FUNCTIONS

A DEFINITIONS

A.1 WRITING FUNCTIONS: LEVEL 1

Ex 1: Consider the following calculation program:

1. Choose a number.
2. Subtract 5 from the chosen number.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \boxed{x - 5}$$

Answer: Given the following program:

1. Choose a number: x.
2. Subtract 5 from the chosen number: $x - 5$.

Thus, the function is:

$$f(x) = x - 5$$

Ex 2: Consider the following calculation program:

1. Choose a number.
2. Multiply the chosen number by three.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \boxed{3x}$$

Answer: Given the following program:

1. Choose a number: x.
2. Multiply the chosen number by three: $3x$.

Thus, the function is:

$$f(x) = 3x$$

Ex 3: Consider the following calculation program:

1. Choose a number.
2. Multiply the chosen number by five.
3. Subtract 2 from the result obtained.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \boxed{5x - 2}$$

Answer: Given the following program:

1. Choose a number: x.
2. Multiply the chosen number by five: $5x$.
3. Subtract 2 from the result obtained: $5x - 2$.

Thus, the function is:

$$f(x) = 5x - 2$$

Ex 4: Consider the following calculation program:

1. Choose a number.
2. Multiply the chosen number by -2.
3. Add 5 to the result obtained.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \boxed{-2x + 5}$$

Answer: Given the following program:

1. Choose a number: x.
2. Multiply the chosen number by -2: $-2x$.
3. Add 5 to the result obtained: $-2x + 5$.

Thus, the function is:

$$f(x) = -2x + 5$$

A.2 WRITING FUNCTIONS: LEVEL 2

Ex 5: Consider the following calculation program:

1. Choose a number.
2. Multiply the chosen number by itself.
3. Subtract 1 from the result obtained.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \boxed{x^2 - 1}$$

Answer: Given the following program:

1. Choose a number: x.
2. Multiply the chosen number by itself: x^2.
3. Subtract 1 from the result obtained: $x^2 - 1$.

Thus, the function is:

$$f(x) = x^2 - 1$$

Ex 6: Consider the following calculation program:

1. Choose a number.
2. Square the chosen number.
3. Multiply the result by 2.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \boxed{2x^2}$$

Answer: Given the following program:

1. Choose a number: x.
2. Square the chosen number: x^2.
3. Multiply the result by 2: $2x^2$.

Thus, the function is:

$$f(x) = 2x^2$$

Ex 7: Consider the following calculation program:

1. Choose a number.
2. Subtract 1 from the chosen number.
3. Multiply the result by the original number chosen.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \boxed{(x-1)x}$$

Answer: Given the following program:

1. Choose a number: x.
2. Subtract 1 from the chosen number: $x - 1$.
3. Multiply the result by the original number: $(x-1)x$.

Thus, the function is:

$$f(x) = (x-1)x$$

A.3 CALCULATING $f(x)$

Ex 8: For $f(x) = x + 3$,

$$f(4) = \boxed{7}$$

Answer:

$$\begin{aligned} f(4) &= (4) + 3 \quad \text{(substituting } x \text{ with (4))} \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

Ex 9: For $f(x) = 2x - 1$,

$$f(5) = \boxed{9}$$

Answer:

$$\begin{aligned} f(5) &= 2 \times (5) - 1 \quad \text{(substituting } x \text{ with (5))} \\ &= 10 - 1 \\ &= 9 \end{aligned}$$

Ex 10: For $f(x) = 3x + 2$,

$$f(2) = \boxed{8}$$

Answer:

$$\begin{aligned} f(2) &= 3 \times (2) + 2 \quad \text{(substituting } x \text{ with (2))} \\ &= 6 + 2 \\ &= 8 \end{aligned}$$

Ex 11: For $f(x) = x^2 - 1$,

$$f(3) = \boxed{8}$$

Answer:

$$\begin{aligned} f(3) &= (3)^2 - 1 \quad \text{(substituting } x \text{ with (3))} \\ &= 9 - 1 \\ &= 8 \end{aligned}$$

Ex 12: For $f(x) = 5x - 3$,

$$f(1) = \boxed{2}$$

Answer:

$$\begin{aligned} f(1) &= 5 \times (1) - 3 \quad \text{(substituting } x \text{ with (1))} \\ &= 5 - 3 \\ &= 2 \end{aligned}$$

Ex 13: For $f(x) = \frac{x}{2} + 4$,

$$f(6) = \boxed{7}$$

Answer:

$$\begin{aligned} f(6) &= \frac{(6)}{2} + 4 \quad \text{(substituting } x \text{ with (6))} \\ &= 3 + 4 \\ &= 7 \end{aligned}$$

Ex 14: For $f(x) = x - 5$,

$$f(10) = \boxed{5}$$

Answer:

$$\begin{aligned} f(10) &= (10) - 5 \quad \text{(substituting } x \text{ with (10))} \\ &= 10 - 5 \\ &= 5 \end{aligned}$$

Ex 15: For $f(x) = 2x - 5$,

$$f(-2) = \boxed{-9}$$

Answer:

$$\begin{aligned} f(-2) &= 2 \times (-2) - 5 \quad \text{(substituting } x \text{ with (-2))} \\ &= -4 - 5 \\ &= -9 \end{aligned}$$

Ex 16: For $f(x) = -x + 4$,

$$f(-3) = \boxed{7}$$

Answer:

$$\begin{aligned} f(-3) &= -(-3) + 4 \quad \text{(substituting } x \text{ with (-3))} \\ &= 3 + 4 \\ &= 7 \end{aligned}$$

Ex 17: For $f(x) = 3x - 7$,

$$f(-1) = \boxed{-10}$$

Answer:

$$\begin{aligned} f(-1) &= 3\times(-1)-7 \quad \text{(substituting } x \text{ with } (-1))\\ &= -3-7\\ &= -10 \end{aligned}$$

Ex 18: For $f(x) = x^2 - 2x$,

$$f(-2) = \boxed{8}$$

Answer:

$$\begin{aligned} f(-2) &= (-2)^2 - 2\times(-2) \quad \text{(substituting } x \text{ with } (-2))\\ &= 4+4\\ &= 8 \end{aligned}$$

Ex 19: For $f(x) = 2x + 3$,

$$f(-3) = \boxed{-3}$$

Answer:

$$\begin{aligned} f(-3) &= 2\times(-3)+3 \quad \text{(substituting } x \text{ with } (-3))\\ &= -6+3\\ &= -3 \end{aligned}$$

Ex 20: For $f(x) = \frac{x}{2} - 4$,

$$f(8) = \boxed{0}$$

Answer:

$$\begin{aligned} f(8) &= \frac{(8)}{2} - 4 \quad \text{(substituting } x \text{ with } (8))\\ &= 4-4\\ &= 0 \end{aligned}$$

Ex 21: For $f(x) = \frac{3x-5}{2}$,

$$f(-1) = \boxed{-4}$$

Answer:

$$\begin{aligned} f(-1) &= \frac{3\times(-1)-5}{2} \quad \text{(substituting } x \text{ with } (-1))\\ &= \frac{-3-5}{2}\\ &= \frac{-8}{2}\\ &= -4 \end{aligned}$$

Ex 22: For $f(x) = \frac{x-6}{2} - 3$,

$$f(10) = \boxed{-1}$$

Answer:

$$\begin{aligned} f(10) &= \frac{(10)-6}{2} - 3 \quad \text{(substituting } x \text{ with } (10))\\ &= \frac{4}{2} - 3\\ &= 2-3\\ &= -1 \end{aligned}$$

A.4 CALCULATING $f(x)$

Ex 23: For $f : x \mapsto x + 3$,

$$f(4) = \boxed{7}$$

Answer:

$$\begin{aligned} f(4) &= (4)+3 \quad \text{(substituting } x \text{ with } (4))\\ &= 4+3\\ &= 7 \end{aligned}$$

Ex 24: For $f : x \mapsto x^2 - 1$,

$$f(2) = \boxed{3}$$

Answer:

$$\begin{aligned} f(2) &= (2)^2 - 1 \quad \text{(substituting } x \text{ with } (2))\\ &= 4-1\\ &= 3 \end{aligned}$$

Ex 25: For $f : x \mapsto (x-1)(x-2)$,

$$f(0) = \boxed{2}$$

Answer:

$$\begin{aligned} f(0) &= (0-1)(0-2) \quad \text{(substituting } x \text{ with } (0))\\ &= (-1)\times(-2)\\ &= 2 \end{aligned}$$

Ex 26: For $f : x \mapsto x^3$,

$$f(-1) = \boxed{-1}$$

Answer:

$$\begin{aligned} f(-1) &= (-1)^3 \quad \text{(substituting } x \text{ with } (-1))\\ &= -1 \end{aligned}$$

A.5 SUBSTITUTING VALUES AND EXPRESSIONS INTO A FUNCTION

Ex 27: For $f : x \mapsto 1 - 3x$, find in simplest form:

1. $f(-2) = \boxed{7}$
2. $f(3) = \boxed{-8}$
3. $f(x+1) = \boxed{-3x-2}$
4. $f(x^2) = \boxed{1-3x^2}$

Answer:

1. $$\begin{aligned} f(-2) &= 1 - 3\times(-2) \quad \text{(substituting } x \text{ with } -2)\\ &= 1+6\\ &= 7 \end{aligned}$$
2. $$\begin{aligned} f(3) &= 1 - 3\times 3 \quad \text{(substituting } x \text{ with } 3)\\ &= 1-9\\ &= -8 \end{aligned}$$

3. $f(x+1) = 1-3(x+1)$ (substituting x with $(x+1)$)
$= 1-3x-3$ (expand)
$= -3x-2$

4. $f(x^2) = 1-3(x^2)$ (substituting x with (x^2))
$= 1-3x^2$

Ex 28: For $f : x \mapsto x^2$, find in simplest form:

1. $f(3) = \boxed{9}$
2. $f(-1) = \boxed{1}$
3. $f(-x) = \boxed{x^2}$
4. $f(x+1) = \boxed{x^2+2x+1}$
5. $f(x+2) = \boxed{x^2+4x+4}$
6. $f(2x) = \boxed{4x^2}$

Answer:

1. $f(3) = 3^2 = 9$
2. $f(-1) = (-1)^2 = 1$
3. $f(-x) = (-x)^2$ (substituting x with $(-x)$)
$= (-1)^2x^2$
$= x^2$
4. $f(x+1) = (x+1)^2$ (substituting x with $(x+1)$)
$= x^2+2x+1$ (binomial expansion)
5. $f(x+2) = (x+2)^2$ (substituting x with $(x+2)$)
$= x^2+4x+4$ (binomial expansion)
6. $f(2x) = (2x)^2$ (substituting x with $(2x)$)
$= 4x^2$

Ex 29: For $g : x \mapsto x^2-2x+1$, find in simplest form:

1. $g(3) = \boxed{4}$
2. $g(-1) = \boxed{4}$
3. $g(-x) = \boxed{x^2+2x+1}$
4. $g(x+1) = \boxed{x^2}$
5. $g(x+2) = \boxed{x^2+2x+1}$
6. $g(2x) = \boxed{4x^2-4x+1}$

Answer:

1. $g(3) = (3)^2 - 2\times(3)+1$ (substituting x with 3)
$= 9-6+1$ (evaluate)
$= 4$
2. $g(-1) = (-1)^2-2\times(-1)+1$ (substituting x with -1)
$= 1+2+1$ (evaluate)
$= 4$
3. $g(-x) = (-x)^2-2\times(-x)+1$ (substituting x with $(-x)$)
$= x^2+2x+1$ (expand)
4. $g(x+1) = (x+1)^2-2(x+1)+1$ (substituting x with $(x+1$
$= (x^2+2x+1)-(2x+2)+1$ (expand)
$= x^2+2x+1-2x-2+1$ (combine)
$= x^2$
5. $g(x+2) = (x+2)^2-2(x+2)+1$ (substituting x with $(x+2$
$= (x^2+4x+4)-(2x+4)+1$ (expand)
$= x^2+4x+4-2x-4+1$ (combine)
$= x^2+2x+1$
6. $g(2x) = (2x)^2-2\times(2x)+1$ (substituting x with $(2x)$)
$= 4x^2-4x+1$ (expand)

B TABLES OF VALUES

B.1 FILLING TABLES OF VALUES

Ex 30: For $f(x) = x^2$, fill in the table of values:

x	-2	-1	0	1	2
$f(x)$	$\boxed{4}$	$\boxed{1}$	$\boxed{0}$	$\boxed{1}$	$\boxed{4}$

Answer:

- $f(-2) = ((-2))^2$ (substituting x with (-2))
$= 4$
- $f(-1) = ((-1))^2$ (substituting x with (-1))
$= 1$
- $f(0) = (0)^2$ (substituting x with (0))
$= 0$
- $f(1) = (1)^2$ (substituting x with (1))
$= 1$
- $f(2) = (2)^2$ (substituting x with (2))
$= 4$

So the table of values is:

x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4

Ex 31: For $f(x) = -2x+1$, fill in the table:

x	-2	-1	0	1	2
$f(x)$	$\boxed{5}$	$\boxed{3}$	$\boxed{1}$	$\boxed{-1}$	$\boxed{-3}$

Answer:

- $f(-2) = -2\times(-2)+1$ (substituting x with (-2))
$= 4+1$
$= 5$
- $f(-1) = -2\times(-1)+1$ (substituting x with (-1))
$= 2+1$
$= 3$

- $f(0) = -2 \times (0) + 1 \quad$ (substituting x with (0))
  $= 0 + 1$
  $= 1$

- $f(1) = -2 \times (1) + 1 \quad$ (substituting x with (1))
  $= -2 + 1$
  $= -1$

- $f(2) = -2 \times (2) + 1 \quad$ (substituting x with (2))
  $= -4 + 1$
  $= -3$

So the table of values is:

x	-2	-1	0	1	2
$f(x)$	5	3	1	-1	-3

Ex 32: For $f(x) = x^2 - 3x + 1$, fill in the table:

x	-2	-1	0	1	2
$f(x)$	11	5	1	-1	-1

Answer:

- $f(-2) = ((-2))^2 - 3 \times (-2) + 1 \quad$ (substituting x with (-2))
  $= 4 + 6 + 1$
  $= 11$

- $f(-1) = ((-1))^2 - 3 \times (-1) + 1 \quad$ (substituting x with (-1))
  $= 1 + 3 + 1$
  $= 5$

- $f(0) = (0)^2 - 3 \times (0) + 1 \quad$ (substituting x with (0))
  $= 0 + 0 + 1$
  $= 1$

- $f(1) = (1)^2 - 3 \times (1) + 1 \quad$ (substituting x with (1))
  $= 1 - 3 + 1$
  $= -1$

- $f(2) = (2)^2 - 3 \times (2) + 1 \quad$ (substituting x with (2))
  $= 4 - 6 + 1$
  $= -1$

So the table of values is:

x	-2	-1	0	1	2
$f(x)$	11	5	1	-1	-1

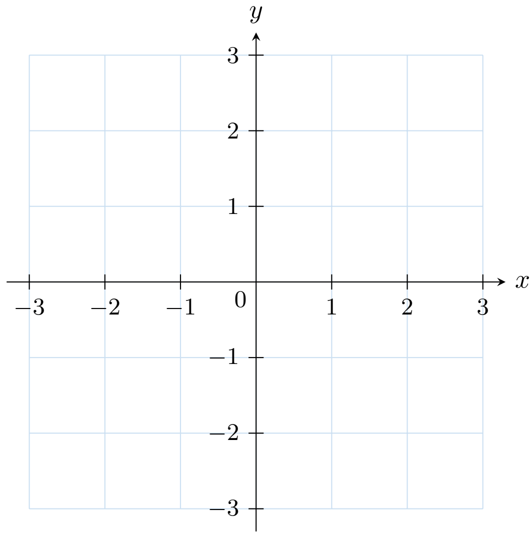
C GRAPHS

C.1 PLOTTING LINE GRAPHS

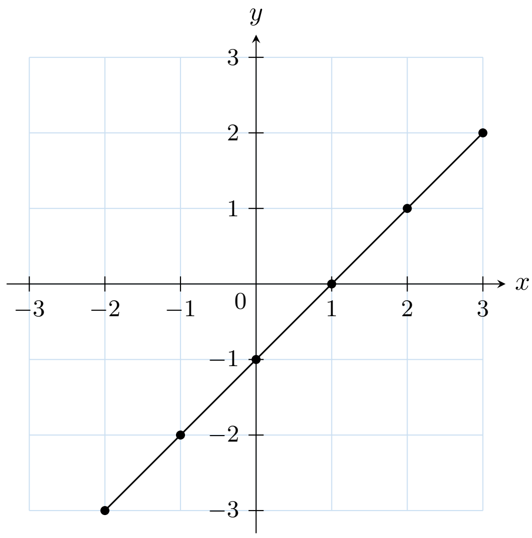
Ex 33: Here is a table of values for the function $f(x) = x - 1$:

x	-2	-1	0	1	2	3
$f(x)$	-3	-2	-1	0	1	2

Plot the line graph of f.



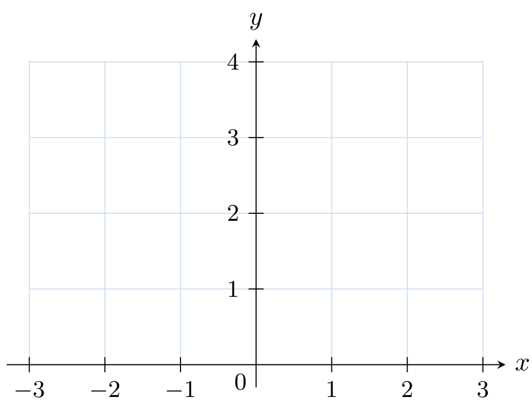
Answer: Plot the points $(-2, -3)$, $(-1, -2)$, $(0, -1)$, $(1, 0)$, $(2, 1)$, and $(3, 2)$. Then, connect the points with straight segments to form the line graph.



