

INTEGERS

A DEFINITION

A.1 COUNTING POSITIVE AND NEGATIVE NUMBERS

Ex 1:

$$\textcircled{+} \textcircled{+} = \boxed{+2}$$

Answer:

- There are 2 positive units.

$$\bullet \textcircled{+} \textcircled{+} = +2$$

Ex 2:

$$\textcircled{-} \textcircled{-} \textcircled{-} = \boxed{-3}$$

Answer:

- There are 3 negative units.

$$\bullet \textcircled{-} \textcircled{-} \textcircled{-} = -3$$

Ex 3:

$$\textcircled{-} \textcircled{-} \textcircled{-} \textcircled{-} \textcircled{-} = \boxed{-5}$$

Answer:

- There are 5 negative units.

$$\bullet \textcircled{-} \textcircled{-} \textcircled{-} \textcircled{-} \textcircled{-} = -5$$

Ex 4:

$$\textcircled{+} \textcircled{+} \textcircled{+} = \boxed{+3}$$

Answer:

- There are 3 positive units.

$$\bullet \textcircled{+} \textcircled{+} \textcircled{+} = +3$$

Ex 5:

$$\textcircled{-} = \boxed{-1}$$

Answer:

- There is 1 negative unit.

$$\bullet \textcircled{-} = -1$$

A.2 WRITING INTEGERS FROM WORDS

Ex 6: Positive two is $\boxed{+2}$.

Answer:

$$\bullet \text{ Positive two is } +2 = \textcircled{+} \textcircled{+}$$

Ex 7: Negative three is $\boxed{-3}$.

Answer:

$$\bullet \text{ Negative three is } -3 = \textcircled{-} \textcircled{-} \textcircled{-}$$

Ex 8: Negative four is $\boxed{-4}$.

Answer:

$$\bullet \text{ Negative four is } -4 = \textcircled{-} \textcircled{-} \textcircled{-} \textcircled{-}$$

Ex 9: Positive five is $\boxed{+5}$.

Answer:

$$\bullet \text{ Positive five is } +5 = \textcircled{+} \textcircled{+} \textcircled{+} \textcircled{+} \textcircled{+}$$

Ex 10: Negative two is $\boxed{-2}$.

Answer:

$$\bullet \text{ Negative two is } -2 = \textcircled{-} \textcircled{-}$$

A.3 FINDING THE OPPOSITE

Ex 11: The opposite of -4 is $\boxed{+4}$.

Answer:

$$\bullet \textcircled{-} \textcircled{-} \textcircled{-} \textcircled{-} + \textcircled{+} \textcircled{+} \textcircled{+} \textcircled{+} = \textcircled{+} \textcircled{+} \textcircled{+} \textcircled{+}$$

$$\bullet (-4) + (+4) = 0$$

- The opposite of -4 is $+4$.

Ex 12: The opposite of -3 is $\boxed{+3}$.

Answer:

$$\bullet \textcircled{-} \textcircled{-} \textcircled{-} + \textcircled{+} \textcircled{+} \textcircled{+} = \textcircled{+} \textcircled{+} \textcircled{+}$$

$$\bullet (-3) + (+3) = 0$$

- The opposite of -3 is $+3$.

Ex 13: The opposite of $+5$ is $\boxed{-5}$.

Answer:

$$\bullet \textcircled{+} \textcircled{+} \textcircled{+} \textcircled{+} \textcircled{+} + \textcircled{-} \textcircled{-} \textcircled{-} \textcircled{-} \textcircled{-} = \textcircled{-} \textcircled{-} \textcircled{-} \textcircled{-} \textcircled{-}$$

$$\bullet (+5) + (-5) = 0$$

- The opposite of $+5$ is -5 .

Ex 14: The opposite of $+1$ is $\boxed{-1}$.

Answer:

$$\bullet \quad \textcircled{+} + \textcircled{-} = \textcircled{\pm}$$

$$\bullet \quad (+1) + (-1) = 0$$

• The opposite of $+1$ is -1 .

Ex 15: The opposite of 0 is $\boxed{0}$.

Answer:

$$\bullet \quad \textcircled{0+0=}$$

$$\bullet \quad 0 + 0 = 0$$

• The opposite of 0 is 0.

A.4 FINDING THE OPPOSITE FOR DECIMAL NUMBERS

Ex 16: The opposite of -4.1 is $\boxed{+4.1}$.

Answer:

$$\bullet \quad (-4.1) + (+4.1) = 0$$

• The opposite of -4.1 is $+4.1$.

Ex 17: The opposite of -0.5 is $\boxed{+0.5}$.

Answer:

$$\bullet \quad (-0.5) + (+0.5) = 0$$

• The opposite of -0.5 is $+0.5$.

Ex 18: The opposite of $+3.5$ is $\boxed{-3.5}$.

Answer:

$$\bullet \quad (+3.5) + (-3.5) = 0$$

• The opposite of $+3.5$ is -3.5 .

Ex 19: The opposite of $+99.5$ is $\boxed{-99.5}$.

Answer:

$$\bullet \quad 99.5 = +99.5$$

$$\bullet \quad (+99.5) + (-99.5) = 0$$

• The opposite of $+99.5$ is -99.5 .

A.5 ADDING SMALL INTEGERS

Ex 20: $(+1) + (-2) = \boxed{-1}$

Answer:

$$\bullet \quad \textcircled{+} + \textcircled{-} \textcircled{-} = \textcircled{-} \textcircled{-}$$

$$\bullet \quad (+1) + (-2) = -1$$

Ex 21: $(+3) + (-1) = \boxed{+2}$

Answer:

$$\bullet \quad \textcircled{+} \textcircled{+} \textcircled{+} + \textcircled{-} = \textcircled{+} \textcircled{+} \textcircled{\pm}$$

$$\bullet \quad (+3) + (-1) = +2$$

Ex 22: $(+2) + (-3) = \boxed{-1}$

Answer:

$$\bullet \quad \textcircled{+} \textcircled{+} + \textcircled{-} \textcircled{-} \textcircled{-} = \textcircled{-} \textcircled{\pm} \textcircled{\pm}$$

$$\bullet \quad (+2) + (-3) = -1$$

Ex 23: $(-2) + (-1) = \boxed{-3}$

Answer:

$$\bullet \quad \textcircled{-} \textcircled{-} + \textcircled{-} = \textcircled{-} \textcircled{-} \textcircled{-}$$

$$\bullet \quad (-2) + (-1) = -3$$

Ex 24: $(-1) + (+3) = \boxed{+2}$

Answer:

$$\bullet \quad \textcircled{-} + \textcircled{+} \textcircled{+} \textcircled{+} = \textcircled{+} \textcircled{+} \textcircled{\pm}$$

