# FUNCTIONS AND GRAPHS OF POLYNOMIALS

## A GENERAL PRINCIPLES OF POLYNOMIAL GRAPHS

### A.1 DETERMINING END BEHAVIOUR

**Ex 1:** For the polynomial $P(x) = -3x^5+4x^2-8x+1$, determine the following limits:

1. $\lim_{x\to\infty} P(x) =$ ☐ $+\infty$ ☐ $-\infty$
2. $\lim_{x\to-\infty} P(x) =$ ☐ $+\infty$ ☐ $-\infty$

**Ex 2:** For the polynomial $P(x) = 2x^4 - 5x^3 + x - 10$, determine the following limits:

1. $\lim_{x\to\infty} P(x) =$ ☐ $+\infty$ ☐ $-\infty$
2. $\lim_{x\to-\infty} P(x) =$ ☐ $+\infty$ ☐ $-\infty$

**Ex 3:** For the polynomial $P(x) = x^7+100x^6-500x^2$, determine the following limits:

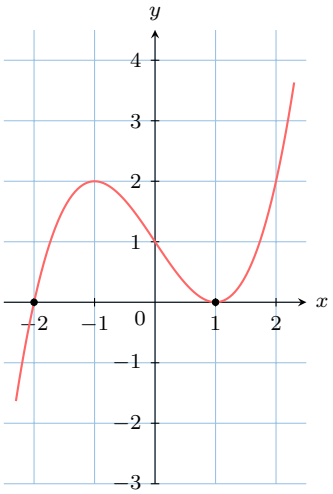
1. $\lim_{x\to\infty} P(x) =$ ☐ $+\infty$ ☐ $-\infty$
2. $\lim_{x\to-\infty} P(x) =$ ☐ $+\infty$ ☐ $-\infty$

**Ex 4:** For the polynomial $P(x) = 50 + x - 2x^6$, determine the following limits:

1. $\lim_{x\to\infty} P(x) =$ ☐ $+\infty$ ☐ $-\infty$
2. $\lim_{x\to-\infty} P(x) =$ ☐ $+\infty$ ☐ $-\infty$

### A.2 INTERPRETING GRAPHS AT THE ROOTS

**MCQ 5:** The graph of a polynomial $P(x)$ is shown below. Which of the following is the most likely factorisation of $P(x)$?



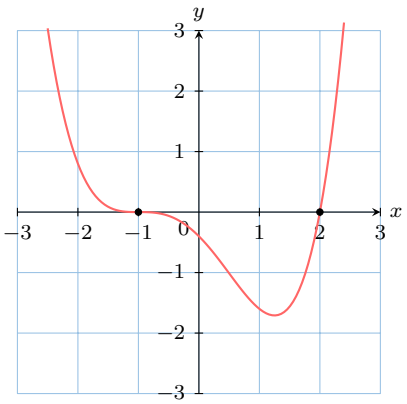

☐ $(x+2)(x-1)$

☐ $(x+2)^2(x-1)$

☐ $(x+2)(x-1)^2$

☐ $(x+2)^3(x-1)$

**MCQ 6:** The graph of a polynomial $P(x)$ is shown below. Which of the following is the most likely factorisation of $P(x)$?



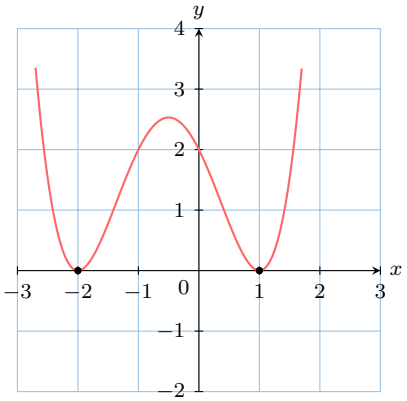

☐ $(x+1)(x-2)$

☐ $(x+1)^2(x-2)$

☐ $(x+1)(x-2)^3$

☐ $(x+1)^3(x-2)$

**MCQ 7:** The graph of a polynomial $P(x)$ is shown below. Which of the following is the most likely factorisation of $P(x)$?