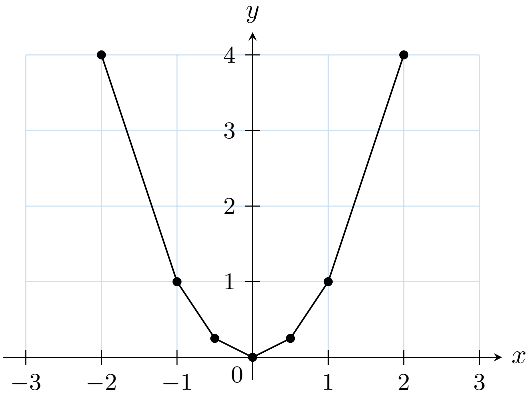
Ex 34: Here is a table of values for the function $f(x) = x^2$:

x	-2	-1	-0.5	0	0.5	1	2
$f(x)$	4	1	0.25	0	0.25	1	4

Plot the line graph of f.



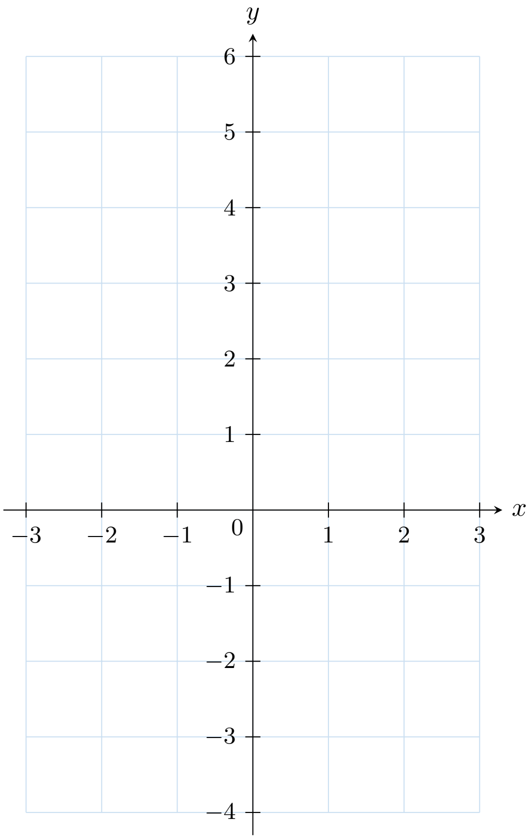
Answer: Plot the points $(-2, 4)$, $(-1, 1)$, $(-0.5, 0.25)$, $(0, 0)$, $(0.5, 0.25)$, $(1, 1)$, and $(2, 4)$. Then, connect the points with straight segments.



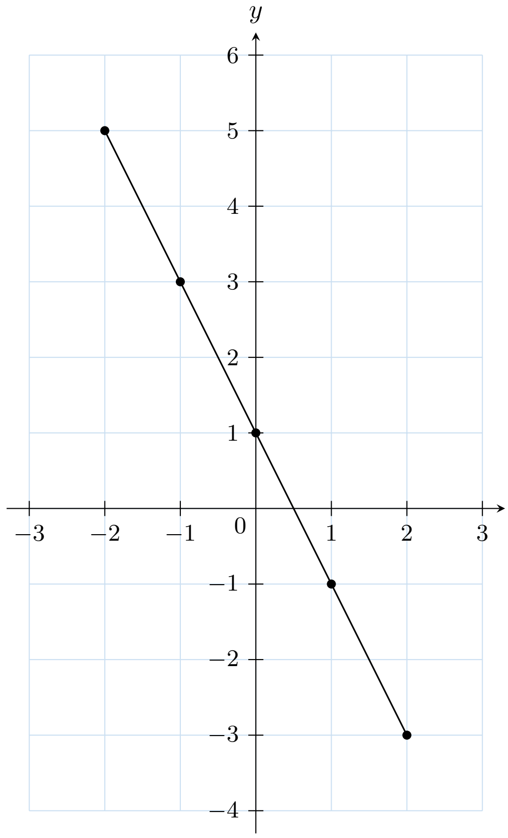
Ex 35: Here is a table of values for the function $f(x) = -2x+1$:

x	-2	-1	0	1	2
$f(x)$	5	3	1	-1	-3

Plot the line graph of f.



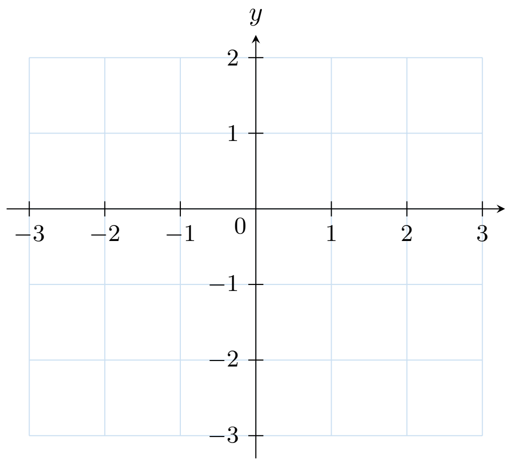
Answer: Plot the points $(-2,5)$, $(-1,3)$, $(0,1)$, $(1,-1)$, $(2,-3)$. Then, connect the points with straight segments to form the line graph.



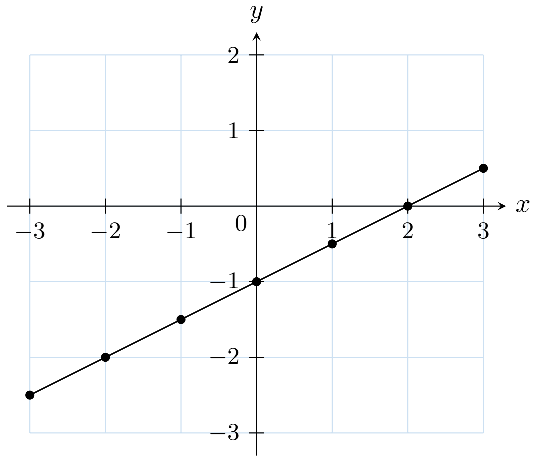
Ex 36: Here is a table of values for the function $f(x) = 0.5x-1$:

x	-3	-2	-1	0	1	2	3
$f(x)$	-2.5	-2	-1.5	-1	-0.5	0	0.5

Plot the line graph of f.



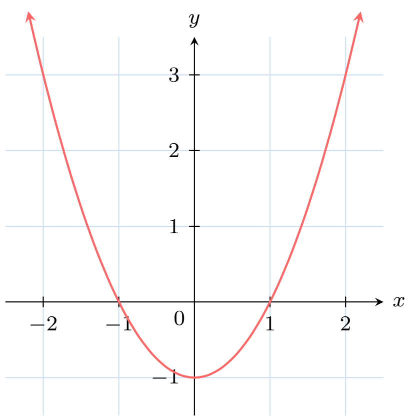
Answer: Plot the points $(-3,-2.5)$, $(-2,-2)$, $(-1,-1.5)$, $(0,-1)$, $(1,-0.5)$, $(2,0)$, $(3,0.5)$. Then, connect the points with straight segments to form the line graph.



D READING VALUES AND SOLVING $f(x) = y$ ON A GRAPH

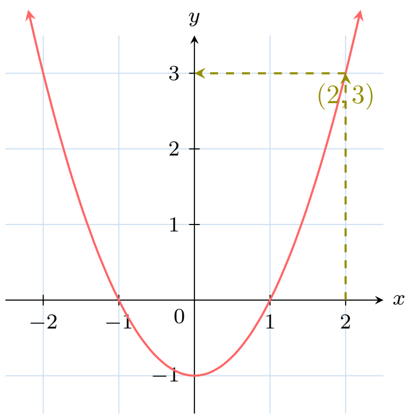
D.1 FINDING $f(x)$

Ex 37: The graph of $y = f(x)$ is:



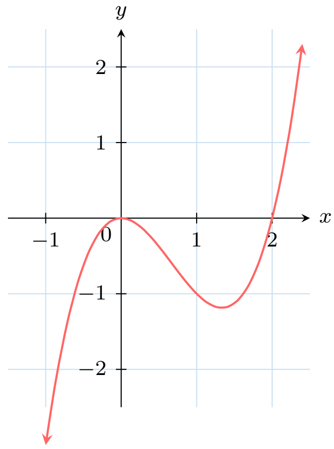
$f(2) = \boxed{3}$

Answer:



