FUNCTIONS

A DEFINITIONS

A.1 WRITING FUNCTIONS: LEVEL 1

Ex 1: Consider the following calculation program:

1. Choose a number.
2. Subtract 5 from the chosen number.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \square$$

Ex 2: Consider the following calculation program:

1. Choose a number.
2. Multiply the chosen number by three.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \square$$

Ex 3: Consider the following calculation program:

1. Choose a number.
2. Multiply the chosen number by five.
3. Subtract 2 from the result obtained.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \square$$

Ex 4: Consider the following calculation program:

1. Choose a number.
2. Multiply the chosen number by -2.
3. Add 5 to the result obtained.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \square$$

A.2 WRITING FUNCTIONS: LEVEL 2

Ex 5: Consider the following calculation program:

1. Choose a number.
2. Multiply the chosen number by itself.
3. Subtract 1 from the result obtained.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \square$$

Ex 6: Consider the following calculation program:

1. Choose a number.
2. Square the chosen number.
3. Multiply the result by 2.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \square$$

Ex 7: Consider the following calculation program:

1. Choose a number.
2. Subtract 1 from the chosen number.
3. Multiply the result by the original number chosen.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \square$$

A.3 CALCULATING $f(x)$

Ex 8: For $f(x) = x + 3$,

$$f(4) = \square$$

Ex 9: For $f(x) = 2x - 1$,

$$f(5) = \square$$

Ex 10: For $f(x) = 3x + 2$,

$$f(2) = \square$$

Ex 11: For $f(x) = x^2 - 1$,

$$f(3) = \square$$

Ex 12: For $f(x) = 5x - 3$,

$$f(1) = \square$$

Ex 13: For $f(x) = \frac{x}{2} + 4$,

$$f(6) = \square$$

Ex 14: For $f(x) = x - 5$,

$$f(10) = \square$$

Ex 15: For $f(x) = 2x - 5$,

$$f(-2) = \square$$

Ex 16: For $f(x) = -x + 4$,

$$f(-3) = \square$$

Ex 17: For $f(x) = 3x - 7$,

$f(-1) =$ ☐

Ex 18: For $f(x) = x^2 - 2x$,

$f(-2) =$ ☐

Ex 19: For $f(x) = 2x + 3$,

$f(-3) =$ ☐

Ex 20: For $f(x) = \frac{x}{2} - 4$,

$f(8) =$ ☐

Ex 21: For $f(x) = \frac{3x-5}{2}$,

$f(-1) =$ ☐

Ex 22: For $f(x) = \frac{x-6}{2} - 3$,

$f(10) =$ ☐

A.4 CALCULATING $f(x)$

Ex 23: For $f : x \mapsto x + 3$,

$f(4) =$ ☐

Ex 24: For $f : x \mapsto x^2 - 1$,

$f(2) =$ ☐

Ex 25: For $f : x \mapsto (x - 1)(x - 2)$,

$f(0) =$ ☐

Ex 26: For $f : x \mapsto x^3$,

$f(-1) =$ ☐

A.5 SUBSTITUTING VALUES AND EXPRESSIONS INTO A FUNCTION

Ex 27: For $f : x \mapsto 1 - 3x$, find in simplest form:

1. $f(-2) =$ ☐
2. $f(3) =$ ☐
3. $f(x + 1) =$ ☐
4. $f(x^2) =$ ☐

Ex 28: For $f : x \mapsto x^2$, find in simplest form:

1. $f(3) =$ ☐
2. $f(-1) =$ ☐
3. $f(-x) =$ ☐
4. $f(x + 1) =$ ☐
5. $f(x + 2) =$ ☐
6. $f(2x) =$ ☐

Ex 29: For $g : x \mapsto x^2 - 2x + 1$, find in simplest form:

1. $g(3) =$ ☐
2. $g(-1) =$ ☐
3. $g(-x) =$ ☐
4. $g(x + 1) =$ ☐
5. $g(x + 2) =$ ☐
6. $g(2x) =$ ☐

