INEQUALITIES

Inequalities are common in real-world contexts, such as budgeting, temperature ranges, or constraints in optimization problems.

A DEFINITION

Definition Inequality

An **inequality** is a mathematical statement that compares two expressions using symbols such as $<$, $>$, $\leq$, or $\geq$. Unlike equations, which state equality, inequalities describe a range of possible values that satisfy the relationship.

Ex: $x + 3 < 5$ means that $x + 3$ is less than 5, and we need to find all values of x that make this true.

B PROPERTIES OF INEQUALITIES

Proposition Properties of Inequality

Inequalities follow specific rules when performing operations:

- **Adding or subtracting** the same number from both sides preserves the inequality direction.
- **Multiplying or dividing** both sides by a **positive number preserves** the inequality direction.
- **Multiplying or dividing** both sides by a **negative number reverses** the inequality direction.
- If $a < b$ and $b < c$, then $a < c$ (**transitive property**).

Ex:

- For $2x < 8$. Dividing both sides by 2 (a positive number) gives $x < 4$.
- For $-2x < 8$, dividing by -2 (a negative number) reverses the inequality, giving $x > -4$.

Method Solving Linear Inequality

To solve a linear inequality, isolate the variable using the properties of inequalities.

Ex: Solve $3x - 5 \leq 10$.

Answer:

$$\begin{aligned} 3x - 5 &\leq 10 \\ 3x &\leq 15 \quad \text{(add 5 to both sides)} \\ x &\leq 5 \quad \text{(divide by 3)} \end{aligned}$$

The solution is $x \leq 5$, which includes all numbers less than or equal to 5.

C TABLE OF SIGNS

The **table of signs** is a visual tool used to study the sign (positive, negative, or zero) of a product or quotient of algebraic expressions.

Method Table of Signs for a Product

To construct a table of signs for a product:

1. **Find the zeros of each factor** (solve when each factor equals zero).
2. **Place these values** in order on a number line, dividing it into intervals.
3. **Determine the sign of each factor** in every interval.
4. **Deduce the sign of the product** in each interval using the sign rule (the product is positive if there is an even number of negative factors, negative if there is an odd number).

Ex: Study the sign of the expression $(x - 2)(x - 1)$.

Answer:

x	$-\infty$		1		2		$+\infty$
$(x-2)$		$-$		$-$	0	$+$	
$(x-1)$		$-$	0	$+$		$+$	
$(x-2)(x-1)$		$+$	0	$-$	0	$+$	

- The sign **changes at the zeros**: for $(x-2)$ at $x=2$, for $(x-1)$ at $x=1$.
- $(x-2) \geqslant 0$ for $x \geqslant 2$ and $(x-1) \geqslant 0$ for $x \geqslant 1$
- The sign of the product $(x-2)(x-1)$ is determined by multiplying the signs of the two factors in each interval.