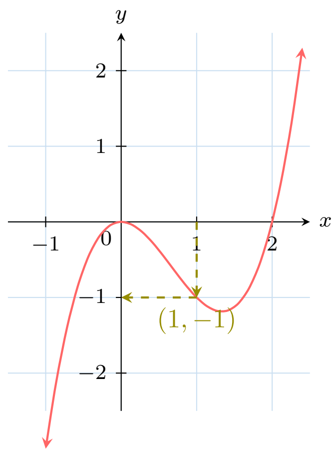
$f(2) = 3$

Ex 38: The graph of $y = f(x)$ is:



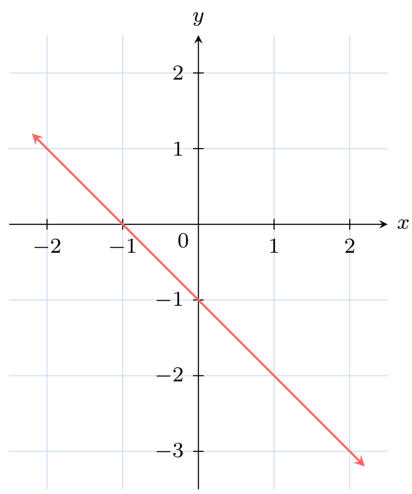
$f(1) = \boxed{-1}$

Answer:



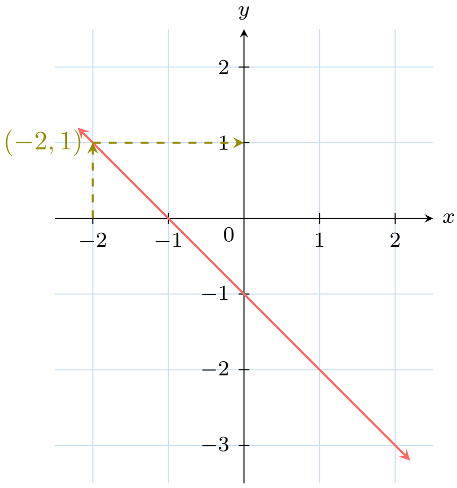
$f(1) = -1$

Ex 39: The graph of $y = f(x)$ is:



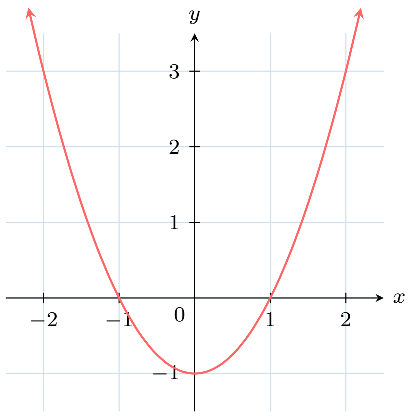
$f(-2) = \boxed{1}$

Answer:



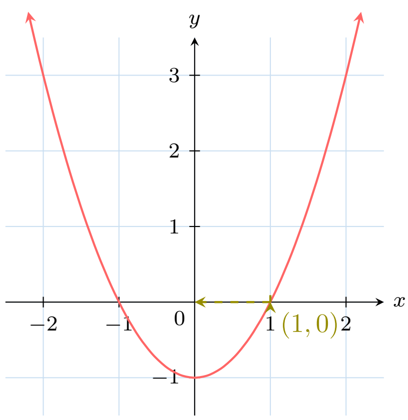
$f(-2) = 1$

Ex 40: The graph of $y = f(x)$ is:



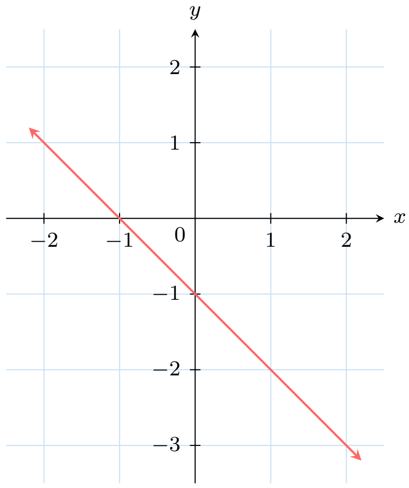
$f(1) = \boxed{0}$

Answer:



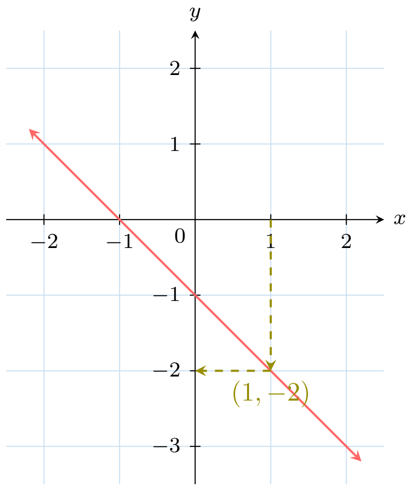
$f(1) = 0$

Ex 41: The graph of $y = f(x)$ is:



$f(1) = \boxed{-2}$

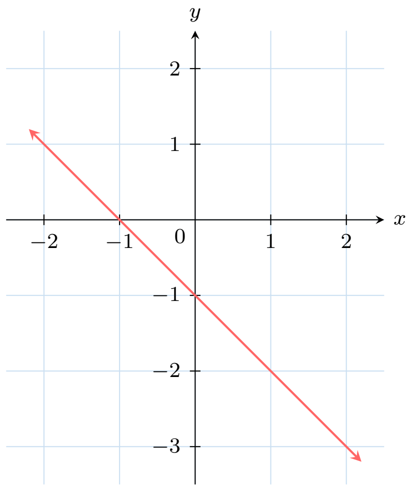
Answer:



$f(1) = -2$

D.2 FINDING x SUCH THAT $f(x) = y$

Ex 42: The graph of $y = f(x)$ is:

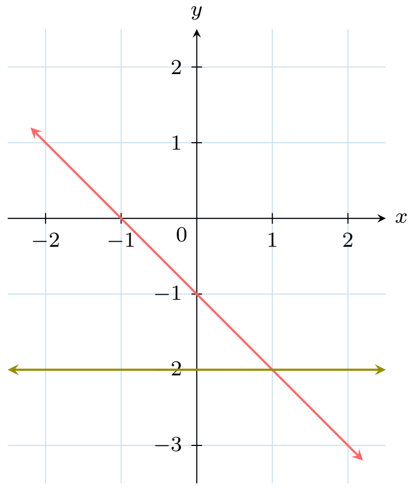


Find x such that $f(x) = -2$.

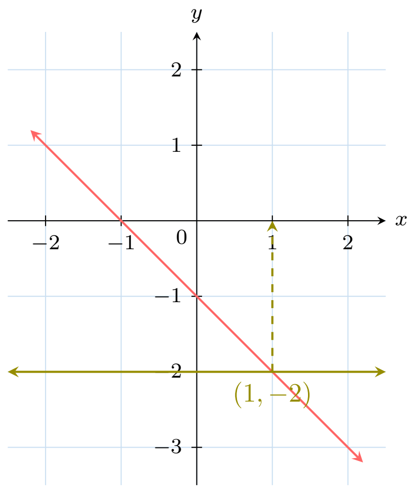
$x = \boxed{1}$

Answer:

- Draw a horizontal line at $y = -2$.

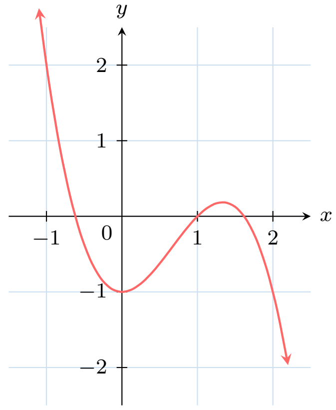


- Identify the intersection point with the curve.



$x = 1$

Ex 43: The graph of $y = f(x)$ is:

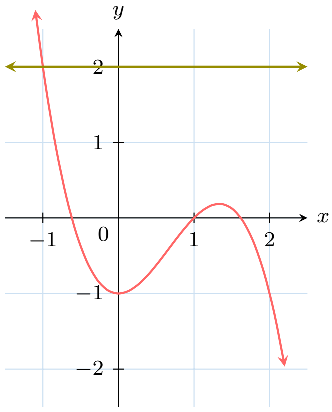


Find x such that $f(x) = 2$.

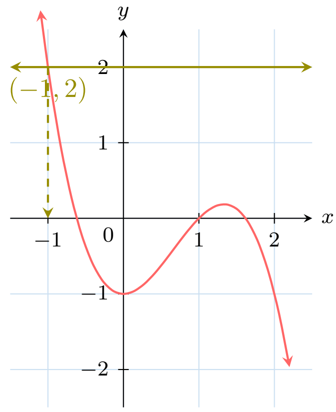
$x = \boxed{-1}$

Answer:

- Draw a horizontal line at $y = 2$.