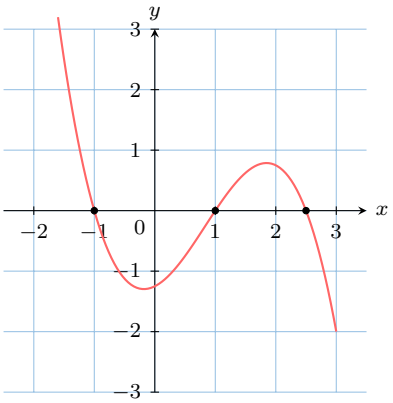

☐ $(x+2)(x-1)$

☐ $(x-2)(x+1)^2$

☐ $(x+2)^2(x-1)^2$

☐ $(x+2)^3(x-1)$

**MCQ 8:** The graph of a polynomial $P(x)$ is shown below. Which of the following is the most likely factorisation of $P(x)$?




☐ $(x+1)(x-1)(x-2.5)$

☐ $-(x+1)(x-1)(x-2.5)$

☐ $-(x-1)(x+1)^2$

☐ $(x+1)(x-1)(x-2.5)^2$

### A.3 INTERPRETING GRAPHS AT THE ROOTS

**Ex 9:** A polynomial function is given by $P(x) = (x-1)^3(x-2)$. Describe the behaviour of the graph of $y = P(x)$ at its x-intercepts.

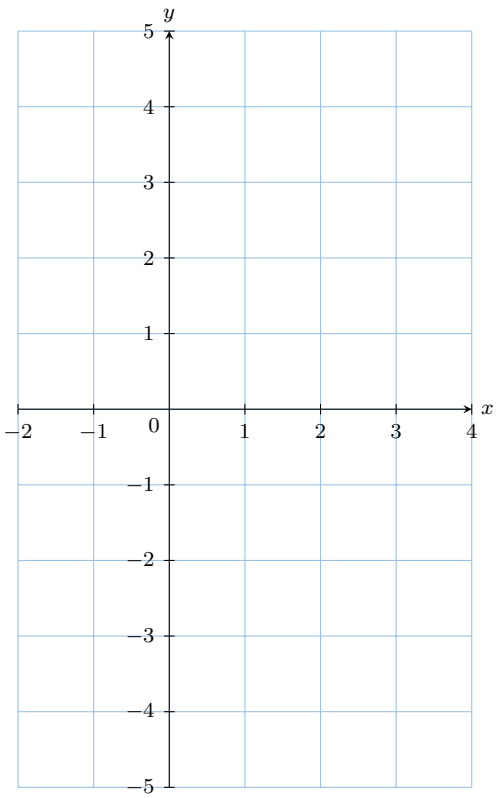
**Ex 10:** A polynomial function is given by $P(x) = (x+3)^2(x-4)$. Describe the behaviour of the graph of $y = P(x)$ at its x-intercepts.

**Ex 11:** A polynomial function is given by $P(x) = x(x-5)^2(x+1)^3$. Describe the behaviour of the graph of $y = P(x)$ at its x-intercepts.