B TABLES OF VALUES

B.1 FILLING TABLES OF VALUES

Ex 30: For $f(x) = x^2$, fill in the table of values:

x	-2	-1	0	1	2
$f(x)$					

Ex 31: For $f(x) = -2x + 1$, fill in the table:

x	-2	-1	0	1	2
$f(x)$					

Ex 32: For $f(x) = x^2 - 3x + 1$, fill in the table:

x	-2	-1	0	1	2
$f(x)$					

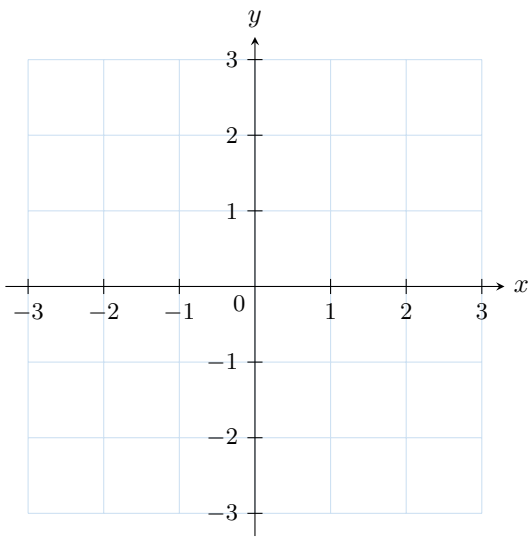
C GRAPHS

C.1 PLOTTING LINE GRAPHS

Ex 33: Here is a table of values for the function $f(x) = x - 1$:

x	-2	-1	0	1	2	3
$f(x)$	-3	-2	-1	0	1	2

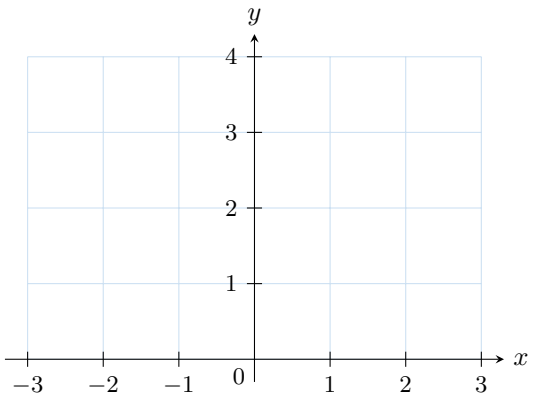
Plot the line graph of f.



Ex 34: Here is a table of values for the function $f(x) = x^2$:

x	-2	-1	-0.5	0	0.5	1	2
$f(x)$	4	1	0.25	0	0.25	1	4

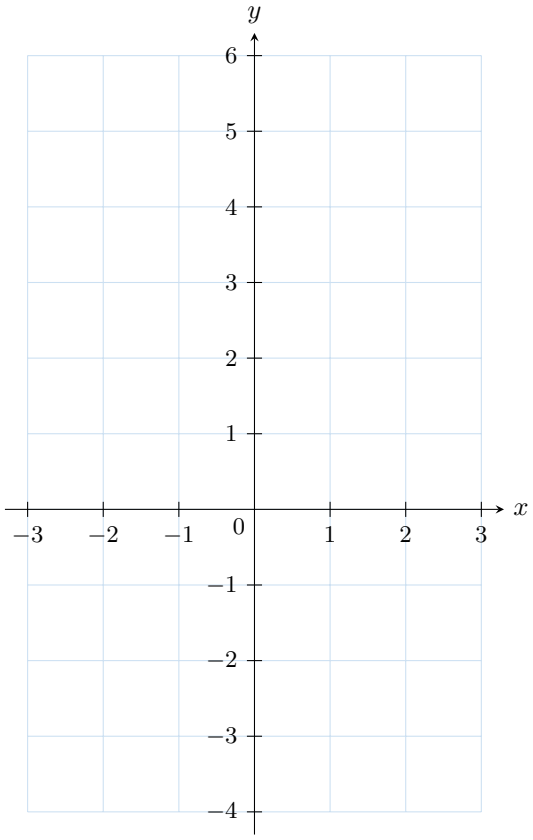
Plot the line graph of f.