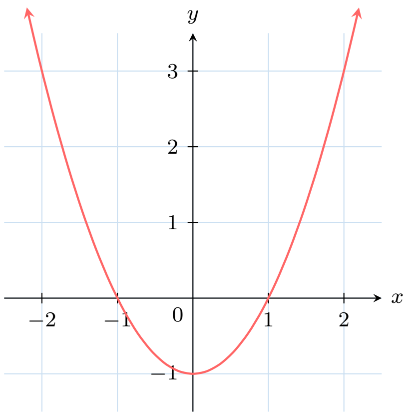


- Identify the intersection point with the curve.



$x = -1$

Ex 44: The graph of $y = f(x)$ is:



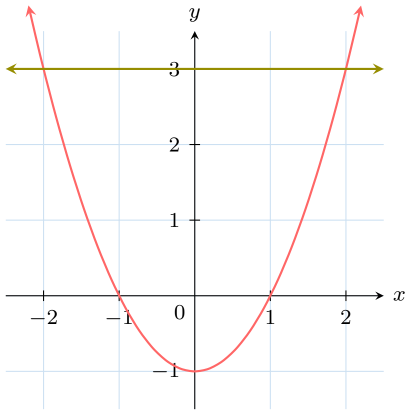
Find all x such that $f(x) = 3$.

Give your answers in increasing order:

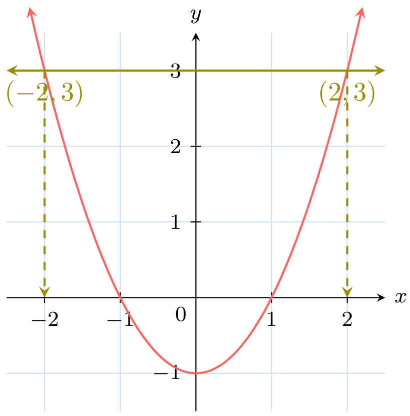
$x = \boxed{-2}$ or $x = \boxed{2}$

Answer:

- Draw a horizontal line at $y = 3$.

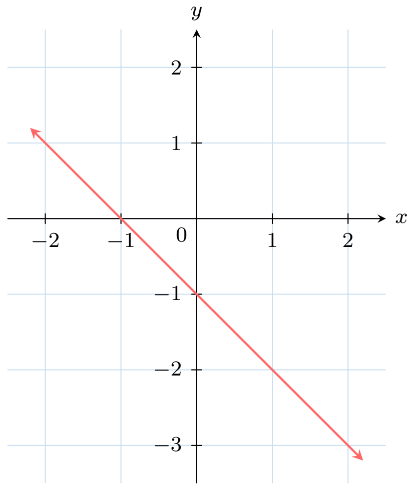


- Identify the intersection points with the curve.



$x = -2$ or $x = 2$

Ex 45: The graph of $y = f(x)$ is:

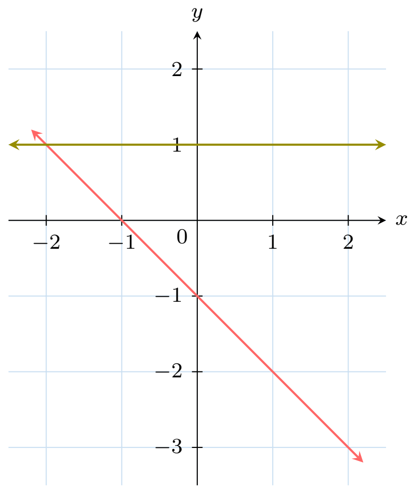


Find x such that $f(x) = 1$.

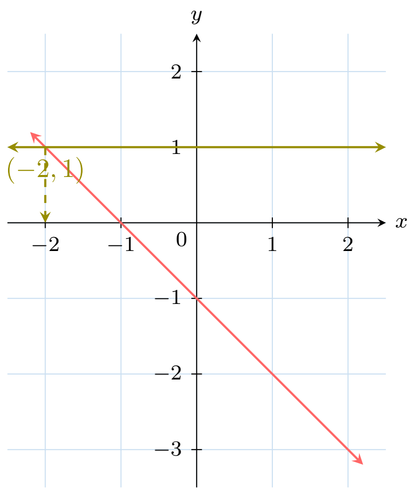
$$x = \boxed{-2}$$

Answer:

- Draw a horizontal line at $y = 1$.



- Identify the intersection point with the curve.



$x = -2$

E SOLVING $f(x) = y$ ALGEBRAICALLY

E.1 SOLVING LINEAR EQUATIONS FOR $f(x) = y$

Ex 46: Let $f(x) = 3x + 12$. Find all x such that $f(x) = 0$. Justify your answer.

Answer: We solve the equation:

$$\begin{aligned} f(x) &= 0 \\ 3x + 12 &= 0 \\ 3x &= -12 \quad \text{(subtract 12 from both sides)} \\ x &= -4 \quad \text{(divide both sides by 3)} \end{aligned}$$

So the solution is $x = -4$.
(Optional) We can check this by calculating $f(-4)$:

$$\begin{aligned} f(-4) &= 3 \times (-4) + 12 \\ &= -12 + 12 \\ &= 0 \end{aligned}$$

Ex 47: Let $f(x) = 2x - 18$. Find all x such that $f(x) = 0$. Justify your answer.

Answer: We solve the equation:

$$\begin{aligned}
f(x) &= 0 \\
2x - 18 &= 0 \\
2x - 18 + 18 &= 0 + 18 \quad \text{(add 18 to both sides)} \\
2x &= 18 \\
\frac{2x}{2} &= \frac{18}{2} \quad \text{(divide both sides by 2)} \\
x &= 9
\end{aligned}$$

So the solution is $x = 9$.*(Optional) We can check this by calculating $f(9)$:*

$$\begin{aligned}
f(9) &= 2 \times 9 - 18 \\
&= 18 - 18 \\
&= 0
\end{aligned}$$

Ex 48: Let $f(x) = 2x + 20$. Find all x such that $f(x) = 10$. Justify your answer.

Answer: We solve the equation:

$$\begin{aligned}
f(x) &= 10 \\
2x + 20 &= 10 \\
2x + 20 - 20 &= 10 - 20 \quad \text{(subtract 20 from both sides)} \\
2x &= -10 \\
\frac{2x}{2} &= \frac{-10}{2} \quad \text{(divide both sides by 2)} \\
x &= -5
\end{aligned}$$

So the solution is $x = -5$.*(Optional) We can check this by calculating $f(-5)$:*

$$\begin{aligned}
f(-5) &= 2 \times (-5) + 20 \\
&= -10 + 20 \\
&= 10
\end{aligned}$$

Ex 49: Let $f(x) = -6x + 7$. Find all x such that $f(x) = 2$. Justify your answer.

Answer: We solve the equation:

$$\begin{aligned}
f(x) &= 2 \\
-6x + 7 &= 2 \\
-6x + 7 - 7 &= 2 - 7 \quad \text{(subtract 7 from both sides)} \\
-6x &= -5 \\
\frac{-6x}{-6} &= \frac{-5}{-6} \quad \text{(divide both sides by } -6\text{)} \\
x &= \frac{5}{6}
\end{aligned}$$

So the solution is $x = \frac{5}{6}$.*(Optional) We can check this by calculating $f\left(\frac{5}{6}\right)$:*

$$\begin{aligned}
f\left(\frac{5}{6}\right) &= -6 \times \frac{5}{6} + 7 \\
&= -5 + 7 \\
&= 2
\end{aligned}$$

E.2 ANALYZING LINEAR MODELS IN CONTEXT

Ex 50: The value of a laptop t years after purchase is given by $V(t) = 1800 - 300t$ dollars.

1. Find $V(3)$

   $\boxed{900}$

   State what this value means
   The value of the laptop after 3 years is $900.

2. Find t when $V(t) = 600$.

   $\boxed{4}$

   Explain what this represents.
   After 4 years, the laptop is worth $600.

3. Find the original purchase price of the laptop.

   $\boxed{1800}$

Answer:

1. $V(3) = 1800 - 300 \times 3 = 1800 - 900 = 900$.
   This means the value of the laptop after 3 years is $900.

2. Solve $1800 - 300t = 600$:

$$\begin{aligned}
1800 - 300t &= 600 \\
1800 - 600 &= 300t \\
1200 &= 300t \\
t &= 4.
\end{aligned}$$

   This represents that after 4 years, the laptop is worth $600.

3. The original purchase price is $V(0) = 1800 - 300 \times 0 = 1800$ dollars.

Ex 51: The height of a plant t weeks after planting is given by $H(t) = 5 + 2t$ cm.

1. Find $H(4)$

   $\boxed{13}$

   State what this value means
   The height of the plant after 4 weeks is 13 cm.

2. Find t when $H(t) = 15$.

   $\boxed{5}$

   Explain what this represents.
   After 5 weeks, the plant is 15 cm tall.

3. Find the initial height of the plant.

   $\boxed{5}$

Answer:

1. $H(4) = 5 + 2 \times 4 = 5 + 8 = 13$.
   This means the height of the plant after 4 weeks is 13 cm.
2. Solve $5 + 2t = 15$:
$$\begin{aligned} 5 + 2t &= 15 \\ 2t &= 10 \\ t &= 5. \end{aligned}$$
   This represents that after 5 weeks, the plant is 15 cm tall.
3. The initial height is $H(0) = 5 + 2 \times 0 = 5$ cm.