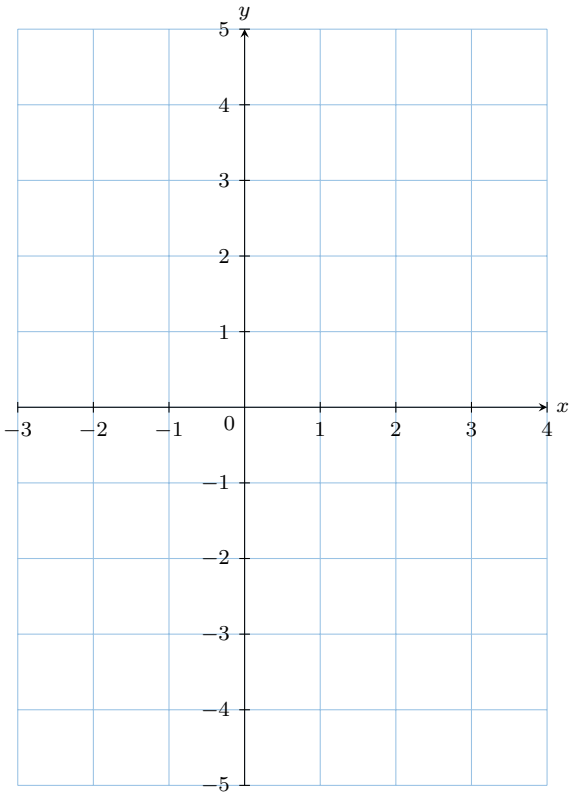
## B GRAPHING CUBIC FUNCTIONS

### B.1 SKETCHING CUBIC FUNCTIONS

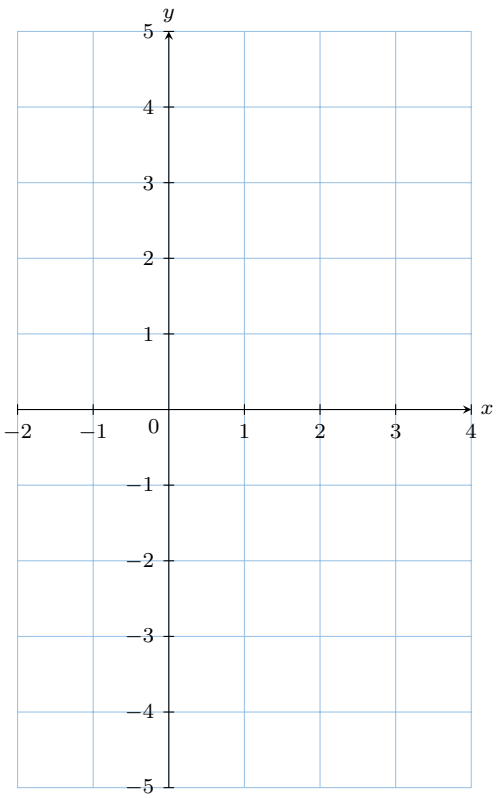
**Ex 12:** Use the axes intercepts to sketch the graph of $y = (x+1)(x-1)(x-3)$.




