INTERESTS

A DEFINITIONS

A.1 FINDING THE INTEREST

Ex 1: Louis lends Hugo \$100. After one year, Hugo repays Louis \$110.
Find the interest paid.

$$\boxed{10}\ \$$$

Answer: The interest paid is the difference between the amount repaid and the original amount lent:

$$\begin{aligned}\text{Interest} &= \text{Amount repaid} - \text{Original amount}\\ &= 110 - 100\\ &= 10\ \$\end{aligned}$$

Ex 2: Maria borrows \$200 from John. After one year, Maria repays John \$230.
Find the interest paid.

$$\boxed{30}\ \$$$

Answer: The interest paid is the difference between the amount repaid and the original amount lent:

$$\begin{aligned}\text{Interest} &= \text{Amount repaid} - \text{Original amount}\\ &= 230 - 200\\ &= 30\ \$\end{aligned}$$

Ex 3: Jack lends Sarah \$500. After one year, Sarah repays Jack \$525.
Find the interest paid.

$$\boxed{25}\ \$$$

Answer: The interest paid is the difference between the amount repaid and the original amount lent:

$$\begin{aligned}\text{Interest} &= \text{Amount repaid} - \text{Original amount}\\ &= 525 - 500\\ &= 25\ \$\end{aligned}$$

Ex 4: A bank lends \$1 000 to a customer. After one year, the customer repays the bank \$1 080.
Find the interest paid.

$$\boxed{80}\ \$$$

Answer: The interest paid is the difference between the amount repaid and the original amount lent:

$$\begin{aligned}\text{Interest} &= \text{Amount repaid} - \text{Original amount}\\ &= 1\,080 - 1\,000\\ &= 80\ \$\end{aligned}$$

A.2 FINDING THE TOTAL AMOUNT

Ex 5: A customer borrows \$2 500 from a bank, with \$150 of interest.
Find the total amount the customer needs to repay the bank.

$$\boxed{2650}\ \$$$

Answer: The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest:

$$\begin{aligned}\text{Amount to repay} &= \text{Principal} + \text{Interest}\\ &= 2\,500 + 150\\ &= 2\,650\ \$\end{aligned}$$

Ex 6: Maria borrows \$300 from John with \$30 of interest.
Find the amount Maria needs to repay.

$$\boxed{330}\ \$$$

Answer: The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest:

$$\begin{aligned}\text{Amount to repay} &= \text{Principal} + \text{Interest}\\ &= 300 + 30\\ &= 330\ \$\end{aligned}$$

Ex 7: Jack lends Sarah \$500 with \$50 of interest.
Find the total amount Sarah needs to repay Jack.

$$\boxed{550}\ \$$$

Answer: The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest:

$$\begin{aligned}\text{Amount to repay} &= \text{Principal} + \text{Interest}\\ &= 500 + 50\\ &= 550\ \$\end{aligned}$$

Ex 8: A bank lends \$1 000 to a customer with \$80 of interest.
Find the total amount the customer needs to repay the bank.

$$\boxed{1080}\ \$$$

Answer: The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest:

$$\begin{aligned}\text{Amount to repay} &= \text{Principal} + \text{Interest}\\ &= 1\,000 + 80\\ &= 1\,080\ \$\end{aligned}$$

A.3 FINDING THE PRINCIPAL

Ex 9: Emma repaid \$330 in total, including \$30 of interest. Find the original amount (principal) that Emma borrowed.

$$\boxed{300}\ \$$$

Answer: The principal is the difference between the total amount repaid and the interest paid:

$$\begin{aligned}\text{Principal} &= \text{Amount repaid} - \text{Interest}\\ &= 330 - 30\\ &= 300\ \$\end{aligned}$$

Ex 10: Lucas repaid \$550 in total, including \$50 of interest. Find the original amount (principal) that Lucas borrowed.

$$\boxed{500}\ \$$$

Answer: The principal is the difference between the total amount repaid and the interest paid:

$$\begin{aligned}\text{Principal} &= \text{Amount repaid} - \text{Interest}\\ &= 550 - 50\\ &= 500\ \$\end{aligned}$$

Ex 11: Sophia repaid \$1 080 in total, including \$80 of interest.
Find the original amount (principal) that Sophia borrowed.

$$\boxed{1000}\ \$$$

Answer: The principal is the difference between the total amount repaid and the interest paid:

$$\begin{aligned}\text{Principal} &= \text{Amount repaid} - \text{Interest}\\ &= 1\,080 - 80\\ &= 1\,000\ \$\end{aligned}$$