Ex 52: The temperature of water t minutes after starting to heat it is given by $T(t) = 25 + 15t°$ degrees Celsius.

1. Find $T(3)$

   70

   State what this value means
   The temperature of the water after 3 minutes is 70°C.
2. Find t when $T(t) = 100$.

   5

   Explain what this represents.
   After 5 minutes, the water reaches boiling point at 100°C.
3. Find the initial temperature of the water.

   25

Answer:

1. $T(3) = 25 + 15 \times 3 = 25 + 45 = 70$.
   This means the temperature of the water after 3 minutes is 70°C.
2. Solve $25 + 15t = 100$:
$$\begin{aligned} 25 + 15t &= 100 \\ 15t &= 75 \\ t &= 5. \end{aligned}$$
   This represents that after 5 minutes, the water reaches boiling point at 100°C.
3. The initial temperature is $T(0) = 25 + 15 \times 0 = 25$°C.

F DOMAIN

F.1 FINDING DOMAINS: LEVEL 1

MCQ 53: Find the domain of the function $f : x \mapsto x^2$.

- ☒ $\mathbb{R}$
- ☐ $\{x \in \mathbb{R} \mid x \neq 0\}$
- ☐ $[0, +\infty)$
- ☐ $(-\infty, 0)$

Answer: The function $f(x) = x^2$ is defined for all real numbers because squaring any real number yields a real result. Therefore, the domain is all real numbers, which is $\mathbb{R}$.

MCQ 54: Find the domain of the function $f : x \mapsto \frac{1}{x}$.

- ☐ $\mathbb{R}$
- ☒ $\{x \in \mathbb{R} \mid x \neq 0\}$
- ☐ $[0, +\infty)$
- ☐ $(-\infty, 0)$

Answer: The function $f(x) = \frac{1}{x}$ is undefined at $x = 0$ because division by zero is not allowed. Therefore, the domain is all real numbers except 0, which is $\{x \in \mathbb{R} \mid x \neq 0\}$.

MCQ 55: Find the domain of the function $f : x \mapsto \sqrt{x}$.

- ☐ $\mathbb{R}$
- ☐ $\{x \in \mathbb{R} \mid x \neq 0\}$
- ☒ $[0, +\infty)$
- ☐ $(-\infty, 0)$

Answer: The function $f(x) = \sqrt{x}$ is undefined for negative real numbers because the square root of a negative number is not real. Therefore, the domain is all non-negative real numbers, which is $[0, +\infty)$.

F.2 FINDING DOMAINS: LEVEL 2

MCQ 56: Find the domain of the function $f : x \mapsto \sqrt{2x - 4}$.

- ☐ $\mathbb{R}$
- ☐ $\{x \in \mathbb{R} \mid x \neq 4\}$
- ☒ $[2, +\infty)$
- ☐ $(-\infty, 4]$

Answer: The function $f(x) = \sqrt{2x - 4}$ is undefined when the expression inside the square root is negative, i.e., when $2x-4 < 0$. Solving this inequality:

$$\begin{aligned} 2x - 4 &< 0 \\ 2x &< 4 \quad \text{(adding 4 to both sides)} \\ x &< 2 \quad \text{(dividing both sides by 2)} \end{aligned}$$

Therefore, the function is defined for $x \geq 2$, so the domain is $[2, +\infty)$.

MCQ 57: Find the domain of the function $f : x \mapsto \frac{x}{x - 3}$.

- ☐ $\mathbb{R}$
- ☐ $\{x \in \mathbb{R} \mid x \neq 3 \text{ and } x \neq 0\}$
- ☐ $[3, +\infty)$
- ☐ $(-\infty, 3)$
- ☒ $\{x \in \mathbb{R} \mid x \neq 3\}$

Answer: The function $f(x) = \dfrac{x}{x-3}$ is undefined when the denominator is zero, i.e., when $x - 3 = 0$. Solving this equation:

$$\begin{aligned} x - 3 &= 0 \\ x &= 3 \end{aligned}$$

Therefore, the function is defined for all real numbers except $x = 3$, so the domain is $\{x \in \mathbb{R} \mid x \neq 3\}$.

MCQ 58: Find the domain of the function $f : x \mapsto \dfrac{1}{x^2 - 9}$.

- ☐ $\mathbb{R}$
- ☐ $(-3, 3)$
- ☐ $[0, +\infty)$
- ☒ $\{x \in \mathbb{R} \mid x \neq -3 \text{ and } x \neq 3\}$
- ☐ $x > 3$

Answer: The function $f(x) = \dfrac{1}{x^2 - 9}$ is undefined when the denominator is zero, i.e., when $x^2 - 9 = 0$. Solving this equation:

$$\begin{aligned} x^2 - 9 &= 0 \\ x^2 &= 9 \\ x &= 3 \quad \text{or} \quad x = -3 \end{aligned}$$

Therefore, the function is defined for all real numbers except $x = 3$ and $x = -3$, so the domain is $\{x \in \mathbb{R} \mid x \neq 3 \text{ and } x \neq -3\}$.

MCQ 59: Find the domain of the function $f : x \mapsto \sqrt{6 - 2x}$.

- ☐ $\mathbb{R}$
- ☒ $(-\infty, 3]$
- ☐ $[3, +\infty)$
- ☐ $(-\infty, 6]$

Answer: The function $f(x) = \sqrt{6 - 2x}$ is undefined when the expression inside the square root is negative, i.e., when $6 - 2x < 0$. Solving this inequality:

$$\begin{aligned} 6 - 2x &< 0 \\ -2x &< -6 \quad \text{(subtract 6 from both sides)} \\ x &> 3 \quad \text{(divide both sides by -2, reverse the sign)} \end{aligned}$$

Therefore, the function is defined for $x \leq 3$, so the domain is $(-\infty, 3]$.

G ALGEBRA OF FUNCTIONS

G.1 ADDING, SUBTRACTING, AND MULTIPLYING FUNCTIONS

Ex 60: For $f(x) = 2x + 2$ and $g(x) = 3 - x$, find in simplest form:

1. $f(3) + g(3) = \boxed{8}$
2. $f(-1) + g(-1) = \boxed{4}$
3. $f(x) + g(x) = \boxed{x + 5}$
4. $g(x) + f(x) = \boxed{x + 5}$

Answer:

1. $$\begin{aligned} f(3) + g(3) &= (2 \times 3 + 2) + (3 - 3) \\ &= (6 + 2) + 0 \\ &= 8 + 0 \\ &= 8 \end{aligned}$$
2. $$\begin{aligned} f(-1) + g(-1) &= (2 \times (-1) + 2) + (3 - (-1)) \\ &= (-2 + 2) + (3 + 1) \\ &= 0 + 4 \\ &= 4 \end{aligned}$$
3. $$\begin{aligned} f(x) + g(x) &= (2x + 2) + (3 - x) \\ &= 2x + 2 + 3 - x \\ &= x + 5 \end{aligned}$$
4. $$\begin{aligned} g(x) + f(x) &= (3 - x) + (2x + 2) \\ &= 3 - x + 2x + 2 \\ &= x + 5 \end{aligned}$$

Ex 61: For $f(x) = x^2 - 2$ and $g(x) = x - 2$, find in simplest form:

1. $f(0) + g(0) = \boxed{-4}$
2. $f(-2) + g(-2) = \boxed{-2}$
3. $f(x) + g(x) = \boxed{x^2 + x - 4}$
4. $f(x) - g(x) = \boxed{x^2 - x}$

Answer:

1. $$\begin{aligned} f(0) + g(0) &= (0^2 - 2) + (0 - 2) \\ &= (-2) + (-2) \\ &= -4 \end{aligned}$$
2. $$\begin{aligned} f(-2) + g(-2) &= ((-2)^2 - 2) + (-2 - 2) \\ &= (4 - 2) + (-4) \\ &= 2 - 4 \\ &= -2 \end{aligned}$$
3. $$\begin{aligned} f(x) + g(x) &= (x^2 - 2) + (x - 2) \\ &= x^2 + x - 4 \end{aligned}$$
4. $$\begin{aligned} f(x) - g(x) &= (x^2 - 2) - (x - 2) \\ &= x^2 - 2 - x + 2 \\ &= x^2 - x \end{aligned}$$

Ex 62: Let $f(x) = 3x - 2$ and $g(x) = x^2$. Find in factorized form:

$$f(x) \times g(x) = \boxed{(3x - 2)x^2}$$

Answer: $$\begin{aligned} f(x) \times g(x) &= (3x - 2) \times x^2 \\ &= (3x - 2)x^2 \end{aligned}$$

Ex 63: Let $f(x) = 2x + 5$ and $g(x) = x - 4$. Find in factorized form:

$$f(x) \times g(x) = \boxed{(2x + 5)(x - 4)}$$

Answer: $$\begin{aligned} f(x) \times g(x) &= (2x + 5) \times (x - 4) \\ &= (2x + 5)(x - 4) \end{aligned}$$

G.2 DECOMPOSING EXPRESSIONS INTO FUNCTIONS

Ex 64: Find two functions f and g such that $f(x) \times g(x) = (x+3)^2(x-2)$.