Ex 35: Here is a table of values for the function $f(x) = -2x+1$:

x	-2	-1	0	1	2
$f(x)$	5	3	1	-1	-3

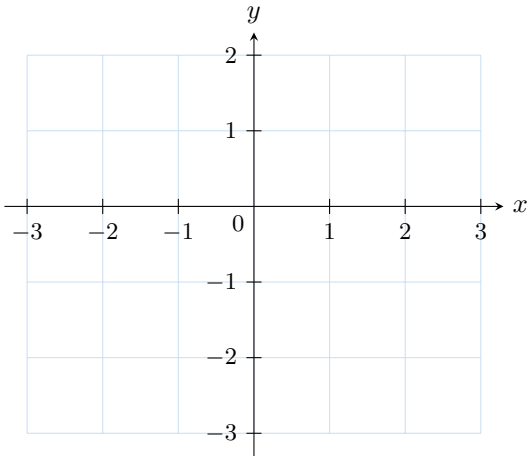
Plot the line graph of f.



Ex 36: Here is a table of values for the function $f(x) = 0.5x-1$:

x	-3	-2	-1	0	1	2	3
$f(x)$	-2.5	-2	-1.5	-1	-0.5	0	0.5

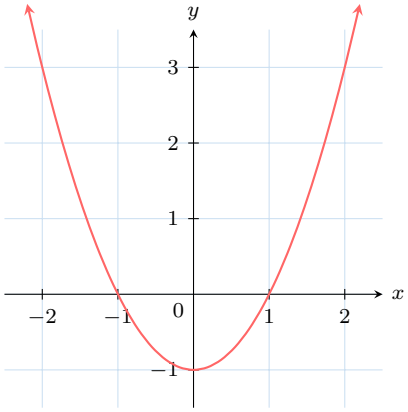
Plot the line graph of f.



D READING VALUES AND SOLVING $f(x) = y$ ON A GRAPH

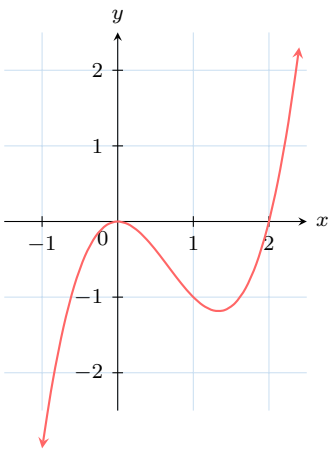
D.1 FINDING $f(x)$

Ex 37: The graph of $y = f(x)$ is:



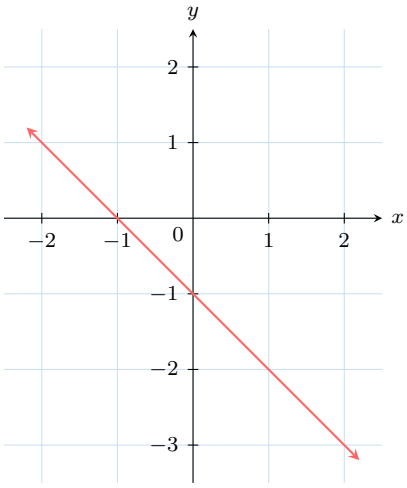
$f(2) =$ ☐

Ex 38: The graph of $y = f(x)$ is:



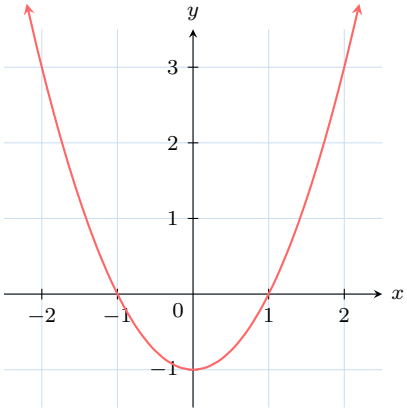
$f(1) =$ ☐

Ex 39: The graph of $y = f(x)$ is:



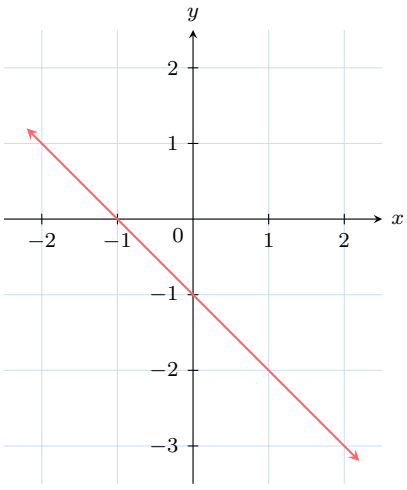
$f(-2) =$ ☐

Ex 40: The graph of $y = f(x)$ is:



$f(1) =$ ☐

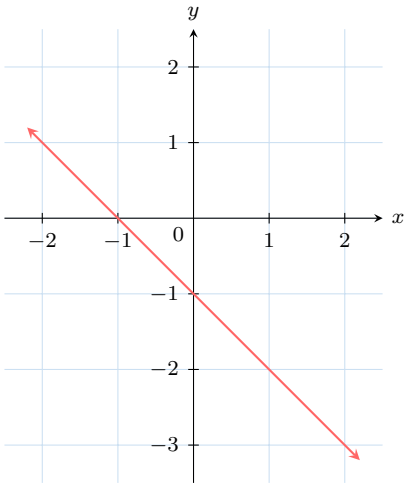
Ex 41: The graph of $y = f(x)$ is:



$f(1) =$ ☐

D.2 FINDING x SUCH THAT $f(x) = y$

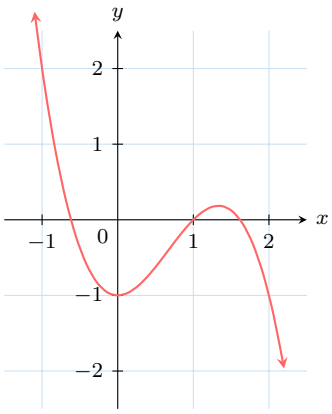
Ex 42: The graph of $y = f(x)$ is:



Find x such that $f(x) = -2$.

$x =$ ☐

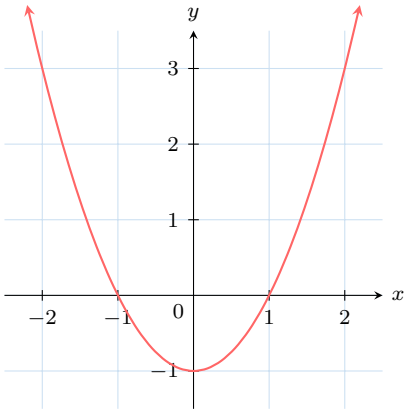
Ex 43: The graph of $y = f(x)$ is:



Find x such that $f(x) = 2$.

$x =$ ☐

Ex 44: The graph of $y = f(x)$ is:

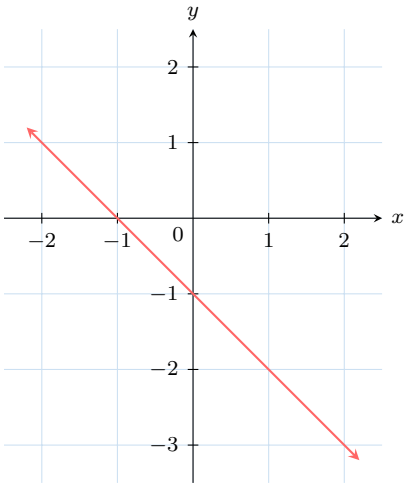


Find all x such that $f(x) = 3$.