$$\bullet \quad (-1) + (+3) = +2$$

Ex 25: $(+2) + (+3) = \boxed{+5}$

Answer:

$$\bullet \quad \textcircled{+} \textcircled{+} + \textcircled{+} \textcircled{+} \textcircled{+} = \textcircled{+} \textcircled{+} \textcircled{+} \textcircled{+} \textcircled{+}$$

$$\bullet \quad (+2) + (+3) = +5$$

A.6 FINDING MISSING NUMBERS IN ADDITION

Ex 26: $(+3) + (\boxed{-2}) = +1$

Answer:

$$\bullet \quad \textcircled{+} \textcircled{+} \textcircled{+} + \textcircled{-} \textcircled{-} = \textcircled{+} \textcircled{\pm} \textcircled{\pm}$$

$$\bullet \quad (+3) + (-2) = +1$$

Ex 27: $(-4) + (\boxed{+2}) = -2$

Answer:

$$\bullet \quad \textcircled{-} \textcircled{-} \textcircled{-} \textcircled{-} + \textcircled{+} \textcircled{+} = \textcircled{-} \textcircled{-} \textcircled{\pm} \textcircled{\pm}$$

$$\bullet \quad (-4) + (+2) = -2$$

Ex 28: $(\boxed{-3}) + (+2) = -1$

Answer:

B RULES OF ADDITION

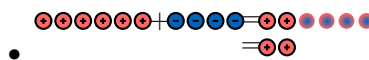
B.1 ADDING INTEGERS

Ex 41: $(+6) + (-4) = \boxed{+2}$

Answer:

- When the signs are opposite, subtract the absolute values ($6 - 4 = 2$), and take the sign of the larger number: $6 > 4$, so the result is $+$.

• $(+6) + (-4) = +2$

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Ex 42: $(+4) + (+7) = \boxed{+11}$

Answer:

- When the signs are the same, add the absolute values ($4 + 7 = 11$), and keep the positive sign.

• $(+4) + (+7) = +11$

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Ex 43: $(-5) + (+8) = \boxed{+3}$

Answer:

- When the signs are opposite, subtract the absolute values ($8 - 5 = 3$), and take the sign of the larger number: $8 > 5$, so the result is $+$.

• $(-5) + (+8) = +3$

• 

Ex 44: $(+6) + (-4) = \boxed{+2}$

Answer:

- When the signs are opposite, subtract the absolute values ($6 - 4 = 2$), and take the sign of the larger number: $6 > 4$, so the result is $+$.

• $(+6) + (-4) = +2$

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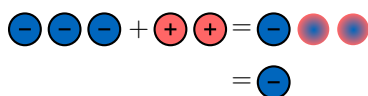
Ex 45: $(-5) + (-4) = \boxed{-9}$

Answer:

- When the signs are the same, add the absolute values ($5 + 4 = 9$), and keep the negative sign.

• $(-5) + (-4) = -9$

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• $(-3) + (+2) = -1$

Ex 29: $(-2) + (-3) = -5$

Answer:

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• $(-2) + (-3) = -5$

Ex 30: $(+2) + (+1) = +3$

Answer:

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• $(+2) + (+1) = +3$

Ex 31: $(+2) + (-5) = -3$

Answer:

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• $(+2) + (-5) = -3$

A.7 FINDING THE ABSOLUTE VALUE

Ex 32: The absolute value of $+2$ is $\boxed{2}$.

Answer: The absolute value of $+2 = \text{two red circles with '+' signs}$ is 2.

Ex 33: The absolute value of -3 is $\boxed{3}$.

Answer: The absolute value of $-3 = \text{three blue circles with '-' signs}$ is 3.

Ex 34: The absolute value of $+5$ is $\boxed{5}$.

Answer: The absolute value of $+5 = \text{five red circles with '+' signs}$ is 5.

Ex 35: The absolute value of -4 is $\boxed{4}$.

Answer: The absolute value of $-4 = \text{four blue circles with '-' signs}$ is 4.

Ex 36: The absolute value of -9 is $\boxed{9}$.

Answer: The absolute value of $-9 = \text{nine blue circles with '-' signs}$ is 9.

A.8 FINDING THE ABSOLUTE VALUE FOR DECIMAL NUMBERS

Ex 37: The absolute value of -2.1 is $\boxed{2.1}$.

Answer: The absolute value of -2.1 is 2.1

Ex 38: The absolute value of -5.4 is $\boxed{5.4}$.

Answer: The absolute value of -5.4 is 5.4.

Ex 39: The absolute value of 3.7 is $\boxed{3.7}$.

Answer: The absolute value of 3.7 is 3.7.

Ex 40: The absolute value of 0 is $\boxed{0}$.

Answer: The absolute value of 0 is 0.

B.2 ADDING INTEGERS WITHOUT EXPLICIT SIGNS

Ex 46: $6 + (-4) = \boxed{+2}$

Answer:

- $6 + (-4) = (+6) + (-4)$
- When the signs are opposite, subtract the absolute values ($6 - 4 = 2$), and take the sign of the larger number: $6 > 4$, so the result is $+$.

- $(+6) + (-4) = +2$



Ex 47: $-5 + 8 = \boxed{+3}$

Answer:

- $-5 + 8 = (-5) + (+8)$
- When the signs are opposite, subtract the absolute values ($8 - 5 = 3$), and take the sign of the larger number: $8 > 5$, so the result is $+$.

- $(-5) + (+8) = +3$



Ex 48: $-2 + (-3) = \boxed{-5}$

Answer:

- $-2 + (-3) = (-2) + (-3)$
- When the signs are the same, add the absolute values ($2 + 3 = 5$), and keep the negative sign.

- $(-2) + (-3) = -5$



Ex 49: $-6 + 0 = \boxed{-6}$

Answer:

- $-6 + 0 = (-6) + 0$
- Adding zero to any number does not change the value, so the result is -6 .

- $(-6) + 0 = -6$

B.3 ADDING SIGNED DECIMAL NUMBERS

Ex 50: $-5 + 8.1 = \boxed{+3.1}$

Answer:

- $-5 + 8.1 = (-5) + (+8.1)$
- When the signs are opposite, subtract the absolute values ($8.1 - 5 = 3.1$), and take the sign of the larger number: $8.1 > 5$, so the result is $+$.