- $f(x) = \boxed{(x+3)^2}$
- $g(x) = \boxed{x-2}$

Answer: One possible pair is $f(x) = (x+3)^2$ and $g(x) = x-2$, since

$$f(x) \times g(x) = (x+3)^2 \times (x-2).$$

Ex 65: Find two functions f and g such that $f(x) \times g(x) = (x^2+4)(3x-7)$.

- $f(x) = \boxed{x^2+4}$
- $g(x) = \boxed{3x-7}$

Answer: One possible pair is $f(x) = x^2+4$ and $g(x) = 3x-7$, since

$$f(x) \times g(x) = (x^2+4) \times (3x-7).$$

Ex 66: Find two functions f and g such that $f(x) + g(x) = (x-2)^2 + \sqrt{x}$.

- $f(x) = \boxed{(x-2)^2}$
- $g(x) = \boxed{\sqrt{x}}$

Answer: One possible pair is $f(x) = (x-2)^2$ and $g(x) = \sqrt{x}$, since

$$f(x) + g(x) = (x-2)^2 + \sqrt{x}.$$

Ex 67: Find two functions f and g such that $f(x) + g(x) = \frac{1}{x} + (x+1)^2$.

- $f(x) = \boxed{\frac{1}{x}}$
- $g(x) = \boxed{(x+1)^2}$

Answer: One possible pair is $f(x) = \frac{1}{x}$ and $g(x) = (x+1)^2$, since

$$f(x) + g(x) = \frac{1}{x} + (x+1)^2.$$

H COMPOSITION

H.1 EVALUATING COMPOSITE FUNCTIONS

Ex 68: For $f(x) = 2x+2$ and $g(x) = 3-x$, find in simplest form:

1. $f(g(3)) = \boxed{2}$
2. $f(g(-1)) = \boxed{10}$
3. $f(g(x)) = \boxed{8-2x}$
4. $g(f(x)) = \boxed{1-2x}$

Answer:

1. $$\begin{aligned} f(g(3)) &= f(3-3) \\ &= f(0) \\ &= 2 \times 0 + 2 \\ &= 2 \end{aligned}$$
2. $$\begin{aligned} f(g(-1)) &= f(3-(-1)) \\ &= f(4) \\ &= 2 \times 4 + 2 \\ &= 8+2 \\ &= 10 \end{aligned}$$
3. $$\begin{aligned} f(g(x)) &= f(3-x) \\ &= 2(3-x)+2 \\ &= 6-2x+2 \\ &= 8-2x \end{aligned}$$
4. $$\begin{aligned} g(f(x)) &= g(2x+2) \\ &= 3-(2x+2) \\ &= 3-2x-2 \\ &= 1-2x \end{aligned}$$

Ex 69: For $f(x) = x^2+2x$ and $g(x) = 2-x$, find in simplest form:

1. $f(g(3)) = \boxed{-1}$
2. $f(g(-1)) = \boxed{15}$
3. $f(g(x)) = \boxed{x^2-6x+8}$
4. $g(f(x)) = \boxed{2-x^2-2x}$

Answer:

1. $$\begin{aligned} f(g(3)) &= f(2-3) \\ &= f(-1) \\ &= (-1)^2 + 2 \times (-1) \\ &= 1-2 \\ &= -1 \end{aligned}$$
2. $$\begin{aligned} f(g(-1)) &= f(2-(-1)) \\ &= f(3) \\ &= 3^2 + 2 \times 3 \\ &= 9+6 \\ &= 15 \end{aligned}$$
3. $$\begin{aligned} f(g(x)) &= f(2-x) \\ &= (2-x)^2 + 2(2-x) \\ &= (4-4x+x^2)+(4-2x) \\ &= x^2-6x+8 \end{aligned}$$
4. $$\begin{aligned} g(f(x)) &= g(x^2+2x) \\ &= 2-(x^2+2x) \\ &= 2-x^2-2x \end{aligned}$$

Ex 70: For $f(x) = 3x - 5$, find in simplest form:

1. $f(f(-1)) = \boxed{-29}$
2. $f(f(x)) = \boxed{9x - 20}$

Answer:

1. $$\begin{aligned} f(f(-1)) &= f(3 \times (-1) - 5) \\ &= f(-8) \\ &= 3 \times (-8) - 5 \\ &= -24 - 5 \\ &= -29 \end{aligned}$$
2. $$\begin{aligned} f(f(x)) &= f(3x - 5) \qquad \text{(substituting } x \text{ with } (3x-5)) \\ &= 3(3x - 5) - 5 \\ &= 9x - 15 - 5 \\ &= 9x - 20 \end{aligned}$$

H.2 DECOMPOSING FUNCTIONS INTO COMPOSITIONS

Ex 71: Find two functions f and g such that $f(g(x)) = \sqrt{2x-1}$ and $g(x) \neq x$.

- $f(x) = \boxed{\sqrt{x}}$
- $g(x) = \boxed{2x - 1}$

Answer: One possible pair is $f(x) = \sqrt{x}$ and $g(x) = 2x - 1$, since

$$\begin{aligned} f(g(x)) &= f(2x - 1) \\ &= \sqrt{2x-1}. \end{aligned}$$

Ex 72: Find two functions f and g such that $f(g(x)) = (x+2)^5$ and $g(x) \neq x$.

- $f(x) = \boxed{x^5}$
- $g(x) = \boxed{x + 2}$

Answer: One possible pair is $f(x) = x^5$ and $g(x) = x + 2$, since

$$\begin{aligned} f(g(x)) &= f(x + 2) \\ &= (x + 2)^5. \end{aligned}$$

Ex 73: Find two functions f and g such that $f(g(x)) = \dfrac{1}{x^2+1}$ and $g(x) \neq x$.

- $f(x) = \boxed{\dfrac{1}{x}}$
- $g(x) = \boxed{x^2 + 1}$

Answer: One possible pair is $f(x) = \dfrac{1}{x}$ and $g(x) = x^2 + 1$, since

$$\begin{aligned} f(g(x)) &= f(x^2 + 1) \\ &= \frac{1}{x^2+1}. \end{aligned}$$

Ex 74: Find two functions f and g such that $f(g(x)) = (x^3 - 2)^{-4}$ and $g(x) \neq x$.

- $f(x) = \boxed{x^{-4}}$
- $g(x) = \boxed{x^3 - 2}$

Answer: One possible pair is $f(x) = x^{-4}$ and $g(x) = x^3 - 2$, since

$$\begin{aligned} f(g(x)) &= f(x^3 - 2) \\ &= (x^3 - 2)^{-4}. \end{aligned}$$

I INVERSE FUNCTION

I.1 FINDING AND CHECKING INVERSES

Ex 75:

1. Find the inverse of $f(x) = x + 3$.

$$f^{-1}(x) = \boxed{x - 3}$$

2. Evaluate

$$\begin{aligned} f^{-1}(f(x)) &= \boxed{x} \\ f(f^{-1}(x)) &= \boxed{x} \end{aligned}$$

Answer:

1. Set $y = x + 3$.
$$\begin{aligned} y &= x + 3 \\ x &= y - 3 \end{aligned}$$
So, the inverse function is $f^{-1}(x) = x - 3$.
2. $$\begin{aligned} f^{-1}(f(x)) &= f^{-1}(x + 3) \\ &= (x + 3) - 3 \\ &= x \end{aligned}$$
3. $$\begin{aligned} f(f^{-1}(x)) &= f(x - 3) \\ &= (x - 3) + 3 \\ &= x \end{aligned}$$

Ex 76:

1. Find the inverse of $f(x) = 4x - 8$.

$$f^{-1}(x) = \boxed{\frac{x+8}{4}}$$

2. Evaluate

$$\begin{aligned} f^{-1}(f(x)) &= \boxed{x} \\ f(f^{-1}(x)) &= \boxed{x} \end{aligned}$$

Answer:

1. Set $y = 4x - 8$.
$$\begin{aligned} y &= 4x - 8 \\ y + 8 &= 4x \\ x &= \frac{y+8}{4} \end{aligned}$$
So, the inverse function is $f^{-1}(x) = \frac{x+8}{4}$.

2.

$$\begin{aligned} f^{-1}(f(x)) &= f^{-1}(4x-8) \\ &= \frac{(4x-8)+8}{4} \\ &= \frac{4x}{4} \\ &= x \end{aligned}$$

3.

$$\begin{aligned} f(f^{-1}(x)) &= f\left(\frac{x+8}{4}\right) \\ &= 4 \times \frac{x+8}{4} - 8 \\ &= (x+8) - 8 \\ &= x \end{aligned}$$

Ex 77:

1. Find the inverse of $f(x) = \dfrac{x}{2} - 3$.

$$f^{-1}(x) = \boxed{2(x+3)}$$

2. Evaluate

$$\begin{aligned} f^{-1}(f(x)) &= \boxed{x} \\ f(f^{-1}(x)) &= \boxed{x} \end{aligned}$$

Answer:

1. Set $y = \dfrac{x}{2} - 3$.

$$\begin{aligned} y &= \frac{x}{2} - 3 \\ y + 3 &= \frac{x}{2} \\ 2(y+3) &= x \\ x &= 2(y+3) \end{aligned}$$

So, the inverse function is $f^{-1}(x) = 2(x+3)$.

2.

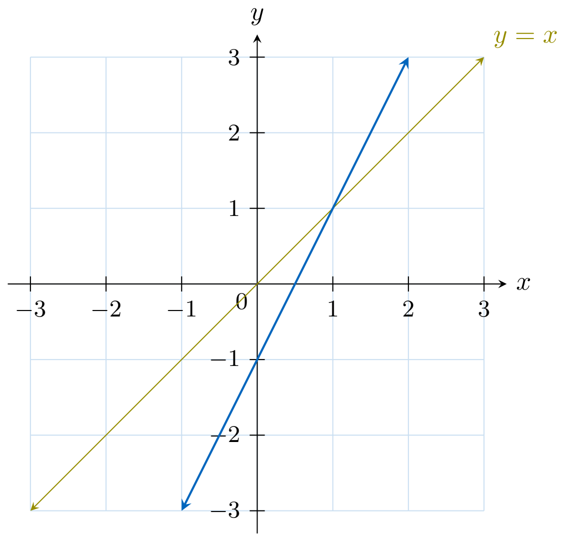
$$\begin{aligned} f^{-1}(f(x)) &= f^{-1}\left(\frac{x}{2} - 3\right) \\ &= 2\left(\frac{x}{2} - 3 + 3\right) \\ &= 2 \times \frac{x}{2} \\ &= x \end{aligned}$$

3.

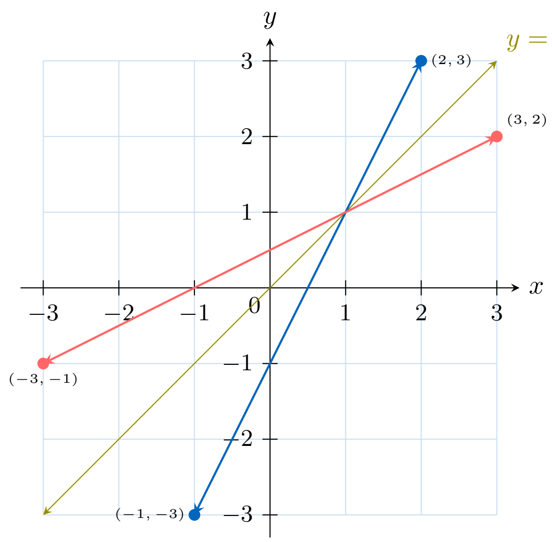
$$\begin{aligned} f(f^{-1}(x)) &= f\left(2(x+3)\right) \\ &= \frac{2(x+3)}{2} - 3 \\ &= (x+3) - 3 \\ &= x \end{aligned}$$

I.2 GRAPHING THE INVERSE FUNCTION BY REFLECTION

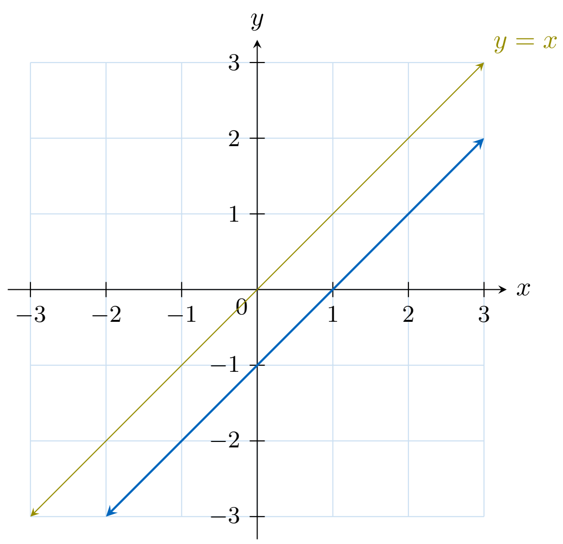
Ex 78: Draw the graph of the inverse function of the blue graph:



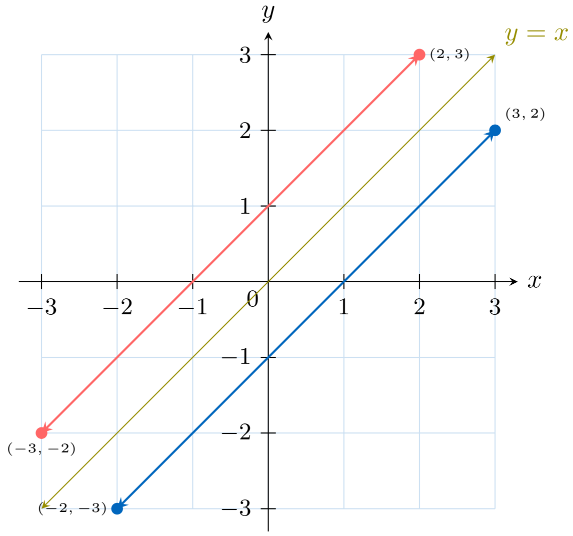
Answer: To draw the inverse, notice that the graph of f^{-1} is the reflection of the graph of f across the line $y = x$. You can plot two points on the blue line (for example, $(-1,-3)$ and $(2,3)$), then swap their coordinates to get points $(-3,-1)$ and $(3,2)$ on the inverse. Draw the line passing through these points: this is $y = \frac{x+1}{2}$, shown below in red.



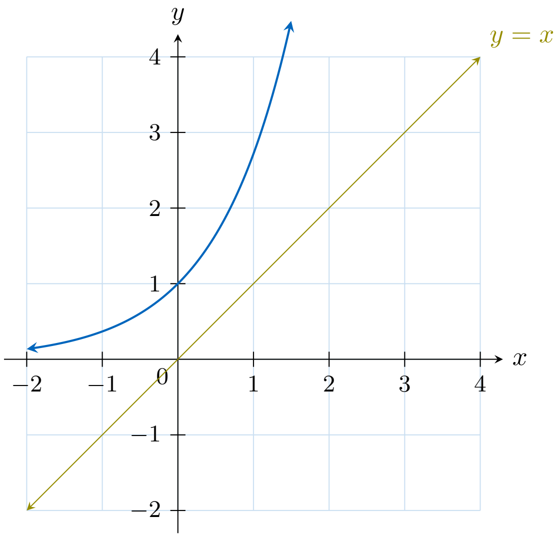
Ex 79: Draw the graph of the inverse function of the blue graph:



Answer: To draw the inverse, notice that the graph of f^{-1} is the reflection of the graph of f across the line $y = x$. For instance, the blue line contains the points $(-2,-3)$ and $(3,2)$. Swap their coordinates to get $(-3,-2)$ and $(2,3)$ on the inverse. Draw the line passing through these points: this is $y=x+1$, shown below in red.



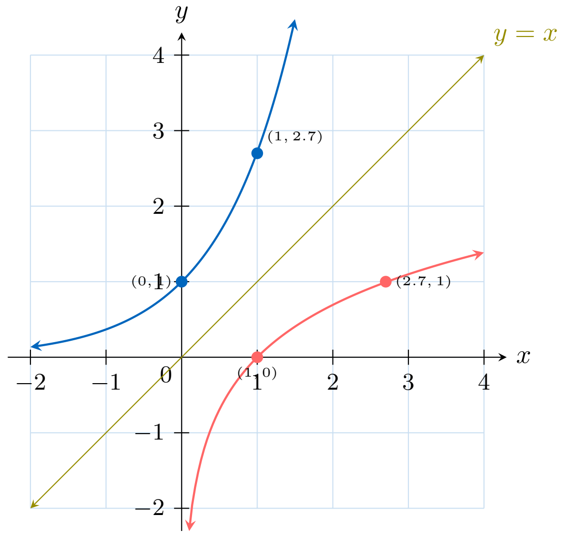
Ex 80: Draw the graph of the inverse function of the blue graph:



Answer: To draw the inverse graph, plot a few symmetric points:

- Take points from the blue curve, such as $(0,1)$ and $(1,2.7)$.
- Find their symmetric points with respect to the line $y=x$, i.e., swap their coordinates: $(1,0)$ and $(2.7,1)$.
- Plot these new points.
- Draw a smooth curve through the symmetric points; this is the graph of the inverse function.

You do **not** need to know the exact equation of the curve!



Remark: The inverse graph is obtained by reflecting each point of the blue curve across the line $y=x$.