Ex 12: Mia repaid \$750 in total, including \$150 of interest. Find the original amount (principal) that Mia borrowed.

$$\boxed{600}\ \$$$

Answer: The principal is the difference between the total amount repaid and the interest paid:

$$\begin{aligned}\text{Principal} &= \text{Amount repaid} - \text{Interest}\\ &= 750 - 150\\ &= 600\ \$\end{aligned}$$

B SIMPLE INTEREST

B.1 FINDING THE INTEREST

Ex 13: Find the simple interest on a principal of \$500 at a rate of 3% per year over 5 years.

$$\boxed{75}\ \$$$

Answer:

$$\begin{aligned}\text{Interest} &= \text{Number of years} \times \text{Percentage of the principal}\\ &= 5 \times 3\% \text{ of } 500\\ &= 5 \times \frac{3}{100} \times 500\\ &= 75\ \$\end{aligned}$$

Ex 14: Find the simple interest on a principal of \$1 000 at a rate of 4% per year over 3 years.

$$\boxed{120}\ \$$$

Answer:

$$\begin{aligned}\text{Interest} &= \text{Number of years} \times \text{Percentage of the principal}\\ &= 3 \times 4\% \text{ of } 1\,000\\ &= 3 \times \frac{4}{100} \times 1\,000\\ &= 120\ \$\end{aligned}$$

Ex 15: Find the simple interest on a principal of \$750 at a rate of 5% per year over 2 years.

$$\boxed{75}\ \$$$

Answer:

$$\begin{aligned}\text{Interest} &= \text{Number of years} \times \text{Percentage of the principal}\\ &= 2 \times 5\% \text{ of } 750\\ &= 2 \times \frac{5}{100} \times 750\\ &= 75\ \$\end{aligned}$$

Ex 16: Find the simple interest on a principal of \$1 200 at a rate of 6% per year over 4 years.

$$\boxed{288}\ \$$$

Answer:

$$\begin{aligned}\text{Interest} &= \text{Number of years} \times \text{Percentage of the principal}\\ &= 4 \times 6\% \text{ of } 1\,200\\ &= 4 \times \frac{6}{100} \times 1\,200\\ &= 288\ \$\end{aligned}$$

B.2 FINDING THE INTEREST OVER MIXED TIME PERIODS

Ex 17: Find the simple interest on a principal of \$600 at a rate of 4% per year over 18 months.

$\boxed{36}$ \$

Answer:

- Convert the time from months to years:

$$\begin{aligned} 18 \text{ months} &= \frac{18}{12} \text{ years} \\ &= 1.5 \text{ years} \end{aligned}$$

- Calculate the interest:

$$\begin{aligned} \text{Interest} &= \text{Number of years} \times \text{Percentage of the principal} \\ &= 1.5 \times 4\% \text{ of } 600 \\ &= 1.5 \times \frac{4}{100} \times 600 \\ &= 36 \text{ \$} \end{aligned}$$

Ex 18: Find the simple interest on a principal of \$700 at a rate of 5% per year over 180 days.

$\boxed{17.26}$ \$ (round at two decimal places)

Answer:

- Convert the time from days to years:

$$\begin{aligned} 180 \text{ days} &= \frac{180}{365} \text{ years} \\ &\approx 0.493 \text{ years} \end{aligned}$$

- Calculate the interest:

$$\begin{aligned} \text{Interest} &= \text{Number of years} \times \text{Percentage of the principal} \\ &= 0.493 \times 5\% \text{ of } 700 \\ &= 0.493 \times \frac{5}{100} \times 700 \\ &= 17.26 \text{ \$} \end{aligned}$$

Ex 19: Find the simple interest on a principal of \$800 at a rate of 4% per year over 9 months.

$\boxed{24}$ \$

Answer:

- Convert the time from months to years:

$$\begin{aligned} 9 \text{ months} &= \frac{9}{12} \text{ years} \\ &= 0.75 \text{ years} \end{aligned}$$

- Calculate the interest:

$$\begin{aligned} \text{Interest} &= \text{Number of years} \times \text{Percentage of the principal} \\ &= 0.75 \times 4\% \text{ of } 800 \\ &= 0.75 \times \frac{4}{100} \times 800 \\ &= 24 \text{ \$} \end{aligned}$$

Ex 20: Find the simple interest on a principal of \$1 200 at a rate of 4% per year over 2 years and 6 months.