Give your answers in increasing order:

$x =$ ☐ or $x =$ ☐

Ex 45: The graph of $y = f(x)$ is:



Find x such that $f(x) = 1$.

$x =$ ☐

E SOLVING $f(x) = y$ ALGEBRAICALLY

E.1 SOLVING LINEAR EQUATIONS FOR $f(x) = y$

Ex 46: Let $f(x) = 3x + 12$. Find all x such that $f(x) = 0$. Justify your answer.

Ex 47: Let $f(x) = 2x - 18$. Find all x such that $f(x) = 0$. Justify your answer.

Ex 48: Let $f(x) = 2x + 20$. Find all x such that $f(x) = 10$. Justify your answer.

Ex 49: Let $f(x) = -6x + 7$. Find all x such that $f(x) = 2$. Justify your answer.

E.2 ANALYZING LINEAR MODELS IN CONTEXT

Ex 50: The value of a laptop t years after purchase is given by $V(t) = 1800 - 300t$ dollars.

1. Find $V(3)$

State what this value means
- ☐ The original purchase price is \$900.
- ☐ The laptop depreciates by \$900 per year.
- ☐ The value of the laptop after 3 years is \$900.

2. Find t when $V(t) = 600$.

Explain what this represents.
- ☐ After 4 years, the laptop is worth \$600.
- ☐ The depreciation rate is \$4 per year.
- ☐ The original price was \$600 after 4 years.

3. Find the original purchase price of the laptop.

Ex 51: The height of a plant t weeks after planting is given by $H(t) = 5 + 2t$ cm.

1. Find $H(4)$

State what this value means
- ☐ The initial height is 13 cm.
- ☐ The plant grows by 13 cm per week.
- ☐ The height of the plant after 4 weeks is 13 cm.

2. Find t when $H(t) = 15$.

Explain what this represents.
☐ After 5 weeks, the plant is 15 cm tall.
☐ The growth rate is 5 cm per week.
☐ The initial height was 15 cm after 5 weeks.

3. Find the initial height of the plant.

Ex 52: The temperature of water t minutes after starting to heat it is given by $T(t) = 25 + 15t^\circ$ degrees Celsius.

1. Find $T(3)$

State what this value means
☐ The temperature of the water after 3 minutes is 70°C.
☐ The initial temperature is 70°C.
☐ The water heats up by 70 degrees per minute.

2. Find t when $T(t) = 100$.

Explain what this represents.
☐ The water is at 100°C after 100 minutes.
☐ The heating rate is 5 degrees per minute.
☐ After 5 minutes, the water reaches boiling point at 100°C.

3. Find the initial temperature of the water.

F DOMAIN

F.1 FINDING DOMAINS: LEVEL 1

MCQ 53: Find the domain of the function $f : x \mapsto x^2$.
☐ $\mathbb{R}$
☐ $\{x \in \mathbb{R} \mid x \neq 0\}$
☐ $[0, +\infty)$
☐ $(-\infty, 0)$

MCQ 54: Find the domain of the function $f : x \mapsto \dfrac{1}{x}$.
☐ $\mathbb{R}$
☐ $\{x \in \mathbb{R} \mid x \neq 0\}$
☐ $[0, +\infty)$
☐ $(-\infty, 0)$

MCQ 55: Find the domain of the function $f : x \mapsto \sqrt{x}$.
☐ $\mathbb{R}$
☐ $\{x \in \mathbb{R} \mid x \neq 0\}$
☐ $[0, +\infty)$
☐ $(-\infty, 0)$

