerator and denominator.

Ex: Simplify $\frac{4}{6}$

Answer: 

E CROSS MULTIPLICATION

Discover: We have learned that two fractions are equal if we can multiply both the numerator and the denominator by the same number.
For example:

$$\frac{2}{3} = \frac{5 \times 2}{5 \times 3} = \frac{10}{15}$$

Now, let's explore another way to check if two fractions are equal.
We can investigate the relationship between their numerators and denominators:

$$\begin{aligned} 2 \times 15 &= 2 \times (5 \times 3) \\ &= 5 \times 2 \times 3 \\ &= 10 \times 3 \end{aligned}$$

So, we can see that:

$$2 \times 15 = 3 \times 10$$

This leads us to a new way of checking if two fractions are equal: by cross multiplying and comparing the products.

$$\frac{2}{3} \overset{\times}{=} \frac{10}{15} \quad \text{if and only if} \quad 2\times 15 = 3\times 10$$

This is known as the cross multiplication property.

Proposition Cross Multiplication Property

$$\frac{a}{b} \overset{\times}{=} \frac{c}{d} \quad \text{if and only if} \quad a\times d = b\times c$$

Ex: Solve x for $\frac{10}{5} = \frac{x}{8}$.

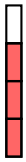
Answer:

$$\frac{10}{5} \overset{\times}{=} \frac{x}{8}$$

$$\begin{aligned} 5\times x &= 10\times 8 && \text{(cross mutiplication)} \\ x &= 10\times 8 \div 5 && \text{(dividing both sides by 5)} \\ x &= 16 \end{aligned}$$

F ORDERING FRACTIONS

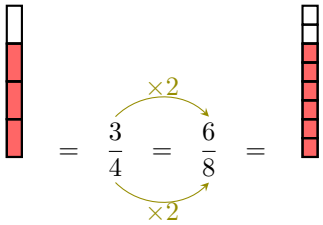
Discover:

- Hugo eats $\frac{3}{4}$ of a cake. 
- Louis eats $\frac{5}{8}$ of the same cake. 

Who eats more cake?

Answer:

- We need to compare the fractions $\frac{3}{4}$ and $\frac{5}{8}$.
- To compare fractions, the pieces must be the same size. We do this by finding a common denominator.
- Convert $\frac{3}{4}$ to an equivalent fraction with denominator 8:



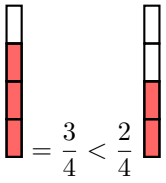
- Now, Hugo eats $\frac{6}{8}$ of the cake and Louis eats $\frac{5}{8}$.
- Since $\frac{6}{8} > \frac{5}{8}$, Hugo eats more cake.

Definition Ordering Fractions with the Same Denominator

For two fractions with the same denominator, the fraction with the larger numerator is larger.

Ex: Compare $\frac{3}{4}$ and $\frac{2}{4}$.

Answer:



$= \frac{3}{4} < \frac{2}{4}$

Method Comparing Fractions with Different Denominators

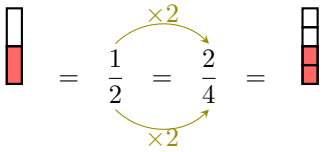
To compare two fractions with different denominators:

- Find a **common denominator**.
- Convert each fraction to an equivalent fraction with that denominator.
- Compare the numerators.

Ex: Compare $\frac{1}{2}$ and $\frac{3}{4}$.

Answer:

- Since $\frac{1}{2}$ and $\frac{3}{4}$ have different denominators, we change $\frac{1}{2}$ into an equivalent fraction with denominator 4:



- Compare the numerators:

$$\frac{2}{4} < \frac{3}{4}$$

- Therefore,

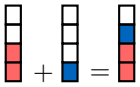
$$\frac{1}{2} < \frac{3}{4}$$

- In pictures:

G ADDITION AND SUBTRACTION WITH COMMON DENOMINATORS

Discover: Hugo eats $\frac{2}{4}$ of a cake:  and Louis eats $\frac{1}{4}$ of the same cake: 
Which fraction of the cake have Hugo and Louis eaten together?

Answer:

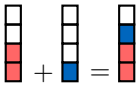


$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

So Hugo and Louis eat $\frac{3}{4}$ of the cake together: 

Definition **Addition of Fractions with Common Denominators**

When we **add** fractions with common denominators, we keep the denominator the same and add the numerators:



$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

Definition **Subtraction of Fractions with Common Denominators**

When we **subtract** fractions with common denominators, we keep the denominator the same and subtract the numerators:

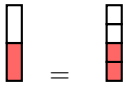
$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$$

H ADDITION AND SUBTRACTION WITH DIFFERENT DENOMINATORS

Discover: Hugo eats $\frac{1}{2}$ of a cake:  and Louis eats $\frac{1}{4}$ of the same cake:  .
What fraction of the cake have Hugo and Louis eaten together?