**Ex 13:** Use the axes intercepts to sketch the graph of $y = -\frac{1}{2}(x+1)^2(x-3)$.

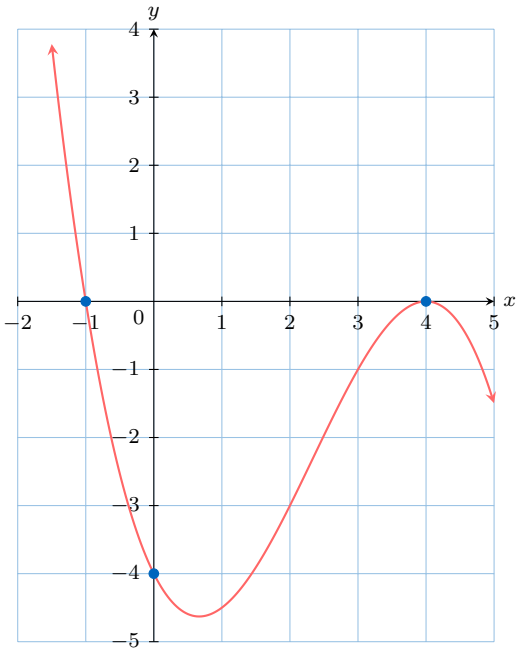



**Ex 14:** Use the axes intercepts to sketch the graph of $y = \frac{1}{2}(x-1)^3$.



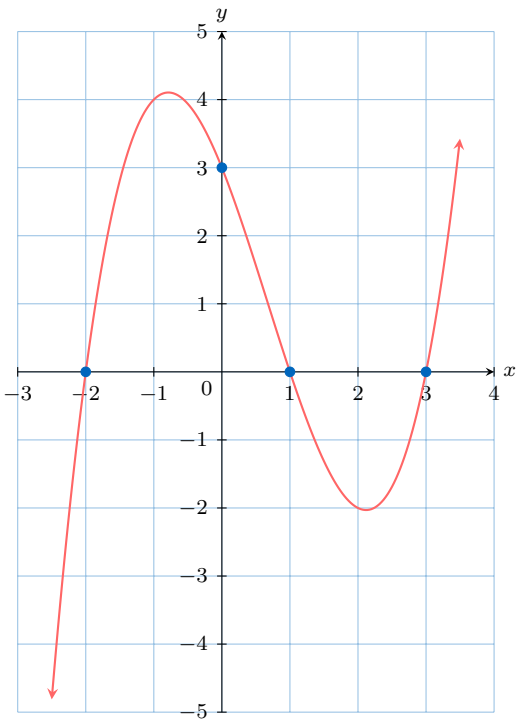

## B.2 FINDING THE FUNCTION FROM A GRAPH

**Ex 15:** Find the equation of the cubic function shown in the graph below.



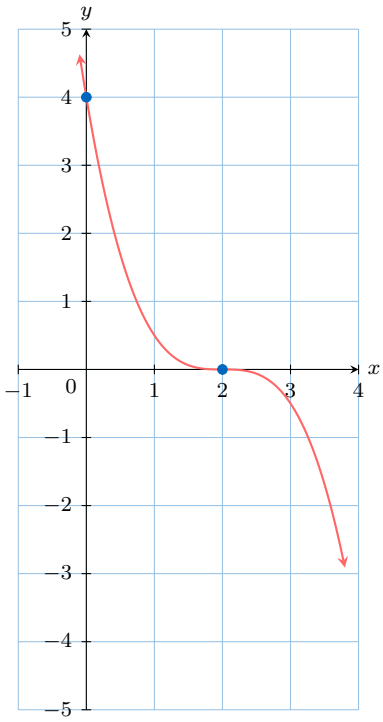

$P(x) =$ ____

**Ex 16:** Find the equation of the cubic function shown in the graph below.




$P(x) =$ ____

**Ex 17:** Find the equation of the cubic function shown in the graph below.

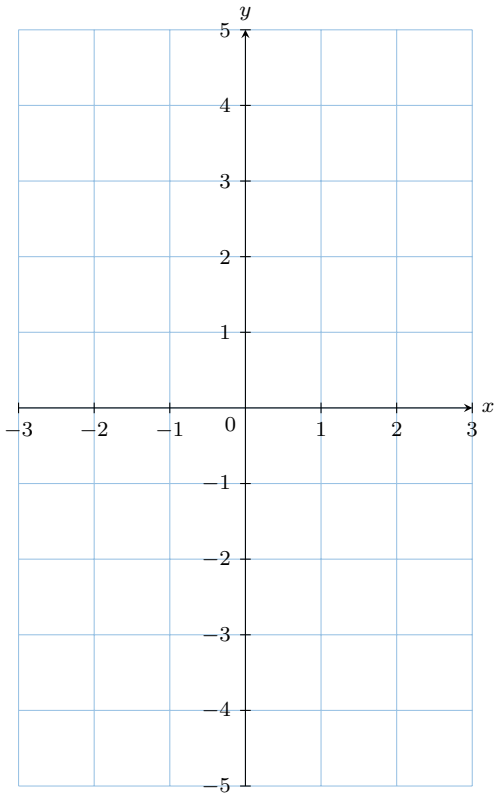



$P(x) =$ ☐

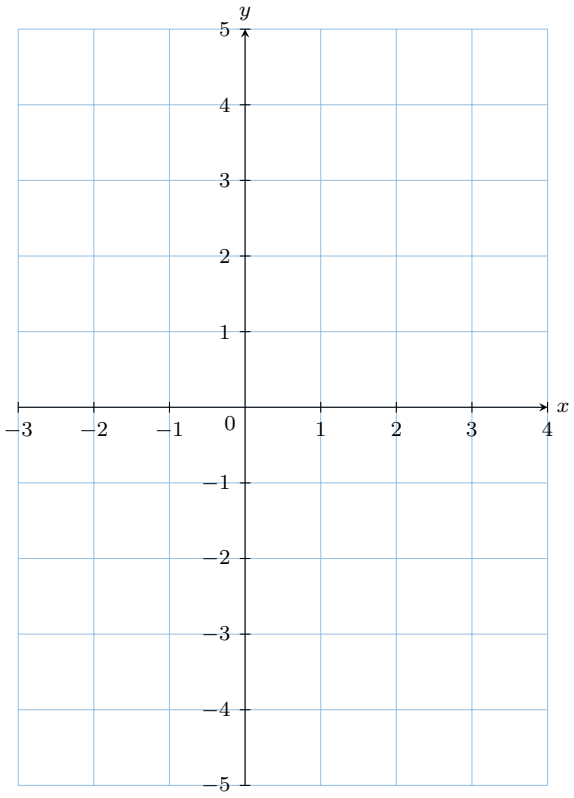
## C GRAPHING QUARTIC FUNCTIONS

### C.1 SKETCHING QUARTIC FUNCTIONS

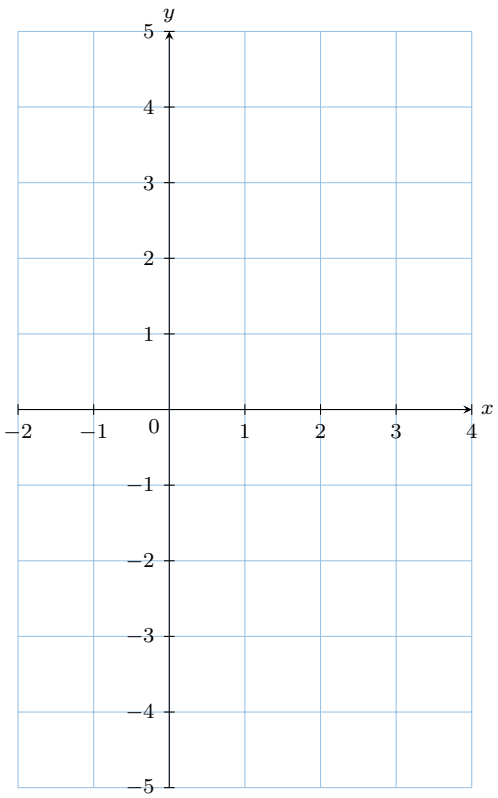
**Ex 18:** Use the axes intercepts to sketch the graph of $y = (x+2)(x+1)(x-1)(x-2)$.




**Ex 19:** Use the axes intercepts to sketch the graph of $y = -\frac{1}{4}(x+2)^2(x-1)(x-3)$.




**Ex 20:** Use the axes intercepts to sketch the graph of $y = \frac{1}{8}(x-2)^3(x+1)$.




## D SOLVING POLYNOMIAL INEQUALITIES

### D.1 SOLVING POLYNOMIAL INEQUALITIES

**MCQ 21:** Which of the following is the solution to the inequality $(x-4)^2(x+1) \leq 0$?

☐ $x \leq -1$

☐ $x \leq -1$ or $x \geq 4$

☐ $x \leq -1$ or $x = 4$

☐ $-1 \leq x \leq 4$

**MCQ 22:** Which of the following is the solution to the inequality $(x+3)(x-1)^3 \geq 0$?

☐ $-3 \leq x \leq 1$

☐ $x \leq -3$

☐ $x \geq 1$

☐ $x \leq -3$ or $x \geq 1$