$\boxed{120}$ \$

Answer:

- Convert the time from years and months to just years:

$$\begin{aligned} 2 \text{ years } 6 \text{ months} &= 2 + \frac{6}{12} \text{ years} \\ &= 2 + 0.5 \text{ years} \\ &= 2.5 \text{ years} \end{aligned}$$

- Calculate the interest:

$$\begin{aligned} \text{Interest} &= \text{Number of years} \times \text{Percentage of the principal} \\ &= 2.5 \times 4\% \text{ of } 1\,200 \\ &= 2.5 \times \frac{4}{100} \times 1\,200 \\ &= 120 \text{ \$} \end{aligned}$$

B.3 FINDING THE TOTAL AMOUNT

Ex 21: Jack lends Sarah \$500 with simple interest over 3 years at a rate of 3% per year.
Find the total amount Sarah needs to repay Jack.

$\boxed{545}$ \$

Answer:

- The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest.
- Calculate the interest:

$$\begin{aligned} \text{Interest} &= \text{Number of years} \times \text{Percentage of the principal} \\ &= 3 \times \frac{3}{100} \times 500 \\ &= 45 \text{ \$} \end{aligned}$$

- Calculate the total amount to repay:

$$\begin{aligned} \text{Amount to repay} &= \text{Principal} + \text{Interest} \\ &= 500 + 45 \\ &= 545 \text{ \$} \end{aligned}$$

Ex 22: Emma borrows \$600 from a bank with simple interest over 4 years at a rate of 2.5% per year.
Find the total amount Emma needs to repay the bank.

$\boxed{660}$ \$

Answer:

- The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest.
- Calculate the interest:

$$\begin{aligned}\text{Interest} &= \text{Number of years} \times \text{Percentage of the principal}\\ &= 4 \times \frac{2.5}{100} \times 600\\ &= 60\ \$\end{aligned}$$

- Calculate the total amount to repay:

$$\begin{aligned}\text{Amount to repay} &= \text{Principal} + \text{Interest}\\ &= 600 + 60\\ &= 660\ \$\end{aligned}$$

Ex 23: Michael lends \$800 to a friend with simple interest over 2 years at a rate of 4% per year.
Find the total amount the friend needs to repay Michael.

$\boxed{864}$ \$

Answer:

- The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest.
- Calculate the interest:

$$\begin{aligned}\text{Interest} &= \text{Number of years} \times \text{Percentage of the principal}\\ &= 2 \times \frac{4}{100} \times 800\\ &= 64\ \$\end{aligned}$$

- Calculate the total amount to repay:

$$\begin{aligned}\text{Amount to repay} &= \text{Principal} + \text{Interest}\\ &= 800 + 64\\ &= 864\ \$\end{aligned}$$

Ex 24: Sophia borrows \$1 200 with simple interest over 5 years at a rate of 2.5% per year.
Find the total amount Sophia needs to repay.

$\boxed{1\,350}$ \$

Answer:

- The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest.
- Calculate the interest:

$$\begin{aligned}\text{Interest} &= \text{Number of years} \times \text{Percentage of the principal}\\ &= 5 \times \frac{2.5}{100} \times 1\,200\\ &= 150\ \$\end{aligned}$$

- Calculate the total amount to repay:

$$\begin{aligned}\text{Amount to repay} &= \text{Principal} + \text{Interest}\\ &= 1\,200 + 150\\ &= 1\,350\ \$\end{aligned}$$

C COMPOUND INTEREST

C.1 FINDING THE TOTAL AMOUNT USING A TABLE

Ex 25: \$1 000 is placed in an account that earns 10% interest per annum (p.a.), and the interest is allowed to compound over three years. This means the account is earning 10% p.a. in compound interest.
Fill in the compound interest table:

Year	Amount	Compound interest
0	\$1 000	10% of \$1 000 = \$100
1	\$1 000 + \$100 = \$1 100	10% of \$1 100 = \$110
2	\$ $\boxed{1210}$	$\boxed{121}$
3	\$ $\boxed{1331}$	

Find the amount at 3 years.

$\boxed{1331}$ dollars

Answer:

Year	Amount	Compound interest
0	\$1 000	10% of \$1 000 = \$100
1	\$1 000 + \$100 = \$1 100	10% of \$1 100 = \$110
2	\$1 100 + \$110 = \$1 210	10% of \$1 210 = \$121
3	\$1 210 + \$121 = \$1 331	

The amount at 3 years is \$1 331.

