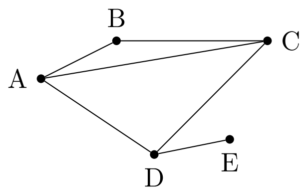
GRAPH THEORY

A DEFINITIONS

A.1 DETERMINING THE DEGREE OF VERTICES

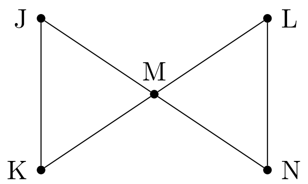
Ex 1:



Count the degree of the vertices:

- $\deg(A) = \square$
- $\deg(B) = \square$
- $\deg(C) = \square$
- $\deg(D) = \square$
- $\deg(E) = \square$

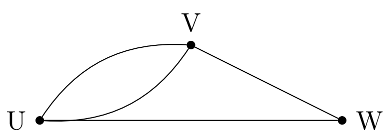
Ex 2:



Count the degree of the vertices:

- $\deg(J) = \square$
- $\deg(K) = \square$
- $\deg(L) = \square$
- $\deg(M) = \square$
- $\deg(N) = \square$

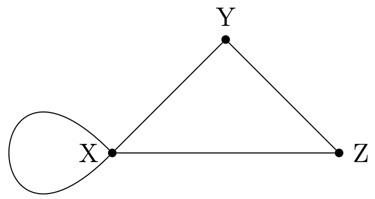
Ex 3:



Count the degree of the vertices:

- $\deg(U) = \square$
- $\deg(V) = \square$
- $\deg(W) = \square$

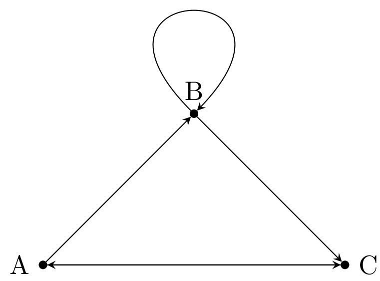
Ex 4:



Count the degree of the vertices (remember that a loop counts for 2):

- $\deg(X) = \square$
- $\deg(Y) = \square$
- $\deg(Z) = \square$

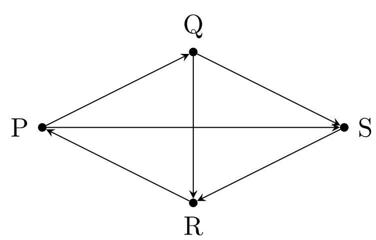
Ex 5:



Determine the in-degree ($\deg_{in}$) and out-degree ($\deg_{out}$) of the vertices:

- Vertex A: $\deg_{in}(A) = \square$, $\deg_{out}(A) = \square$
- Vertex B: $\deg_{in}(B) = \square$, $\deg_{out}(B) = \square$
- Vertex C: $\deg_{in}(C) = \square$, $\deg_{out}(C) = \square$

Ex 6:

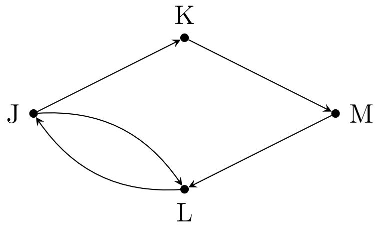


Count the in-degree and out-degree:

- $\deg_{in}(P) = \square$, $\deg_{out}(P) = \square$
- $\deg_{in}(Q) = \square$, $\deg_{out}(Q) = \square$
- $\deg_{in}(R) = \square$, $\deg_{out}(R) = \square$
- $\deg_{in}(S) = \square$, $\deg_{out}(S) = \square$

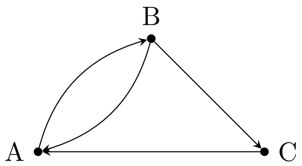
A.2 IDENTIFYING PATHS AND CIRCUITS

Ex 7: Consider the following directed graph (digraph):



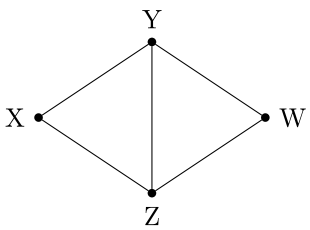
1. Is it possible to go directly from K to J?
2. Find a path from K to J.

Ex 8: Consider the following directed graph (digraph):



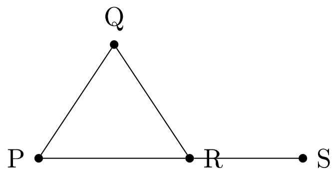
1. Is it possible to go directly from A to C?
2. Find a circuit starting and ending at A.

Ex 9: Consider the following undirected graph:



1. Is it possible to go directly from X to W?
2. Find a cycle starting and ending at Y that passes through Z.

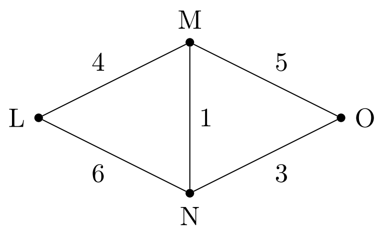
Ex 10: Consider the following undirected graph:



1. Is it possible to go from P to S without passing through R?
2. Find a path from Q to S.

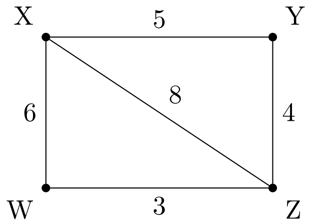
A.3 ANALYZING PATHS IN WEIGHTED GRAPHS

Ex 11: Consider the following weighted graph representing travel costs between cities:



1. What is the weight of edge MN?
2. Calculate the total weight of the path $L \to M \to O \to N \to L$.
3. Find the lowest total weight of the path from L to O.

Ex 12: Consider the following weighted graph representing driving times (in hours) between towns:

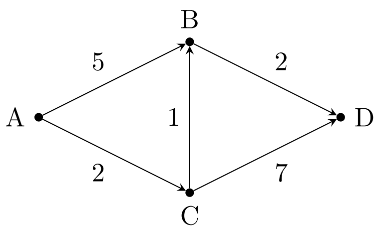


1. What is the weight of edge XZ?

2. Calculate the total weight of the path $W \to X \to Y \to Z$.

3. Find the lowest total weight of the path from W to Y.

Ex 13: Consider the following weighted directed graph representing flight costs between airports. Note that the connections are one-way (indicated by arrows).

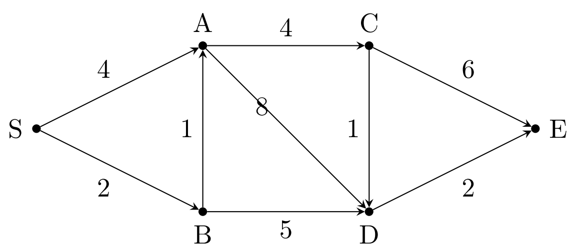


1. What is the weight of the directed edge from C to B?

2. Calculate the total weight of the path $A \to B \to D$.

3. Find the lowest total weight of a path from A to D.

Ex 14: Consider the following weighted directed graph representing latency (in ms) in a complex computer network from a source server S to a destination server E.



1. What is the weight of the directed edge from B to A?

2. Calculate the total weight of the path $S \to B \to A \to C \to D \to E$.

3. Find the lowest total weight of a path from S to E.

A.4 MODELLING SITUATIONS WITH GRAPHS

Ex 15: A computer network consists of 4 computers (labeled A, B, C, and D) connected to a central server (labeled S). Every computer is connected directly to the server, but not to each other.
Draw a graph to illustrate this network.

Ex 16: Four friends (Alice, Bob, Charlie, and David) meet for dinner. Everyone shakes hands with everyone else exactly once. Draw a graph to represent these handshakes, where vertices represent the people and edges represent the handshakes.

Ex 17: A train line serves four stations: North Station, Central Station, South Station, and Airport. The track goes from North to Central, then from Central to South, and finally from South to Airport. The train travels the same path in both directions.
Draw a graph to illustrate the connections between these stations.

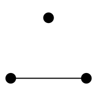
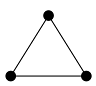
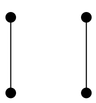
Ex 18: In a small business, a CEO manages two team leaders (L_1 and L_2). Team leader L_1 supervises two employees (E_1 and E_2), while team leader L_2 supervises one employee (E_3).
Draw a graph representing this hierarchy.

Ex 19: A food delivery company has a distribution center (D) and three delivery zones (Z_1, Z_2, Z_3). The delivery trucks leave the distribution center to go to Z_1 or Z_2. From Z_1, they can go to Z_2 or Z_3. From Z_2, they can only go to Z_3. Once in Z_3, they return directly to the distribution center D.
Draw a directed graph to illustrate these delivery routes.

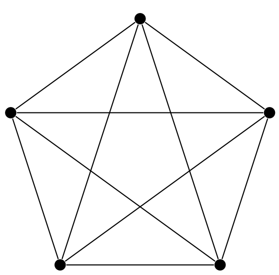
B PROPERTIES OF GRAPHS

B.1 IDENTIFYING GRAPH PROPERTIES

MCQ 20: Which of the following graphs is **connected**?

- ☐ 
- ☐ 
- ☐
- ☐ 

Ex 21: Consider the complete graph K_5 shown below.



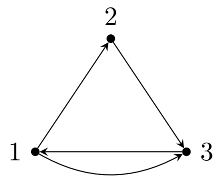
1. How many vertices does it have?

2. How many edges does it have?

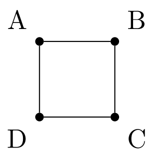
C ADJACENCY MATRICES

C.1 WRITING ADJACENCY MATRICES

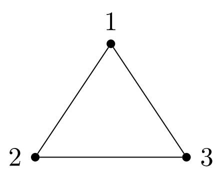
Ex 22: Write the adjacency matrix **M** for the following directed graph:



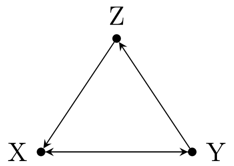
Ex 23: Write the adjacency matrix **M** for the following undirected graph:



Ex 24: Write the adjacency matrix **M** for the following undirected graph:



Ex 25: Write the adjacency matrix $\mathbf{M}$ for the following directed graph:



C.2 DETERMINING THE NUMBER OF WALKS

Ex 26: Consider the directed graph H given by the following adjacency matrix (vertices are ordered alphabetically: P, Q, R):

$$\mathbf{M} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

1. Draw the directed graph H.
2. Find the number of walks of length 2 from vertex P to vertex R.
3. Find the number of walks of length 3 from vertex R to vertex Q.

Ex 27: Consider the undirected graph K given by the following adjacency matrix (vertices are ordered alphabetically: X, Y, Z):

$$\mathbf{M} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

1. Draw the undirected graph K.
2. Find the number of walks of length 2 from vertex X to vertex Y.
3. Find the number of walks of length 3 from vertex Y to vertex Z.

Ex 28: Consider the graph G given by the following adjacency matrix (vertices are ordered alphabetically: A, B, C, D):

$$\mathbf{M} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

1. Draw the non-directed graph G.
2. Find the number of walks of length 2 from vertex A to vertex D.
3. Find the number of walks of length 3 from vertex B to itself.