F.2 FINDING DOMAINS: LEVEL 2

MCQ 56: Find the domain of the function $f : x \mapsto \sqrt{2x - 4}$.
☐ $\mathbb{R}$
☐ $\{x \in \mathbb{R} \mid x \neq 4\}$
☐ $[2, +\infty)$
☐ $(-\infty, 4]$

MCQ 57: Find the domain of the function $f : x \mapsto \dfrac{x}{x - 3}$.
☐ $\mathbb{R}$
☐ $\{x \in \mathbb{R} \mid x \neq 3 \text{ and } x \neq 0\}$
☐ $[3, +\infty)$
☐ $(-\infty, 3)$
☐ $\{x \in \mathbb{R} \mid x \neq 3\}$

MCQ 58: Find the domain of the function $f : x \mapsto \dfrac{1}{x^2 - 9}$.
☐ $\mathbb{R}$
☐ $(-3, 3)$
☐ $[0, +\infty)$
☐ $\{x \in \mathbb{R} \mid x \neq -3 \text{ and } x \neq 3\}$
☐ $x > 3$

MCQ 59: Find the domain of the function $f : x \mapsto \sqrt{6 - 2x}$.
☐ $\mathbb{R}$
☐ $(-\infty, 3]$
☐ $[3, +\infty)$
☐ $(-\infty, 6]$

G ALGEBRA OF FUNCTIONS

G.1 ADDING, SUBTRACTING, AND MULTIPLYING FUNCTIONS

Ex 60: For $f(x) = 2x + 2$ and $g(x) = 3 - x$, find in simplest form:

1. $f(3) + g(3) =$
2. $f(-1) + g(-1) =$
3. $f(x) + g(x) =$
4. $g(x) + f(x) =$

Ex 61: For $f(x) = x^2 - 2$ and $g(x) = x - 2$, find in simplest form:

1. $f(0) + g(0) =$

2. $f(-2)+g(-2)=$ $\square$

3. $f(x)+g(x)=$ $\square$

4. $f(x)-g(x)=$ $\square$

Ex 62: Let $f(x)=3x-2$ and $g(x)=x^2$. Find in factorized form:

$$f(x)\times g(x)=\square$$

Ex 63: Let $f(x)=2x+5$ and $g(x)=x-4$. Find in factorized form:

$$f(x)\times g(x)=\square$$

G.2 DECOMPOSING EXPRESSIONS INTO FUNCTIONS

Ex 64: Find two functions f and g such that $f(x)\times g(x)=(x+3)^2(x-2)$.

- $f(x)=$ $\square$
- $g(x)=$ $\square$

Ex 65: Find two functions f and g such that $f(x)\times g(x)=(x^2+4)(3x-7)$.

- $f(x)=$ $\square$
- $g(x)=$ $\square$

Ex 66: Find two functions f and g such that $f(x)+g(x)=(x-2)^2+\sqrt{x}$.

- $f(x)=$ $\square$
- $g(x)=$ $\square$

Ex 67: Find two functions f and g such that $f(x)+g(x)=\dfrac{1}{x}+(x+1)^2$.

- $f(x)=$ $\square$
- $g(x)=$ $\square$

H COMPOSITION

H.1 EVALUATING COMPOSITE FUNCTIONS

Ex 68: For $f(x)=2x+2$ and $g(x)=3-x$, find in simplest form:

1. $f(g(3))=$ $\square$
2. $f(g(-1))=$ $\square$
3. $f(g(x))=$ $\square$
4. $g(f(x))=$ $\square$

Ex 69: For $f(x)=x^2+2x$ and $g(x)=2-x$, find in simplest form:

1. $f(g(3))=$ $\square$
2. $f(g(-1))=$ $\square$
3. $f(g(x))=$ $\square$
4. $g(f(x))=$ $\square$

Ex 70: For $f(x)=3x-5$, find in simplest form:

1. $f(f(-1))=$ $\square$
2. $f(f(x))=$ $\square$

H.2 DECOMPOSING FUNCTIONS INTO COMPOSITIONS

Ex 71: Find two functions f and g such that $f(g(x))=\sqrt{2x-1}$ and $g(x)\neq x$.