Ex 26: \$3 000 is placed in an account that earns 20% interest per annum (p.a.), and the interest is allowed to compound over three years. This means the account is earning 20% p.a. in compound interest.
Fill in the compound interest table:

Year	Amount	Compound interest
0	\$3 000	20% of \$3 000 = \$600
1	\$3 000 + \$600 = \$3 600	20% of \$3 600 = \$720
2	\$ $\boxed{4320}$	$\boxed{864}$
3	\$ $\boxed{5184}$	

Find the amount at 3 years.

$\boxed{5184}$ dollars

Answer:

Year	Amount	Compound interest
0	\$3 000	20% of \$3 000 = \$600
1	\$3 000 + \$600 = \$3 600	20% of \$3 600 = \$720
2	\$3 600 + \$720 = \$4 320	20% of \$4 320 = \$864
3	\$4 320 + \$864 = \$5 184	

The amount at 3 years is \$5 184.

Ex 27: \$1 000 is placed in an account that earns 5% interest per annum (p.a.), and the interest is allowed to compound over two years.
Fill in the compound interest table:

Year	Amount	Compound interest
0	\$1 000	50
1	\$ 1050	52.5
2	\$ 1102.5	—-

Find the amount after 2 years.

1102.5 dollars

Answer:

Year	Amount	Compound interest
0	\$1 000	5% of \$1 000 = \$50
1	\$1 000 + \$50 = \$1 050	5% of \$1 050 = \$52.5
2	\$1 050 + \$52.5 = \$1 102.5	—-

The amount after 2 years is \$1 102.5.

C.2 FINDING THE TOTAL AMOUNT

Ex 28: Find the final amount on a principal of \$10 000 at a rate of 10% per year over 3 years compounded yearly.

13310 dollars

Answer:

- **Method 1: Amount year by year**
  - Year 0: Initial amount = \$10 000
  - Year 1: \$10 000 + 10% of 10 000 = \$11 000
  - Year 2: \$11 000 + 10% of 11 000 = \$12 100
  - Year 3: \$12 100 + 10% of 12 100 = \$13 310

  So, the final amount after 3 years is \$13 310.
- **Method 2: Using the compound interest formula**

$$\begin{aligned} A &= P(1+r)^t \\ &= 10\,000 \times (1+0.10)^3 \\ &= 10\,000 \times 1.1^3 \\ &= 13\,310 \end{aligned}$$

Thus, the final amount after 3 years is \$13 310.

Ex 29: Find the final amount on a principal of \$200 000 at a rate of 5% per year over 3 years compounded yearly.

231525 dollars

Answer:

- **Method 1: Amount year by year**
  - Year 0: Initial amount = \$200 000
  - Year 1: \$200 000 + 5% of 200 000 = \$210 000
  - Year 2: \$210 000 + 5% of 210 000 = \$220 500
  - Year 3: \$220 500 + 5% of 220 500 = \$231 525

  So, the final amount after 3 years is \$231 525.
- **Method 2: Using the compound interest formula**

$$\begin{aligned} A &= P(1+r)^t \\ &= 200\,000 \times (1+0.05)^3 \\ &= 200\,000 \times 1.157625 \\ &= 231\,525 \end{aligned}$$

Thus, the final amount after 3 years is \$231 525.

Ex 30: Find the final amount on a principal of \$5 000 at a rate of 8% per year over 2 years compounded yearly.

5832 dollars

Answer:

- **Method 1: Amount year by year**
  - Year 0: Initial amount = \$5 000
  - Year 1: \$5 000 + 8% of 5 000 = \$5 400
  - Year 2: \$5 400 + 8% of 5 400 = \$5 832

  So, the final amount after 2 years is \$5 832.
- **Method 2: Using the compound interest formula**

$$\begin{aligned} A &= P(1+r)^t \\ &= 5\,000 \times (1+0.08)^2 \\ &= 5\,000 \times 1.1664 \\ &= 5\,832 \end{aligned}$$

Thus, the final amount after 2 years is \$5 832.

Ex 31: Find the final amount on a principal of \$5 000 at a rate of 8% per year over 20 years compounded yearly (round to the nearest integer).