**MCQ 23:** Which of the following is the solution to the inequality $(x+3)(x-1)^3 \geq 0$?

☐ $-3 \leq x \leq 1$

☐ $x \leq -3$

☐ $x \geq 1$

☐ $x \leq -3$ or $x \geq 1$

**MCQ 24:** Which of the following is the solution to the inequality $x^3 + 2x^2 < 3x$?

☐ $x < -3$ or $x > 1$

☐ $x < -3$ or $0 < x < 1$

☐ $-3 < x < 0$

☐ $-3 < x < 0$ or $x > 1$

**Ex 25:** Find the set of values for which $(x+2)(x-1)(x-3) < 0$.

**Ex 26:** Find the set of values for which $x^2 - 3x + 2 > 0$.

## D.2 SOLVING POLYNOMIAL AND RATIONAL INEQUALITIES

**Ex 27:** Consider the inequality $\frac{x-4}{2x+1} \geq 1$.

1. Rewrite the inequality in the form $f(x) \geq 0$, where $f(x)$ is a single rational expression.
2. Find the critical values for the inequality.
3. Hence, solve the inequality $\frac{x-4}{2x+1} \geq 1$.

**Ex 28:** Solve the inequality $x \leq \frac{3}{x-2}$.

1. Rewrite the inequality in the form $f(x) \leq 0$, where $f(x)$ is a single rational expression.
2. Find the critical values for the inequality.
3. Hence, solve the inequality $x \leq \frac{3}{x-2}$.

**Ex 29:** Solve the inequality $\frac{2}{x+3} < \frac{1}{x-2}$.

1. Rewrite the inequality in the form $f(x) < 0$, where $f(x)$ is a single rational expression.
2. Find the critical values for the inequality.
3. Hence, solve the inequality $\frac{2}{x+3} < \frac{1}{x-2}$.

**Ex 30:** Let $P(x) = x^3 - 2x^2 - 5x + 6$.

1. Show that $(x - 1)$ is a factor of $P(x)$.
2. Hence, fully factorize $P(x)$.
3. Using the factors of $P(x)$, solve the inequality $x^3 - 2x^2 - 5x + 6 > 0$.