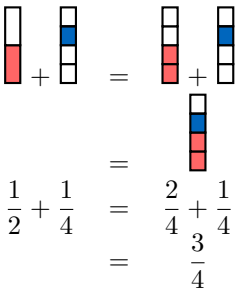
Answer:

- **Step 1: Find a common denominator**: To add the fractions, we need equal-sized parts. Divide each of Hugo's parts into two smaller parts:

  So, Hugo eats $\frac{1}{2} = \frac{2}{4}$ of the cake.

- **Step 2: Add the fractions using the common denominator**: Now, we can add the two fractions:

  $$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

- **Step 3: Final Answer**: Hugo and Louis eat $\frac{3}{4}$ of the cake together: 

Method **Addition or Subtraction of Fractions with Different Denominators**

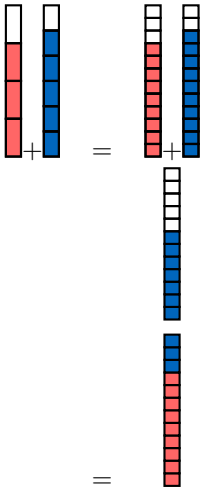
To add or subtract fractions with different denominators:

- **Find a common denominator**: Choose a common multiple of the denominators.
- **Convert each fraction**: Rewrite each fraction so it has the common denominator.
- **Add or subtract the numerators**: Add or subtract the numerators and keep the denominator the same.

Ex: Calculate $\frac{3}{4} + \frac{5}{6}$.

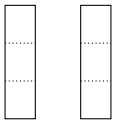
Answer:

- **Find a common denominator**:To add fractions, they must have the same denominator.
  - Multiples of 4: 4, 8, **12**, 16, 20, ...
  - Multiples of 6: 6, **12**, 18, 24, ...
  - The smallest common denominator is **12**.
- $\dfrac{3}{4}+\dfrac{5}{6}=\dfrac{3\times 3}{4\times 3}+\dfrac{5\times 2}{6\times 2}$
  $=\dfrac{9}{12}+\dfrac{10}{12}$ (common denominator = 12)
  $=\dfrac{9+10}{12}$ (adding numerators)
  $=\dfrac{19}{12}$
- **Visual representation**:



I FRACTION AS QUOTIENT

Discover: Two cakes are shared equally among three people.

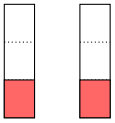


1. Use the figure to determine what fraction of the cakes each person receives.
2. Copy and complete: ... cakes ÷ ... people = $\dfrac{\ldots}{\ldots}$ of a cake each.

Answer:

1. Each cake is divided into three equal parts. Each person receives one piece from each cake, totaling two pieces. Since each cake is divided into three parts, each piece represents $\dfrac{1}{3}$ of a cake. Therefore, each person receives:

$$\frac{1}{3}+\frac{1}{3}=\frac{2}{3} \text{ of the cakes.}$$



2. 2 cakes ÷ 3 people = $\dfrac{2}{3}$ of a cake each.

Proposition **Fraction as Quotient**

A fraction is a quotient that represents the result of **division**. It tells us how much of something we have when we divide it into equal parts.

- **The top number (numerator)** is the whole.
- **The bottom number (denominator)** is the number of equal parts the whole is divided into.

The fraction $\frac{2}{3}$ is the same as saying **"2 divided by 3"**.

$$2 \div 3 = \quad = \frac{2}{3}$$

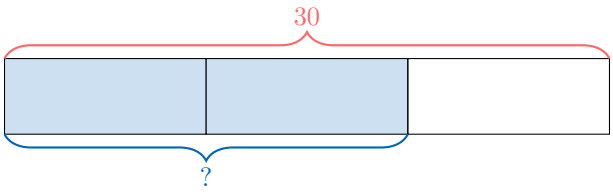
The fraction $\frac{2}{3}$ is the number which, when multiplied by 3, gives 2:

$$\frac{2}{3} \times 3 = 2$$

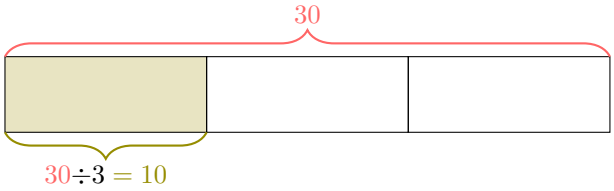
J FRACTION AS RATIO

Discover: In a class of 30 students, $\frac{2}{3}$ of the students are girls. How many students are girls?

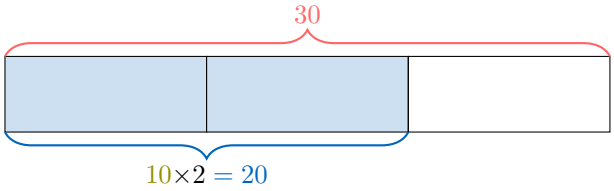
Answer: The fraction $\frac{2}{3}$ represents the ratio of girls to the total number of students.
We can visualize this problem using a bar model:



- **Method 1 (unitary method)**:
  - Divide the total number of students by the denominator of the fraction to find how many students are in each part:



    This means each part contains 10 students.
  - Multiply the result by the numerator to find how many students are girls:



    So, there are 20 girls.