- $f(x)=$ $\square$
- $g(x)=$ $\square$

Ex 72: Find two functions f and g such that $f(g(x))=(x+2)^5$ and $g(x)\neq x$.

- $f(x)=$ $\square$
- $g(x)=$ $\square$

Ex 73: Find two functions f and g such that $f(g(x))=\dfrac{1}{x^2+1}$ and $g(x)\neq x$.

- $f(x)=$ $\square$
- $g(x)=$ $\square$

Ex 74: Find two functions f and g such that $f(g(x)) = (x^3 - 2)^{-4}$ and $g(x) \neq x$.

- $f(x) =$ ☐
- $g(x) =$ ☐

I INVERSE FUNCTION

I.1 FINDING AND CHECKING INVERSES

Ex 75:

1. Find the inverse of $f(x) = x + 3$.

$$f^{-1}(x) = \square$$

2. Evaluate

$$f^{-1}(f(x)) = \square$$
$$f(f^{-1}(x)) = \square$$

Ex 76:

1. Find the inverse of $f(x) = 4x - 8$.

$$f^{-1}(x) = \square$$

2. Evaluate

$$f^{-1}(f(x)) = \square$$
$$f(f^{-1}(x)) = \square$$

Ex 77:

1. Find the inverse of $f(x) = \dfrac{x}{2} - 3$.

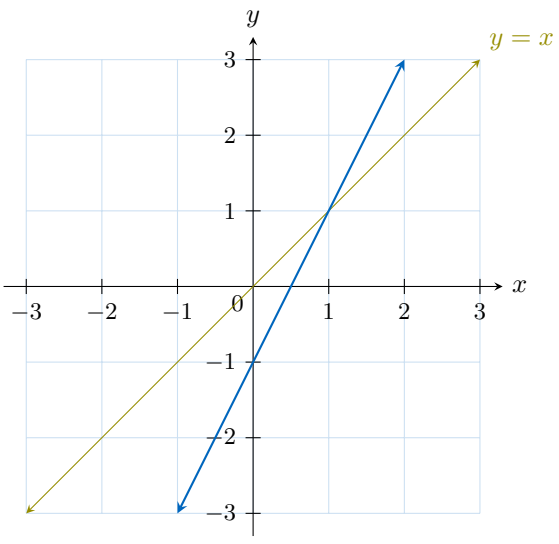
$$f^{-1}(x) = \square$$

2. Evaluate

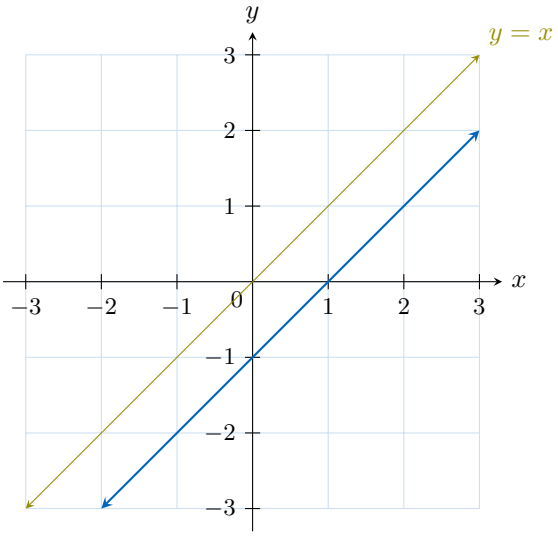
$$f^{-1}(f(x)) = \square$$
$$f(f^{-1}(x)) = \square$$

I.2 GRAPHING THE INVERSE FUNCTION BY REFLECTION

Ex 78: Draw the graph of the inverse function of the blue graph:



Ex 79: Draw the graph of the inverse function of the blue graph:



Ex 80: Draw the graph of the inverse function of the blue graph: