INTERESTS

A DEFINITIONS

Discover: We've all heard of interest rates—whether on a mortgage, a credit card, or a loan. But what does it really mean?
Interest is essentially the "rent" you pay for borrowing money. It's the extra amount you pay to use someone else's money for a certain period.
Example of interest:
Imagine you borrow \$100 today and promise to pay it back in one year. If you return exactly \$100 after one year, there's no interest. However, the lender might want to be compensated for letting you use their money.
They may ask for a percentage as interest. For example, at a 10% interest rate per year, the interest paid is:

$$\begin{aligned}\text{Interest Paid} &= \text{Percentage of the Original Amount}\\ &= \text{Interest Rate} \times \text{Original Amount}\\ &= 10\% \times 100\\ &= \frac{10}{100} \times 100\\ &= 10 \text{ dollars}\end{aligned}$$

Therefore, after one year, you owe:

$$\begin{aligned}\text{Amount at Year 1} &= \text{Original Amount} + \text{Interest Paid}\\ &= 100 + 10\\ &= 110 \text{ dollars}\end{aligned}$$

So you would pay back \$110 instead of \$100. The extra \$10 is the interest—the cost of borrowing for a year.

Definition **Principal**

The **principal** is the original amount of money that is either invested or loaned.

Definition **Interest**

Interest is the cost paid for borrowing money or the amount earned from lending or investing money.

B SIMPLE INTEREST

Discover: Suppose you borrow \$100 with an interest rate of 10% per year. With simple interest, the interest is calculated only on the initial amount each year.

- Total interest after 1 year $= 10\% \times 100$
$$= \frac{10}{100} \times 100$$
$$= 10 \text{ dollars}$$
- Total interest after 2 years $= 2 \times 10\% \times 100$
$$= 2 \times \frac{10}{100} \times 100$$
$$= 20 \text{ dollars}$$
- Total interest after 3 years $= 3 \times 10\% \times 100$
$$= 3 \times \frac{10}{100} \times 100$$
$$= 30 \text{ dollars}$$

These observations lead to the simple interest formula:

$$\text{Simple Interest} = \text{Number of years} \times \text{Interest rate} \times \text{Principal}$$

Definition **Simple Interest**

The **simple interest** is calculated each year as a fixed percentage of the principal (original amount) borrowed or invested.

Proposition **Simple Interest Formula**

The simple interest, denoted by I, is calculated as:

$$I = t \times r \times P$$

where:

- P is the principal (original amount)
- r is the interest rate per year (expressed as a decimal)
- t is the time (in years)

The final amount, denoted by A, is:

$$A = P + I$$

Ex: Find the simple interest on a principal of \$500 at a rate of 3% per year over 5 years.

Answer:

$$\begin{aligned}\text{Interest} &= 5 \times 3\% \times 500 \\ &= 5 \times \frac{3}{100} \times 500 \\ &= 75 \text{ dollars}\end{aligned}$$

C COMPOUND INTEREST

Discover: If you leave money in the bank for a period of time, the interest earned is automatically added to your account. After the interest is added, it also begins to earn interest in the next time period. This process is called **compound interest**.
Example of compound interest: \$1 000 is placed in an account that earns 10% interest per annum (p.a.), and the interest is allowed to compound over three years. This means the account is earning 10% p.a. in compound interest.
We can illustrate this in a table:

Year	Amount	Interest Earned
0	\$1 000	10% of \$1 000 = \$100
1	\$1 000 + \$100 = \$1 100	10% of \$1 100 = \$110
2	\$1 100 + \$110 = \$1 210	10% of \$1 210 = \$121
3	\$1 210 + \$121 = \$1 331	—

After 3 years, there will be a total of \$1 331 in the account, meaning you have earned \$331 in compound interest.
You can also calculate the final amount using a different method:

- $\begin{aligned}\text{Amount after 1 year} &= \text{Initial amount} + \text{Interest on the initial amount} \\ &= 1\,000 + 0.1 \times 1\,000 \\ &= (1 + 0.1) \times 1\,000 \\ &= 1.1 \times 1\,000\end{aligned}$
- $\begin{aligned}\text{Amount after 2 years} &= \text{Amount after 1 year} + \text{Interest on the amount after 1 year} \\ &= 1.1 \times 1\,000 + 0.1 \times 1.1 \times 1\,000 \\ &= (1 + 0.1) \times 1.1 \times 1\,000 \\ &= 1.1^2 \times 1\,000\end{aligned}$
- $\begin{aligned}\text{Amount after 3 years} &= \text{Amount after 2 years} + \text{Interest on the amount after 2 years} \\ &= 1.1^2 \times 1\,000 + 0.1 \times 1.1^2 \times 1\,000 \\ &= (1 + 0.1) \times 1.1^2 \times 1\,000 \\ &= 1.1^3 \times 1\,000\end{aligned}$

These observations lead to the compound interest formula:

$$\text{Final amount} = (1 + \text{Interest rate})^{\text{Number of years}} \times \text{Initial amount}$$

Definition **Compound Interest**

Compound interest is interest that accumulates on both the principal sum and the previously accumulated interest.

Proposition **Annual Compound Interest Formula**

The final amount of an investment with interest compounded annually is:

$$A = P(1+r)^t$$

where:

- P is the principal,
- r is the annual interest rate (as a decimal),
- t is the time (in years).

Ex: Find the final amount for compound interest on a principal of \$500 at a rate of 3% per year over 5 years.

Answer:

$$\begin{aligned} A &= P(1+r)^t \qquad \textit{The final amount is approximately} \\ &= 500 \times (1+0.03)^5 \\ &\approx \$580.81 \end{aligned}$$

\$580.81.