- **Method 2** (calculation using a formula):

$$\begin{aligned} \text{Number of girls} &= \frac{2}{3} \text{ of } 30 \\ &= \frac{2}{3} \times 30 \\ &= (2 \div 3) \times 30 \\ &= 20 \end{aligned}$$

Definition **Fractions as Ratios**

A fraction can represent the ratio of part to the whole:

$$\frac{\text{Part}}{\text{Whole}}$$

Ex: There are 3 apples in Hugo's basket. 2 of the apples are red.

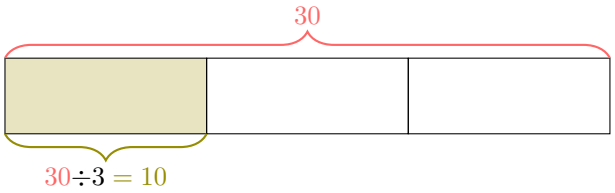
The fraction (ratio) of red apples is:

$$\frac{2}{3}$$

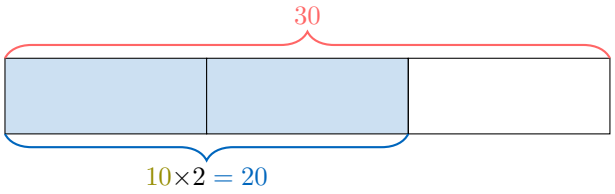
Method **Finding a Quantity from a Fraction (Ratio)**

To calculate $\frac{2}{3}$ of 30:

- **Method 1 (unitary method)**:
  - Divide the total by the denominator to find the amount for one part:



  - Multiply the result by the numerator to find the desired quantity:



- **Method 2** (calculation using a formula):

$$\begin{aligned}\frac{2}{3} \text{ of } 30 &= \frac{2}{3} \times 30 \\ &= (2 \div 3) \times 30 \\ &= 20\end{aligned}$$

K FRACTION AS DECIMAL NUMBER

Discover: Decimals and fractions can both be used to describe values between whole numbers. We can convert:

- **Fraction into Decimal**: Perform the division of the numerator by the denominator. For example,

$$\begin{aligned}\frac{1}{2} &= 1 \div 2 \\ &= 0.5\end{aligned}$$

- **Decimal into Fraction**: Multiply the decimal by a power of 10 (10, 100, 1000, ...) to eliminate the decimal point. Then, write the result over the same power of 10 to form a fraction. For example:

$$\begin{aligned}1.3 &= \frac{1.3 \times 10}{10} \\ &= \frac{13}{10}\end{aligned}$$

Method **Converting a Fraction to a Decimal**

- **Division Method**: Perform the division of the numerator by the denominator.
- **Power of 10 Denominator Method**: Find an equivalent fraction where the denominator is a power of 10.

Ex: Convert $\frac{3}{4}$ to a decimal number.

Answer:

- **Division Method**:

$$\begin{aligned}\frac{3}{4} &= 3 \div 4 \\ &= 0.75\end{aligned}$$

$$\begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ \underline{2.8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

- **Power of 10 Denominator Method**:

$$\begin{aligned}\frac{3}{4} &= \frac{3\times 25}{4\times 25} \\ &= \frac{75}{100} \\ &= 75 \div 100 \\ &= 0.75\end{aligned}$$

Method **Converting Decimal to Fraction**

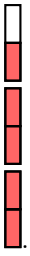
- Multiply the decimal by a power of 10 (10, 100, 1000, ...) to eliminate the decimal point.
- Write the result over the same power of 10 to form a fraction.

Ex: Convert 1.3 to a fraction.

Answer:

$$\begin{aligned}1.3 &= \frac{1.3 \times 10}{10} \\ &= \frac{13}{10}\end{aligned}$$

L PROPER AND IMPROPER FRACTIONS

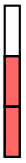
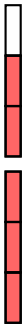
Discover: You have $\frac{5}{2}$ of a pain au chocolat:  .
How can you represent this amount in simple way?

Answer: You have 2 whole pains au chocolat and $\frac{1}{2}$ of another pain au chocolat. Is it easier to think of $\frac{5}{2}$ as $2+\frac{1}{2}$? This is the concept of a mixed number.

Definition **Proper and improper fractions**

A fraction which has numerator less than its denominator is called a **proper fraction**.
A fraction which has numerator greater than its denominator is called an **improper fraction**.

Ex:

- $\frac{2}{3} =$  is a proper fraction.
- $\frac{5}{3} = 1 + \frac{2}{3} =$  is an improper fraction.

Definition **Mixed Number**

A **mixed number** is a representation of a number that combines a whole number and a proper fraction.
By standard convention, the addition symbol is implied and thus not explicitly written:

$1\frac{2}{3}$ is understood as $1 + \frac{2}{3} =$