23305 dollars

Answer: Using the compound interest formula,

$$\begin{aligned} A &= P(1+r)^t \\ &= 5\,000 \times (1+0.08)^{20} \\ &\approx 23\,305 \end{aligned}$$

Thus, the final amount after 20 years is \$23 305.

C.3 FINDING THE BEST OPTION OF INVESTMENT

Ex 32: You have \$8 000 to invest for 5 years and there are 2 possible options you have been offered:

- Option 1: Invest at 9% p.a. simple interest.
- Option 2: Invest at 8% p.a. compound interest.
- Calculate the amount accumulated at the end of the 5 years for option 1 (round to the nearest integer):

11600 dollars

- Calculate the amount accumulated at the end of the 5 years for option 2 (round to the nearest integer):

$\boxed{11755}$ dollars

- Decide which option to take.
Option 2

Answer:

- **Option 1: Simple Interest**
Use the simple interest formula:

$$\begin{aligned} A &= (1 + t \times r) \times P \\ &= (1 + 5 \times 0.09) \times 8\,000 \\ &= (1 + 0.45) \times 8\,000 \\ &= 1.45 \times 8\,000 \\ &= 11\,600 \text{ dollars} \end{aligned}$$

Thus, the accumulated amount for option 1 after 5 years is \$11 600.

- **Option 2: Compound Interest**
Use the compound interest formula:

$$\begin{aligned} A &= (1 + r)^t \times P \\ &= (1 + 0.08)^5 \times 8\,000 \\ &\approx 1.4693 \times 8\,000 \\ &\approx 11\,755 \text{ dollars} \end{aligned}$$

Thus, the accumulated amount for option 2 after 5 years is \$11 755.

- **Conclusion**
Option 1 (simple interest): \$11 600
Option 2 (compound interest): \$11 755
Since option 2 gives a higher final amount, it is better to choose option 2 with compound interest.

Ex 33: You have \$20 000 to invest for 5 years and there are 2 possible options you have been offered:

- Option 1: Invest at 7% p.a. simple interest.
- Option 2: Invest at 6% p.a. compound interest.
- Calculate the amount accumulated at the end of the 5 years for option 1 (round to the nearest integer):

$\boxed{27000}$ dollars

- Calculate the amount accumulated at the end of the 5 years for option 2 (round to the nearest integer):

$\boxed{26765}$ dollars

- Decide which option to take.
Option 1

Answer:

- **Option 1: Simple Interest**

$$\begin{aligned} A &= (1 + t \times r) \times P \\ &= (1 + 5 \times 0.07) \times 20\,000 \\ &= (1 + 0.35) \times 20\,000 \\ &= 1.35 \times 20\,000 \\ &= 27\,000 \text{ dollars} \end{aligned}$$

- **Option 2: Compound Interest**

$$\begin{aligned} A &= (1 + r)^t \times P \\ &= (1 + 0.06)^5 \times 20\,000 \\ &\approx 26\,765 \text{ dollars} \end{aligned}$$

- **Conclusion**
Option 1 (simple interest): \$27 000
Option 2 (compound interest): \$26 765
Since option 1 gives a higher final amount, it is better to choose option 1 with simple interest.

Ex 34: You have \$50 000 to invest for 30 years and there are 2 possible options you have been offered:

- Option 1: Invest at 10% p.a. simple interest.
- Option 2: Invest at 9% p.a. compound interest.
- Calculate the amount accumulated at the end of the 30 years for option 1 (round to the nearest integer):

$\boxed{200000}$ dollars

- Calculate the amount accumulated at the end of the 30 years for option 2 (round to the nearest integer):

$\boxed{663384}$ dollars

- Decide which option to take.
Option 2

Answer:

- **Option 1: Simple Interest**

$$\begin{aligned} A &= (1 + t \times r) \times P \\ &= (1 + 30 \times 0.10) \times 50\,000 \\ &= (1 + 3.0) \times 50\,000 \\ &= 4.0 \times 50\,000 \\ &= 200\,000 \text{ dollars} \end{aligned}$$

- **Option 2: Compound Interest**

$$\begin{aligned} A &= (1 + r)^t \times P \\ &= (1 + 0.09)^{30} \times 50\,000 \\ &\approx 13.2677 \times 50\,000 \\ &\approx 663\,384 \text{ dollars} \end{aligned}$$

- **Conclusion**
Option 1 (simple interest): \$200 000
Option 2 (compound interest): \$663 384
Since option 2 gives a much higher final amount, it is better to choose option 2 with compound interest.