INTERESTS

A DEFINITIONS

> **Definition Principal**
>
> The **principal** is the original amount of money that is either invested or loaned.

> **Definition Interest**
>
> **Interest** is the cost paid for borrowing money or the amount earned from lending or investing money.

B SIMPLE INTEREST

> **Definition Simple Interest**
>
> The **simple interest** is calculated each year as a fixed percentage on the principal (original amount) of money borrowed or invested.

> **Proposition Simple Interest Formula**
>
> The simple interest, denoted by I, is calculated as:
>
> $$I = t \times r \times P$$
>
> where:
>
> - P is the principal (original amount)
> - r is the interest rate per year
> - t is the time (in years)
>
> The final amount, denoted by A, is:
> $$\begin{aligned} A &= P + I \\ &= P + t \times r \times P \\ &= (1 + t \times r) \times P \end{aligned}$$

Ex: Find the simple interest on a principal of \$500 at a rate of 3% per year over 5 years.

Answer:

$$\begin{aligned} \text{Interest} &= 5 \times 3\% \text{ of } 500 \\ &= 5 \times \frac{3}{100} \times 500 \\ &= 75 \text{ dollars} \end{aligned}$$

> **Method Finding the Principal**
>
> To find the principal P, use the formula:
> $$P = \frac{I}{t \times r}$$

Ex: An investment earns \$100 over 5 years at a rate of 5% per year. Find the principal.

Answer:

$$\begin{aligned} P &= \frac{I}{t \times r} \\ &= \frac{100}{5 \times \frac{5}{100}} \\ &= 400 \text{ dollars} \end{aligned}$$

The principal is \$400.

Method Finding the Rate

To find the rate r, use the formula:

$$r = \frac{I}{t \times P}$$

Ex: A principal of $500 earns $60 over 5 years. Find the interest rate per year.

Answer:

$$
\begin{aligned}
r &= \frac{I}{t \times P} \\
&= \frac{60}{5 \times 500} \\
&= 0.024 \\
&= \frac{2.4}{100} \\
&= 2.4\%
\end{aligned}
$$

The interest rate per year is 2.4%.

Method Finding the Number of Years

To find the number of years t, use the formula:

$$t = \frac{I}{r \times P}$$

Ex: A principal of $1\,000 earns $300 at an interest rate of 3% per year. Find the number of years.

Answer:

$$
\begin{aligned}
t &= \frac{I}{r \times P} \\
&= \frac{300}{\frac{3}{100} \times 1\,000} \\
&= 10 \text{ years}
\end{aligned}
$$

The number of years is 10.

C COMPOUND INTEREST

Definition Compound Interest

Compound interest is interest that accumulates on both the principal sum and the previously accumulated interest.

Proposition Annual Compound Interest Formula

The final amount of an investment with interest compounded annually is:

$$A = P(1 + r)^t$$

where:

- P is the principal,
- r is the annual interest rate,
- t is the time (in years).

Ex: Find the final amount for compound interest on a principal of $500 at a rate of 3% per year over 5 years.

Answer:

$$
\begin{aligned}
A &= P(1 + r)^t \\
&= 500 \times (1 + 0.03)^5 \\
&\approx \$580.81
\end{aligned}
$$

The final amount is approximately $580.81.