- $(-5) + (+8.1) = +3.1$

Ex 51: $-3 + (-2.5) = \boxed{-5.5}$

Answer:

- $-3 + (-2.5) = (-3) + (-2.5)$
- When the signs are the same, add the absolute values ($3 + 2.5 = 5.5$), and keep the negative sign.
- $(-3) + (-2.5) = -5.5$

Ex 52: $-1.6 + (+2.6) = \boxed{+1}$

Answer:

- $-1.6 + (+2.6) = (-1.6) + (+2.6)$
- When the signs are opposite, subtract the absolute values ($2.6 - 1.6 = 1.0$), and take the sign of the larger number: $2.6 > 1.6$, so the result is $+$.
- $(-1.6) + (+2.6) = +1.0$

Ex 53: $-3.5 + (+1.5) = \boxed{-2}$

Answer:

- $-3.5 + (+1.5) = (-3.5) + (+1.5)$
- When the signs are opposite, subtract the absolute values ($3.5 - 1.5 = 2.0$), and take the sign of the larger number: $3.5 > 1.5$, so the result is $-$.
- $(-3.5) + (+1.5) = -2.0$

B.4 ADDING MULTIPLE INTEGERS

Ex 54: Calculate:

$$(+3) + (-7) + (-5) = \boxed{-9}$$

Answer:

$$\begin{aligned} (+3) + (-7) + (-5) &= (-4) + (-5) & ((+3) + (-7) = (-4)) \\ &= (-9) & ((-4) + (-5) = (-9)) \end{aligned}$$

Ex 55: Calculate:

$$(-2) + (-4) + (+7) = \boxed{+1}$$

Answer:

$$\begin{aligned} (-2) + (-4) + (+7) &= (-6) + (+7) & ((-2) + (-4) = (-6)) \\ &= +1 & ((-6) + (+7) = +1) \end{aligned}$$

Ex 56: Calculate:

$$(-2) + (+4) + (-2) = \boxed{0}$$

Answer:

$$\begin{aligned} (-2) + (+4) + (-2) &= (+2) + (-2) & ((-2) + (+4) = +2) \\ &= 0 & ((+2) + (-2) = 0) \end{aligned}$$

Ex 57: Calculate:

$$(-10) + (+3) + (-7) = \boxed{-14}$$

Answer:

$$\begin{aligned} (-10) + (+3) + (-7) &= (-7) + (-7) & ((-10) + (+3) = -7) \\ &= -14 & ((-7) + (-7) = -14) \end{aligned}$$

B.5 ADDING INTEGERS IN REAL-WORLD PROBLEMS

Ex 58: During a hike, the hiker experiences altitude changes as follows. Positive numbers indicate climbing (gaining altitude), while negative numbers indicate descending (losing altitude):

- The hiker starts at an altitude of +300 meters.
- They climb +150 meters in the morning.
- In the afternoon, they descend by -200 meters.

What is the hiker's final altitude at the end of the day?

$$\boxed{250} \text{ meters}$$

Answer:

$$\begin{aligned} (+300) + (+150) + (-200) &= (+450) + (-200) & ((+300) + (+150) &= +450) \\ &= +250 & ((+450) + (-200) &= +250) \end{aligned}$$

Ex 59: A person keeps track of their bank account balance as follows. Positive numbers indicate deposits (money added), while negative numbers indicate withdrawals (money taken out):

- The person starts with +50 dollars in their account.
- They deposit +30 dollars.
- Later, they withdraw -40 dollars.

What is the person's final balance?

$$\boxed{40} \text{ dollars}$$

Answer:

$$\begin{aligned} (+50) + (+30) + (-40) &= (+80) + (-40) & ((+50) + (+30) &= +80) \\ &= +40 & ((+80) + (-40) &= +40) \end{aligned}$$

Ex 60: In a round of golf, each hole has a "par" score, and a player's score is based on how many strokes they take compared to par. A score of 0 means the player made par, a positive number means they took extra strokes (over par), and a negative number means they made fewer strokes (under par).

- Hole 1: Par 3, player scored -1 (under par)
- Hole 2: Par 4, player scored +2 (over par)
- Hole 3: Par 5, player scored 0 (made par)
- Hole 4: Par 3, player scored +1 (over par)
- Hole 5: Par 4, player scored -2 (under par)

What is the player's total score?

$$\boxed{0}$$

Answer:

$$\begin{aligned} (-1) + (+2) + (0) + (+1) + (-2) \\ &= (+1) + (0) + (+1) + (-2) & ((-1) + (+2) &= (+1)) \\ &= (+1) + (+1) + (-2) & ((+1) + (0) &= (+1)) \\ &= (+2) + (-2) & ((+1) + (+1) &= (+2)) \\ &= 0 & ((+2) + (-2) &= 0) \end{aligned}$$

Ex 61: Throughout the day, the temperature in a city changes as follows. Positive numbers indicate a rise in temperature, while negative numbers indicate a drop in temperature:

- In the morning, the temperature started at +5°C and dropped by -3°C.
- In the afternoon, the temperature rose by +4°C.
- In the evening, the temperature dropped again by -6°C.
- At night, it dropped further by -1°C.

What is the temperature at the end of the day?

$$\boxed{-1}^{\circ}\text{C}$$

Answer:

$$\begin{aligned} (+5) + (-3) + (+4) + (-6) + (-1) \\ &= (+2) + (+4) + (-6) + (-1) & ((+5) + (-3) &= (+2)) \\ &= (+6) + (-6) + (-1) & ((+2) + (+4) &= (+6)) \\ &= 0 + (-1) & ((+6) + (-6) &= 0) \\ &= -1 & (0 + (-1) &= -1) \end{aligned}$$

C SUBTRACTION

C.1 CONVERTING SUBTRACTION TO ADDITION

Ex 62: Convert the subtracting in addition:

$$(+4) - (+2) = \boxed{+4} + \boxed{-2}$$

Answer:

$$\begin{aligned} &\bullet (+4) - (+2) = (+4) + (-2) \\ &\bullet \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \end{aligned}$$

Ex 63: Convert the subtraction into addition:

$$(-5) - (-3) = \boxed{-5} + \boxed{+3}$$

Answer:

$$\begin{aligned} &\bullet (-5) - (-3) = (-5) + (+3) \\ &\bullet \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \end{aligned}$$

Ex 64: Convert the subtraction into addition:

$$(+4) - (-2) = \boxed{+4} + \boxed{+2}$$

Answer:

$$\begin{aligned} &\bullet (+4) - (-2) = (+4) + (+2) \\ &\bullet \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \end{aligned}$$

Ex 65: Convert the subtraction into addition:

$$(-1) - (+2) = \boxed{-1} + \boxed{-2}$$

Answer:

$$\begin{aligned} &\bullet (-1) - (+2) = (-1) + (-2) \\ &\bullet \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \end{aligned}$$

C.2 SUBTRACTING INTEGERS STEP BY STEP

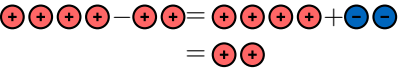
Ex 66: Calculate:

$$(+4) - (+2) = (\boxed{+4}) + (\boxed{-2})$$

$$= \boxed{2}$$

Answer:

- $(+4) - (+2) = (+4) + (-2)$
- $= (+2)$

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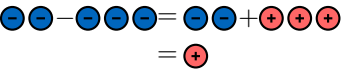
Ex 67: Calculate:

$$(-2) - (-3) = (\boxed{-2}) + (\boxed{+3})$$

$$= \boxed{+1}$$

Answer:

- $(-2) - (-3) = (-2) + (+3)$
- $= (+1)$

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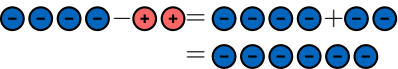
Ex 68: Calculate:

$$(-4) - (+2) = (\boxed{-4}) + (\boxed{-2})$$

$$= \boxed{-6}$$

Answer:

- $(-4) - (+2) = (-4) + (-2)$
- $= (-6)$

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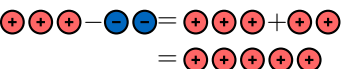
Ex 69: Calculate:

$$(+3) - (-2) = (\boxed{+3}) + (\boxed{+2})$$

$$= \boxed{+5}$$

Answer:

- $(+3) - (-2) = (+3) + (+2)$
- $= (+5)$

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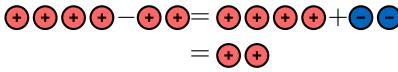
C.3 SUBTRACTING INTEGERS

Ex 70: Calculate:

$$(+4) - (+2) = \boxed{2}$$

Answer:

- $(+4) - (+2) = (+4) + (-2)$
- $= (+2)$

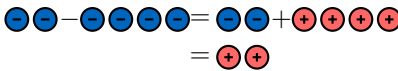
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Ex 71: Calculate:

$$(-2) - (-4) = \boxed{+2}$$

Answer:

- $(-2) - (-4) = (-2) + (+4)$
- $= (+2)$

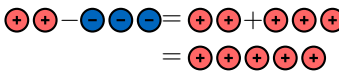
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Ex 72: Calculate:

$$(+2) - (-3) = \boxed{+5}$$

Answer:

- $(+2) - (-3) = (+2) + (+3)$
- $= (+5)$

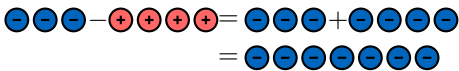
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Ex 73: Calculate:

$$(-3) - (+4) = \boxed{-7}$$

Answer:

- $(-3) - (+4) = (-3) + (-4)$
- $= (-7)$

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C.4 SUBTRACTING INTEGERS WITHOUT EXPLICIT SIGNS

Ex 74: Calculate:

$$3 - (-2) = \boxed{5}$$

Answer:

- $3 - (-2) = (+3) - (-2)$
- $= (+3) + (+2)$
- $= (+5)$

$$\begin{array}{c} \bullet \\ \begin{array}{c} \text{(+)} \text{(+)} \text{(+)} - \text{(-)} \text{(-)} = \text{(+)} \text{(+)} \text{(+)} + \text{(+)} \text{(+)} \\ = \text{(+)} \text{(+)} \text{(+)} \text{(+)} \text{(+)} \end{array} \end{array}$$

Ex 75: Calculate:

$$(-2) - 3 = \boxed{-5}$$

Answer:

$$\begin{array}{l} \bullet \quad (-2) - 3 = \text{(-2)} - \text{(+3)} \\ \quad \quad \quad = \text{(-2)} + \text{(-3)} \\ \quad \quad \quad = \text{(-5)} \end{array}$$

$$\begin{array}{c} \bullet \\ \begin{array}{c} \text{(-)} \text{(-)} - \text{(+)} \text{(+)} \text{(+)} = \text{(-)} \text{(-)} + \text{(-)} \text{(-)} \text{(-)} \\ = \text{(-)} \text{(-)} \text{(-)} \text{(-)} \end{array} \end{array}$$

Ex 76: Calculate:

$$(-3) - (-5) = \boxed{+2}$$

Answer:

$$\begin{array}{l} \bullet \quad (-3) - (-5) = \text{(-3)} - \text{(-5)} \\ \quad \quad \quad = \text{(-3)} + \text{(+5)} \\ \quad \quad \quad = \text{(+2)} \end{array}$$

$$\begin{array}{c} \bullet \\ \begin{array}{c} \text{(-)} \text{(-)} \text{(-)} - \text{(-)} \text{(-)} \text{(-)} \text{(-)} = \text{(-)} \text{(-)} \text{(-)} + \text{(+)} \text{(+)} \text{(+)} \text{(+)} \\ = \text{(+)} \text{(+)} \end{array} \end{array}$$

Ex 77: Calculate:

$$3 - 5 = \boxed{-2}$$

Answer:

$$\begin{array}{l} \bullet \quad 3 - 5 = \text{(+3)} - \text{(+5)} \\ \quad \quad \quad = \text{(+3)} + \text{(-5)} \\ \quad \quad \quad = \text{(-2)} \end{array}$$

$$\begin{array}{c} \bullet \\ \begin{array}{c} \text{(+)} \text{(+)} \text{(+)} - \text{(+)} \text{(+)} \text{(+)} \text{(+)} = \text{(+)} \text{(+)} \text{(+)} + \text{(-)} \text{(-)} \text{(-)} \text{(-)} \\ = \text{(-)} \text{(-)} \end{array} \end{array}$$

C.5 ADDING/SUBTRACTING MULTIPLE INTEGERS

Ex 78: Calculate:

$$(+3) - (-7) - (+5) = \boxed{+5}$$

Answer:

$$\begin{array}{ll} (+3) - (-7) - (-5) = (+3) + (+7) + (-5) & \text{(subtraction to addition)} \\ \quad \quad \quad = (+10) + (-5) & ((+3)+(+7)=(+10)) \\ \quad \quad \quad = (+5) & ((+10)+(-5)=(+5)) \end{array}$$

Ex 79: Calculate:

$$(-2) - (-3) + (-2) = \boxed{-1}$$

Answer:

$$\begin{array}{ll} (-2) - (-3) + (-2) = (-2) + (+3) + (-2) & \text{(subtraction to addition)} \\ \quad \quad \quad = (+1) + (-2) & ((-2)+(+3)=(+1)) \\ \quad \quad \quad = (-1) & ((+1)+(-2)=(-1)) \end{array}$$

Ex 80: Calculate:

$$(-5) - (-4) + (-3) = \boxed{-4}$$

Answer:

$$\begin{array}{ll} (-5) - (-4) + (-3) = (-5) + (+4) + (-3) & \text{(subtraction to addition)} \\ \quad \quad \quad = (-1) + (-3) & ((-5)+(+4)=(-1)) \\ \quad \quad \quad = (-4) & ((-1)+(-3)=(-4)) \end{array}$$

Ex 81: Calculate:

$$(+6) - (-3) + (-4) = \boxed{+5}$$

Answer:

$$\begin{array}{ll} (+6) - (-3) + (-4) = (+6) + (+3) + (-4) & \text{(subtraction to addition)} \\ \quad \quad \quad = (+9) + (-4) & ((+6)+(+3)=(+9)) \\ \quad \quad \quad = (+5) & ((+9)+(-4)=(+5)) \end{array}$$

C.6 SUBTRACTING INTEGERS IN REAL-WORLD PROBLEMS

Ex 82: In the morning, the temperature was -7°C , and by the evening, the temperature was -2°C . Find the variation of temperature.

$$\boxed{5}^{\circ}\text{C}$$

Answer:

$$\begin{array}{l} \text{Temperature variation} = \text{Final Temperature} - \text{Initial Temperature} \\ \quad \quad \quad = (-2) - (-7) \\ \quad \quad \quad = (-2) + (+7) \quad \text{(subtraction to addition)} \\ \quad \quad \quad = +5^{\circ}\text{C} \end{array}$$

Ex 83: In the morning, your bank account balance was -50 dollars, and by the evening, it was $+30$ dollars. Find the change in your bank account balance.

$$\boxed{80} \text{ dollars}$$

Answer:

$$\begin{array}{l} \text{Change in Balance} = \text{Final Balance} - \text{Initial Balance} \\ \quad \quad \quad = 30 - (-50) \\ \quad \quad \quad = 30 + 50 \quad \text{(subtraction to addition)} \\ \quad \quad \quad = 80 \text{ dollars} \end{array}$$

Ex 84: A lift starts at the 5th floor and descends to the -2nd floor (below ground level). Find the change in the lift's position.

$$\boxed{-7} \text{ floors}$$

Answer:

$$\begin{array}{l} \text{Change in Position} = \text{Final Floor} - \text{Initial Floor} \\ \quad \quad \quad = (-2) - 5 \\ \quad \quad \quad = (-2) + (-5) \quad \text{(subtraction to addition)} \\ \quad \quad \quad = -7 \text{ floors} \end{array}$$

Ex 85: The GDP (Gross Domestic Product) growth rate of a country was -2% in 2024. It was $+3\%$ in 2025. Find the variation in the GDP growth rate.

$$\boxed{5}\%$$

Answer:

$$\text{GDP Variation} = \text{Final GDP} - \text{Initial GDP}$$

$$\text{Variation du PIB} = \text{Taux de croissance final du PIB} - \text{Taux de croissance initial du PIB}$$

$$= (+3) - (-2)$$

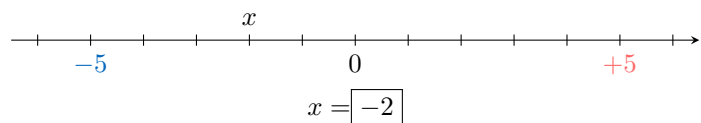
$$= (+3) + (+2) \quad (\text{subtraction to addition})$$

$$= +5\%$$

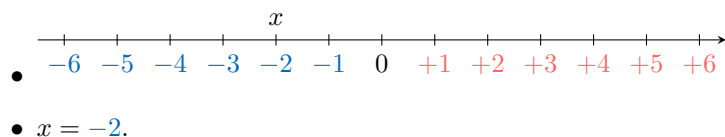
D ON THE NUMBER LINE

D.1 FINDING X ON THE NUMBER LINE

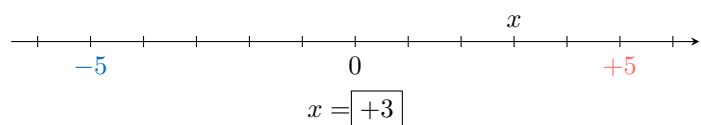
Ex 86: Find the value of x .



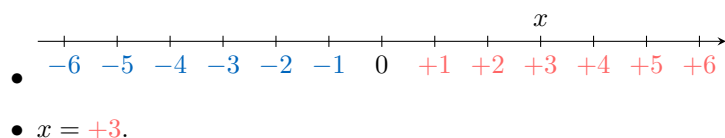
Answer:



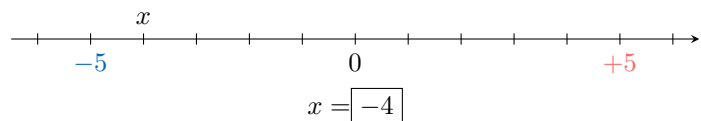
Ex 87: Find the value of x .



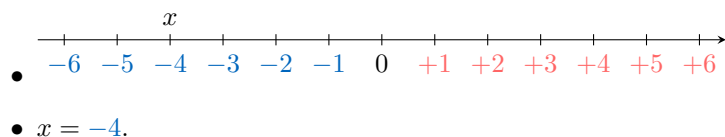
Answer:



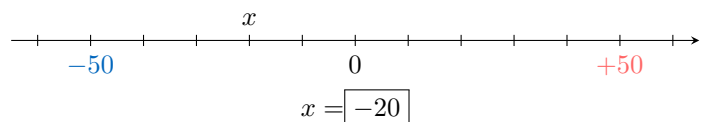
Ex 88: Find the value of x .



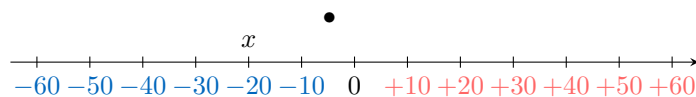
Answer:



Ex 89: Find the value of x .

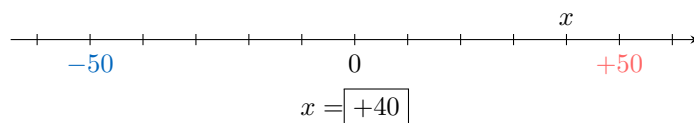


Answer:

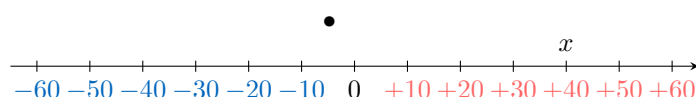


$$\bullet x = -20.$$

Ex 90: Find the value of x .

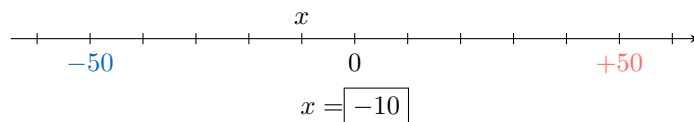


Answer:

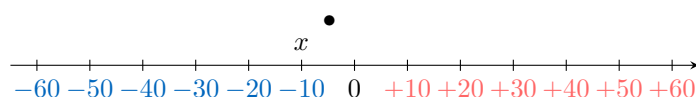


$$\bullet x = +40.$$

Ex 91: Find the value of x .



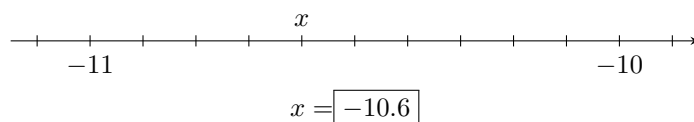
Answer:



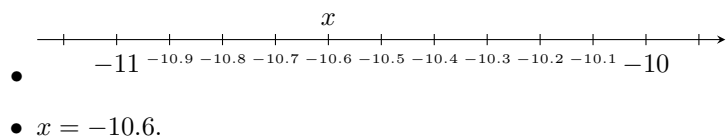
$$\bullet x = -10.$$

D.2 FINDING DECIMAL NUMBERS ON THE NUMBER LINE

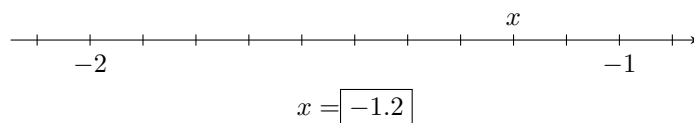
Ex 92: Find the value of x .



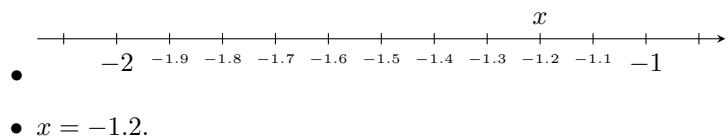
Answer:



Ex 93: Find the value of x .

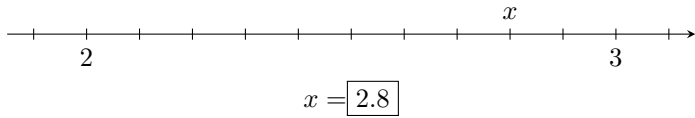


Answer:

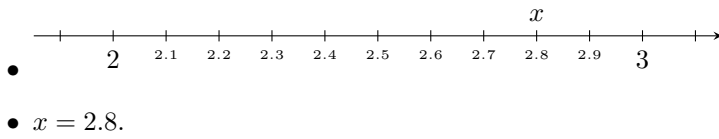


$$\bullet x = -1.2.$$

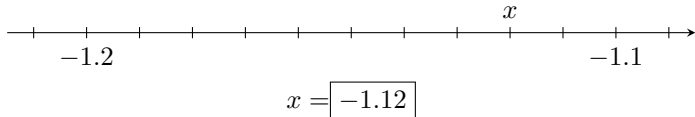
Ex 94: Find the value of x .



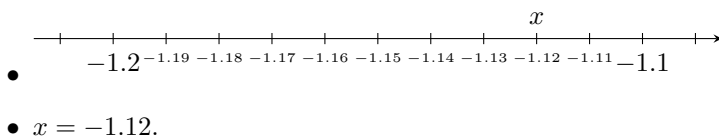
Answer:



Ex 95: Find the value of x .



Answer:



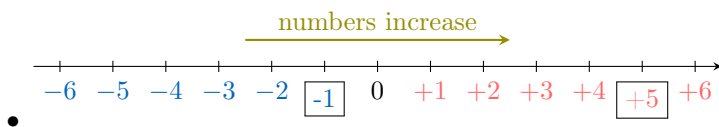
E ORDERING

E.1 COMPARING SMALL INTEGERS

Ex 96: Compare: -1 $<$ $+5$

Answer:

- As $+5$ is positive and -1 is negative, the positive number is greater than the negative number: $-1 < +5$

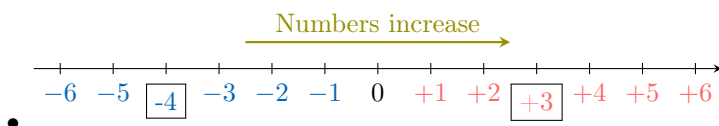


Ex 97: Compare the numbers: -4 $<$ $+3$

Answer:

- $+3$ is positive, and -4 is negative. A positive number is always greater than a negative number:

$$-4 < +3$$



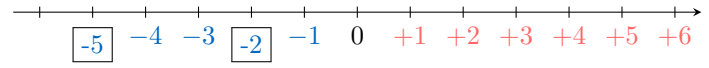
Ex 98: Compare the numbers: -2 $>$ -5

Answer:

- Both numbers are negative, but -2 is closer to zero than -5 . Therefore, -2 is greater than -5 :

$$-2 > -5$$

Numbers increase

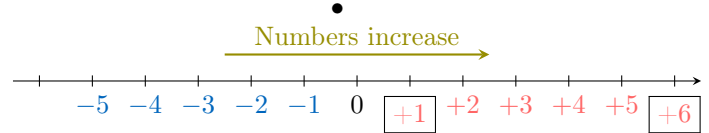


Ex 99: Compare the numbers: $+6$ $>$ $+1$

Answer:

- Both numbers are positive, but $+6$ is farther from zero than $+1$. Therefore, $+6$ is greater than $+1$:

$$+6 > +1$$

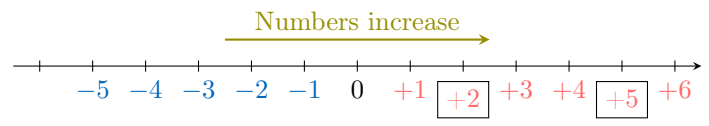


Ex 100: Compare the numbers: $+2$ $<$ $+5$

Answer:

- Both numbers are positive, but $+5$ is farther from zero than $+2$:

$$+2 < +5$$

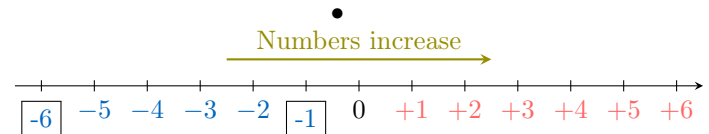


Ex 101: Compare the numbers: -6 $<$ -1

Answer:

- Both numbers are negative, but -1 is closer to zero than -6 :

$$-6 < -1$$

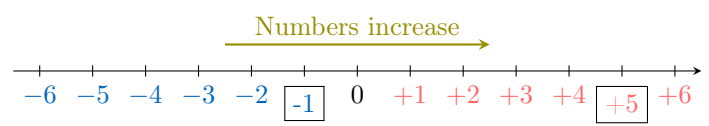


Ex 102: Compare the numbers: -1 $<$ $+5$

Answer:

- $+5$ is positive and -1 is negative. A positive number is always greater than a negative number:

$$-1 < +5$$



E.2 COMPARING INTEGERS

Ex 103: Compare the numbers: $-20 \square 1$

Answer:

- 1 is positive and -20 is negative. A positive number is always greater than a negative number:

$$-20 < +1$$

Ex 104: Compare the numbers: $-99 \square -100$

Answer:

- Both numbers are negative, but -99 is closer to zero than -100 :

$$-99 > -100$$

Ex 105: Compare the numbers: $234 \square -1200$

Answer:

- $+234$ is positive and -1200 is negative. A positive number is always greater than a negative number:

$$+234 > -1200$$

Ex 106: Compare the numbers: $-18 \square -3$

Answer:

- Both numbers are negative, but -3 is closer to zero than -18 :

$$-18 < -3$$

Ex 107: Compare the numbers: $230 \square 200$

Answer:

- Both numbers are positive, but $+230$ is farther from zero than $+200$:

$$+230 > +200$$

Ex 108: Compare the numbers: $99 \square -100$

Answer:

- $+99$ is positive and -100 is negative. A positive number is always greater than a negative number:

$$+99 > -100$$

E.3 COMPARING INTEGERS IN REAL-WORLD PROBLEMS

MCQ 109: During a golf tournament, the scores relative to par for five different holes were:

$$-2, -1, 0, +1, -3$$

Order these scores from the best performance (most under par) to the worst performance (above par).

Choose one answer:

☐ $0 < +1 < -1 < -2 < -3$

☒ $-3 < -2 < -1 < 0 < +1$

☐ $+1 > 0 > -1 > -2 > -3$

☐ $-1 < -2 < -3 < 0 < +1$

Answer:

- From the best performance to the worst: $-3 < -2 < -1 < 0 < +1$

MCQ 110: Given the depths of various lakes below sea level:

Lake	Depth below sea level
Lake Assal, Djibouti	-155 m
Death Valley, USA	-86 m
Caspian Sea, Central Asia	-28 m
Sea of Galilee, Israel	-214 m

Which lake is the deepest below sea level?

Choose one answer:

☐ Lake Assal, Djibouti

☐ Death Valley, USA

☐ Caspian Sea, Central Asia

☒ Sea of Galilee, Israel

Answer: The lake that is the deepest below sea level is the Sea of Galilee, Israel, with a depth of -214 m.

MCQ 111: The recorded temperatures in a particular week were:

$$-2.5^{\circ}\text{C}, 1.2^{\circ}\text{C}, -0.8^{\circ}\text{C}, 0.5^{\circ}\text{C}, -3.2^{\circ}\text{C}$$

Order these temperatures from coldest to warmest.

Choose one answer:

☐ $1.2^{\circ}\text{C} < 0.5^{\circ}\text{C} < -0.8^{\circ}\text{C} < -2.5^{\circ}\text{C}$

☒ $-3.2^{\circ}\text{C} < -2.5^{\circ}\text{C} < -0.8^{\circ}\text{C} < 0.5^{\circ}\text{C} < 1.2^{\circ}\text{C}$

☐ $1.2^{\circ}\text{C} > 0.5^{\circ}\text{C} > -0.8^{\circ}\text{C} > -3.2^{\circ}\text{C}$

☐ $-0.8^{\circ}\text{C} < -2.5^{\circ}\text{C} < -3.2^{\circ}\text{C} < 0.5^{\circ}\text{C} < 1.2^{\circ}\text{C}$

Answer:

- From coldest to warmest: $-3.2^{\circ}\text{C} < -2.5^{\circ}\text{C} < -0.8^{\circ}\text{C} < 0.5^{\circ}\text{C} < 1.2^{\circ}\text{C}$

MCQ 112: Given the years of significant events in Ancient Roman history:

Event	Year
Founding of Rome	-753
End of the Roman Republic	-27
Sacking of Rome by the Gauls	-390
Julius Caesar's assassination	-44

Which event happened the earliest?

Choose one answer:

☒ Founding of Rome

☐ End of the Roman Republic

☐ Sacking of Rome by the Gauls

☐ Julius Caesar's assassination

Answer: The event that happened the earliest was the Founding of Rome in -753 .

F MULTIPLICATION

F.1 MULTIPLYING SMALL INTEGERS

Ex 113: Calculate:

$$(-2) \times (-7) = \boxed{+14}$$

Answer:

- Multiply the absolute values: $2 \times 7 = 14$.
- When you multiply two negative numbers, the result is positive: $(-) \times (-) = (+)$.
- Therefore, $(-2) \times (-7) = +14$.

Ex 114: Calculate:

$$(-4) \times (+6) = \boxed{-24}$$

Answer:

- Multiply the absolute values: $4 \times 6 = 24$.
- When you multiply a negative number by a positive number, the result is negative: $(-) \times (+) = (-)$.
- Therefore, $(-4) \times (+6) = -24$.

Ex 115: Calculate:

$$(+5) \times (-3) = \boxed{-15}$$

Answer:

- Multiply the absolute values: $5 \times 3 = 15$.
- When you multiply a positive number by a negative number, the result is negative: $(+) \times (-) = (-)$.
- Therefore, $(+5) \times (-3) = -15$.

Ex 116: Calculate:

$$(-6) \times (-2) = \boxed{+12}$$

Answer:

- Multiply the absolute values: $6 \times 2 = 12$.
- When you multiply two negative numbers, the result is positive: $(-) \times (-) = (+)$.
- Therefore, $(-6) \times (-2) = +12$.

Ex 117: Calculate:

$$(+3) \times (+5) = \boxed{+15}$$

Answer:

- Multiply the absolute values: $3 \times 5 = 15$.
- Multiplying two positive numbers gives a positive result: $(+) \times (+) = (+)$.
- Therefore, $(+3) \times (+5) = +15$.

Ex 118: Calculate:

$$(-1) \times (-1) = \boxed{+1}$$

Answer:

- Multiply the absolute values: $1 \times 1 = 1$.
- When you multiply two negative numbers, the result is positive: $(-) \times (-) = (+)$.
- Therefore, $(-1) \times (-1) = +1$.

F.2 MULTIPLYING INTEGERS WITHOUT EXPLICIT SIGNS

Ex 119: Calculate:

$$3 \times (-2) = \boxed{-6}$$

Answer:

$$\begin{aligned} 3 \times (-2) &= (+3) \times (-2) \\ &= -6 \quad ((+) \times (-) = (-)) \end{aligned}$$

Ex 120: Calculate:

$$(-3) \times 8 = \boxed{-24}$$

Answer:

$$\begin{aligned} (-3) \times 8 &= (-3) \times (+8) \\ &= -24 \quad ((-) \times (+) = (-)) \end{aligned}$$

Ex 121: Calculate:

$$(-5) \times (-8) = \boxed{+40}$$

Answer:

$$\begin{aligned} (-5) \times (-8) &= (-5) \times (-8) \\ &= +40 \quad ((-) \times (-) = (+)) \end{aligned}$$

Ex 122: Calculate:

$$(-6) \times 9 = \boxed{-54}$$

Answer:

$$\begin{aligned} (-6) \times 9 &= (-6) \times (+9) \\ &= -54 \quad ((-) \times (+) = (-)) \end{aligned}$$

F.3 CALCULATING POWERS OF NEGATIVE NUMBERS

Ex 123: Calculate:

$$(-2)^2 = \boxed{4}$$

Answer:

$$\begin{aligned} (-2)^2 &= (-2) \times (-2) \\ &= +4 \quad ((-) \times (-) = (+)) \end{aligned}$$

Ex 124: Calculate:

$$(-4)^2 = \boxed{16}$$

Answer:

$$\begin{aligned} (-4)^2 &= (-4) \times (-4) \\ &= +16 \quad ((-) \times (-) = (+)) \end{aligned}$$

Ex 125: Calculate:

$$(-1)^3 = \boxed{-1}$$

Answer:

$$\begin{aligned} (-1)^3 &= (-1) \times (-1) \times (-1) \\ &= (+1) \times (-1) \quad ((-1) \times (-1) = (+1)) \\ &= -1 \end{aligned}$$

Ex 126: Calculate:

$$(-2)^3 = \boxed{-8}$$

Answer:

$$\begin{aligned} (-2)^3 &= (-2) \times (-2) \times (-2) \\ &= (+4) \times (-2) \quad ((-2) \times (-2) = (+4)) \\ &= -8 \end{aligned}$$

F.4 MULTIPLYING INTEGERS IN REAL-WORLD PROBLEMS

Ex 127: At midnight, the temperature was 0 degrees. The temperature continued to change by -3 degrees every hour. What was the temperature 4 hours later?

$$\boxed{-12}^{\circ}\text{C}$$

Answer:

- The temperature changes by -3°C every hour.
- In 4 hours, the total change in temperature is $4 \times -3 = -12^{\circ}\text{C}$.
- Since the temperature started at 0°C , 4 hours later, the temperature will be $0 + (-12) = -12^{\circ}\text{C}$.

Ex 128: A diver starts at sea level (0 meters). The diver descends 5 meters every minute. How deep is the diver after 6 minutes?

$$\boxed{-30} \text{ meters}$$

Answer:

- The diver descends by 5 meters every minute.
- In 6 minutes, the total change in depth is $6 \times -5 = -30$ meters.
- Since the diver started at 0 meters, after 6 minutes, the diver will be $0 + (-30) = -30$ meters deep.

Ex 129: A hiker is at an altitude of 150 meters. The hiker descends by 10 meters every minute. What is the hiker's altitude after 7 minutes?

$$\boxed{80} \text{ meters}$$

Answer:

- The hiker descends by 10 meters every minute.
- In 7 minutes, the total descent is $7 \times -10 = -70$ meters.
- Starting at 150 meters, the hiker's altitude will be $150 + (-70) = 80$ meters after 7 minutes.

Ex 130: Your bank account has a balance of 90 euros. You make a withdrawal of 40 euros every day for 5 days. What is your account balance after 5 days?

$$\boxed{-110} \text{ euros}$$

Answer:

- You withdraw 40 euros every day.
- In 5 days, the total withdrawal is $5 \times -40 = -200$ euros.
- Starting with 90 euros, your balance will be $90 + (-200) = -110$ euros after 5 days.

G DIVISION

G.1 DIVIDING SMALL INTEGERS

Ex 131: Calculate:

$$(+8) \div (-2) = \boxed{-4}$$

Answer:

- Divide the absolute values: $8 \div 2 = 4$.
- When you divide a positive number by a negative number, the result is negative: $(+) \div (-) = (-)$.
- Therefore, $(+8) \div (-2) = -4$.

Ex 132: Calculate:

$$(-12) \div (-3) = \boxed{+4}$$

Answer:

- Divide the absolute values: $12 \div 3 = 4$.
- When you divide two negative numbers, the result is positive: $(-) \div (-) = (+)$.
- Therefore, $(-12) \div (-3) = +4$.

Ex 133: Calculate:

$$(-15) \div (+5) = \boxed{-3}$$

Answer:

- Divide the absolute values: $15 \div 5 = 3$.
- When you divide a negative number by a positive number, the result is negative: $(-) \div (+) = (-)$.
- Therefore, $(-15) \div (+5) = -3$.

Ex 134: Calculate:

$$(+20) \div (+4) = \boxed{+5}$$

Answer:

- Divide the absolute values: $20 \div 4 = 5$.
- When you divide two positive numbers, the result is positive:
 $(+) \div (+) = (+)$.
- Therefore, $(+20) \div (+4) = +5$.

G.2 DIVIDING INTEGERS WITHOUT EXPLICIT SIGNS

Ex 135: Calculate:

$$8 \div (-2) = \boxed{-4}$$

Answer:

$$\begin{aligned} 8 \div (-2) &= (+8) \div (-2) \\ &= -4 \quad ((+) \div (-) = (-)) \end{aligned}$$

Ex 136: Calculate:

$$-12 \div (-3) = \boxed{+4}$$

Answer:

$$\begin{aligned} -12 \div (-3) &= (-12) \div (-3) \\ &= +4 \quad ((-) \div (-) = (+)) \end{aligned}$$

Ex 137: Calculate:

$$-18 \div 6 = \boxed{-3}$$

Answer:

$$\begin{aligned} -18 \div 6 &= (-18) \div (+6) \\ &= -3 \quad ((-) \div (+) = (-)) \end{aligned}$$

Ex 138: Calculate:

$$24 \div 4 = \boxed{6}$$

Answer:

$$\begin{aligned} 24 \div 4 &= (+24) \div (+4) \\ &= +6 \quad ((+) \div (+) = (+)) \end{aligned}$$