LINEAR FUNCTIONS

A DEFINITION

A.1 FINDING THE FUNCTION FROM A MACHINE PROCESS

Ex 1: Consider the following calculation program:

1. Choose a number.
2. Multiply by 5.
3. Add 2.

Let x be the chosen number. Find the function that gives the output of this program.

$$f(x) = \boxed{5x+2}$$

Answer: Step by step:

1. Choose a number: x
2. Multiply by 5: $5x$
3. Add 2: $5x+2$

So the function is:

$$f(x) = 5x+2$$

Ex 2: Consider the following calculation program:

1. Choose a number.
2. Multiply by 2.
3. Subtract 3.

Let x be the chosen number. Find the function that gives the output of this program.

$$f(x) = \boxed{2x-3}$$

Answer: Step by step:

1. Choose a number: x
2. Multiply by 2: $2x$
3. Subtract 3: $2x-3$

So the function is:

$$f(x) = 2x-3$$

Ex 3: Consider the following calculation program:

1. Choose a number.
2. Divide by 2.
3. Add 2.

Let x be the chosen number. Find the function that gives the output of this program.

$$f(x) = \boxed{\frac{x}{2}+2}$$

Answer: Step by step:

1. Choose a number: x
2. Divide by 2: $\frac{x}{2}$
3. Add 2: $\frac{x}{2}+2$

So the function is:

$$f(x) = \frac{x}{2}+2$$

Ex 4: Consider the following calculation program:

1. Choose a number.
2. Multiply by $\frac{2}{3}$.
3. Subtract 2.

Let x be the chosen number. Find the function that gives the output of this program.

$$f(x) = \boxed{\frac{2}{3}x-2}$$

Answer: Step by step:

1. Choose a number: x
2. Multiply by $\frac{2}{3}$: $\frac{2}{3}x$
3. Subtract 2: $\frac{2}{3}x-2$

So the function is:

$$f(x) = \frac{2}{3}x-2$$

A.2 MODELING SITUATIONS WITH LINEAR FUNCTIONS

Ex 5: A mechanic charges a \$40 call-out fee and \$30 per hour thereafter.
Find the mechanic's fee $M(x)$ for a job which takes x hours.

$$M(x) = \boxed{40+30x}$$

Answer: The total fee, $M(x)$, is the sum of the fixed call-out fee and the variable cost, which depends on the number of hours, x. We can write this as an equation:

$$M(x) = (\text{Fixed Fee}) + (\text{Hourly Rate}) \times (\text{Number of hours})$$

Substituting the values from the problem:

$$\begin{aligned} M(x) &= \underbrace{40}_{\text{Fixed Fee}} + \underbrace{30\times x}_{\text{Cost for } x \text{ hours}} \\ &= 40+30x \end{aligned}$$

Therefore, the formula is $\boldsymbol{M(x) = 30x+40}$.

Ex 6: A taxi company charges a \$5 pick-up fee and \$2 per kilometer traveled.
Find the total fare $T(x)$ for a trip of x kilometers.

$$T(x) = \boxed{5+2x}$$

Answer: The total fare, $T(x)$, is the sum of the fixed pick-up fee and the variable cost, which depends on the distance in kilometers, x.
We can write this as an equation:

$$T(x) = (\text{Pick-up Fee}) + (\text{Cost per km}) \times (\text{Number of km})$$

Substituting the values from the problem:

$$\begin{aligned} T(x) &= \underbrace{5}_{\text{Pick-up Fee}} + \underbrace{2 \times x}_{\text{Cost for } x \text{ km}} \\ &= 5 + 2x \end{aligned}$$

Therefore, the formula is $\boldsymbol{T(x) = 2x + 5}$.

Ex 7: The temperature T in degrees Fahrenheit (°F) is related to the temperature x in degrees Celsius (°C) by the following rule:
Multiply by 1.8, then add 32.
Write the function that expresses T as a function of x.

$$T(x) = \boxed{1.8x + 32}$$

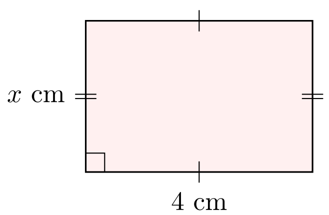
Answer: To convert Celsius to Fahrenheit:

- Multiply x by 1.8: $1.8x$
- Add 32: $1.8x + 32$

So, the function is:

$$T(x) = 1.8x + 32$$

Ex 8: A rectangle has a fixed width of 4 cm. Its length is x cm.



Express the perimeter $P(x)$ of the rectangle as a function of its length x.

$$P(x) = \boxed{2x + 8}$$

Answer: The perimeter of a rectangle is given by

$$P = 2 \times \text{length} + 2 \times \text{width}$$

Here, the width is 4, and the length is x:

$$\begin{aligned} P(x) &= 2 \times x + 2 \times 4 \\ &= 2x + 8 \end{aligned}$$

Ex 9: A water tank already contains 35 liters of water and fills at a rate of 10 liters per minute. Let x be the number of minutes the tank has been filling. Find $f(x)$ be the total amount of water in the tank in liters.

$$f(x) = \boxed{10x + 35} \text{ liters}$$

Answer:

- After 0 minutes, $f(0) = 10 \times 0 + 35 = 35$ liters.
- After 1 minute, $f(1) = 10 \times 1 + 35 = 45$ liters.
- After 2 minutes, $f(2) = 10 \times 2 + 35 = 55$ liters.
- $\vdots$
- After x minutes, $f(x) = 10x + 35$ liters.

Ex 10: A person starts walking at a constant speed of 5 kilometers per hour from a starting point that is 10 kilometers away from their destination. Let x be the number of hours they have been walking. Find $f(x)$ be the distance remaining to their destination in kilometers.

$$f(x) = \boxed{10 - 5x} \text{ kilometers}$$

Answer: The remaining distance, $f(x)$, is the initial distance minus the distance that has been walked. The distance walked is calculated by multiplying the speed by the time, x.
We can write this as an equation:

$$f(x) = (\text{Initial Distance}) - (\text{Speed} \times \text{Time})$$

Substituting the values from the problem:

$$\begin{aligned} f(x) &= \underbrace{10}_{\text{Initial Distance}} - \underbrace{5 \times x}_{\text{Distance Walked in } x \text{ hours}} \\ &= 10 - 5x \end{aligned}$$

Therefore, the formula is $\boldsymbol{f(x) = 10 - 5x}$.

A.3 FINDING a AND b

Ex 11: For the linear function $f(x) = 2x+1$, find the coefficients in the form $f(x) = ax + b$:

$$a = \boxed{2} \text{ and } b = \boxed{1}$$

Answer:

- The function $f(x) = 2x + 1$ is in the form $f(x) = ax + b$.
- The coefficients are $a = 2$ and $b = 1$.

Ex 12: For the linear function $f(x) = 5x-2$, find the coefficients in the form $f(x) = ax + b$:

$$a = \boxed{5} \text{ and } b = \boxed{-2}$$

Answer:

- The function $f(x) = 5x + -2$ is in the form $f(x) = ax + b$.
- The coefficients are $a = 5$ and $b = -2$.

Ex 13: For the linear function $f(x) = -x - 3$, find the coefficients in the form $f(x) = ax + b$:

$$a = \boxed{-1} \text{ and } b = \boxed{-3}$$

Answer:

- The function $f(x) = -1x + -3$ is in the form $f(x) = ax + b$.
- The coefficients are $a = -1$ and $b = -3$.

Ex 14: For the linear function $f(x) = 3-2x$, find the coefficients in the form $f(x) = ax + b$:

$$a = \boxed{-2} \text{ and } b = \boxed{3}$$

Answer: First, rewrite $f(x) = 3 - 2x$ as $f(x) = -2x + 3$ so it matches the form $ax + b$.

- The function $f(x) = -2x + 3$ is in the form $f(x) = ax + b$.
- The coefficients are $a = -2$ and $b = 3$.

A.4 FINDING $f(x)$

Ex 15: For $f(x) = 3x + 4$, find:

$$f(2) = \boxed{10}$$

Answer:

$$\begin{aligned} f(2) &= 3 \times 2 + 4 \\ &= 6 + 4 \\ &= 10 \end{aligned}$$

Ex 16: For $f(x) = -2x + 8$, find:

$$f(3) = \boxed{2}$$

Answer:

$$\begin{aligned} f(3) &= -2 \times 3 + 8 \\ &= -6 + 8 \\ &= 2 \end{aligned}$$

Ex 17: For $f(x) = \frac{1}{2}x + \frac{1}{2}$, find:

$$f(3) = \boxed{2}$$

Answer:

$$\begin{aligned} f(3) &= \frac{1}{2} \times 3 + \frac{1}{2} \\ &= \frac{3}{2} + \frac{1}{2} \\ &= \frac{4}{2} \\ &= 2 \end{aligned}$$

Ex 18: For $f(x) = -x - 1$, find:

$$f(-1) = \boxed{0}$$

Answer:

$$\begin{aligned} f(-1) &= -(-1) - 1 \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

A.5 FINDING x FOR $f(x) = c$

Ex 19: For $f(x) = 3x + 2$, find x for $f(x) = 14$:

$$x = \boxed{4}$$

Answer:

$$\begin{aligned} f(x) &= 3x + 2 \\ 14 &= 3x + 2 && \text{(substitute } f(x) = 14\text{)} \\ 3x &= 14 - 2 && \text{(subtract 2 from both sides)} \\ 3x &= 12 \\ x &= \frac{12}{3} && \text{(divide both sides by 3)} \\ x &= 4 \end{aligned}$$

Ex 20: For $f(x) = 5x - 3$, find x for $f(x) = 32$:

$$x = \boxed{7}$$

Answer:

$$\begin{aligned} f(x) &= 5x - 3 \\ 32 &= 5x - 3 && \text{(substitute } f(x) = 32\text{)} \\ 5x &= 32 + 3 && \text{(add 3 to both sides)} \\ 5x &= 35 \\ x &= \frac{35}{5} && \text{(divide both sides by 5)} \\ x &= 7 \end{aligned}$$

Ex 21: For $f(x) = -2x + 1$, find x for $f(x) = 5$:

$$x = \boxed{-2}$$

Answer:

$$\begin{aligned} f(x) &= -2x + 1 \\ 5 &= -2x + 1 && \text{(substitute } f(x) = 5\text{)} \\ -2x &= 5 - 1 && \text{(subtract 1 from both sides)} \\ -2x &= 4 \\ x &= \frac{4}{-2} && \text{(divide both sides by } -2\text{)} \\ x &= -2 \end{aligned}$$

Ex 22: For $f(x) = 6x + 1$, find x for $f(x) = 10$:

$$x = \boxed{\frac{3}{2}}$$

Answer:

$$\begin{aligned} f(x) &= 6x + 1 \\ 10 &= 6x + 1 && \text{(substitute } f(x) = 10\text{)} \\ 6x &= 10 - 1 && \text{(subtract 1 from both sides)} \\ 6x &= 9 \\ x &= \frac{9}{6} && \text{(divide both sides by 6)} \\ x &= \frac{3}{2} \end{aligned}$$

A.6 IDENTIFYING LINEAR FUNCTIONS

MCQ 23: Is $f(x) = 2x + 1$ a linear function?

☒ Yes

☐ No

Answer: Yes, $f(x) = 2x + 1$ is a linear function. It has the form $f(x) = ax + b$ with $a = 2$ and $b = 1$.

MCQ 24: Is $f(x) = x^2 + 2x - 1$ a linear function?

☐ Yes

☒ No

Answer: No, $f(x) = x^2 + 2x - 1$ is not a linear function. A linear function is of the form $f(x) = ax + b$. This function has a x^2 term, so it is quadratic.

MCQ 25: Is $f(x) = -2 + 2x$ a linear function?

☒ Yes

☐ No

Answer: Yes, $f(x) = -2+2x$ (or $f(x) = 2x-2$) is a linear function. It is of the form $f(x) = ax + b$ with $a = 2$ and $b = -2$.

MCQ 26: Is $f(x) = \frac{2}{x}$ a linear function?

☐ Yes

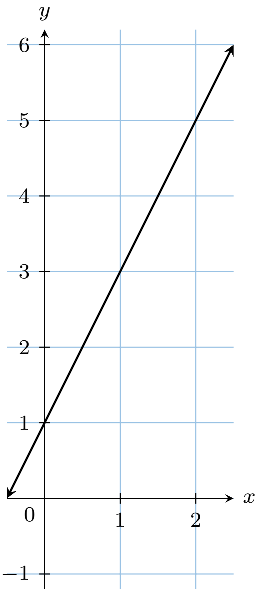
☒ No

Answer: No, $f(x) = \frac{2}{x}$ is not a linear function. Linear functions have the form $f(x) = ax + b$ with a and b constants. Here, the variable x is in the denominator.

B GRAPH OF A LINEAR FUNCTION

B.1 FINDING THE FUNCTION FROM THE GRAPH

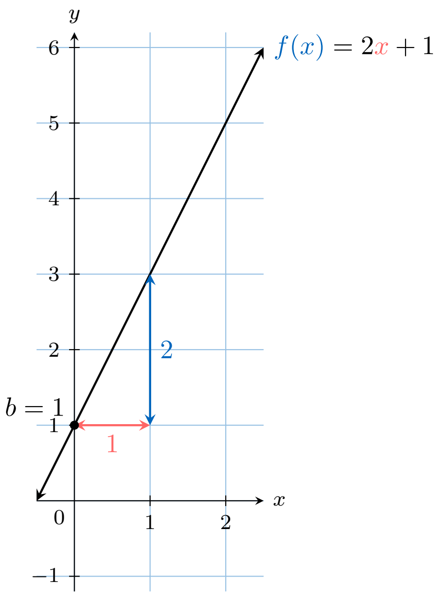
Ex 27: The graph of the function is shown below:



Find the function:

$$f(x) = \boxed{2x+1}$$

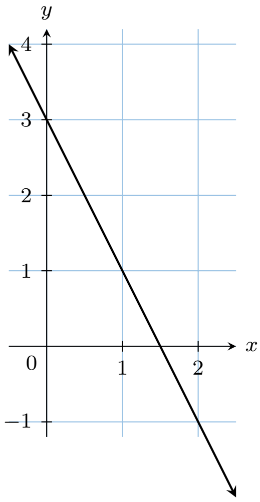
Answer:



- The graph is a line with a slope $a = \frac{2}{1} = 2$ and a y-intercept $b = 1$. Therefore, the function is:

$$f(x) = 2x + 1$$

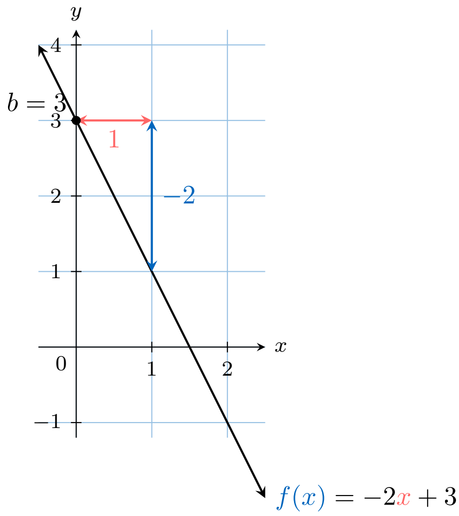
Ex 28: The graph of the function is shown below:



Find the function:

$$f(x) = \boxed{-2x+3}$$

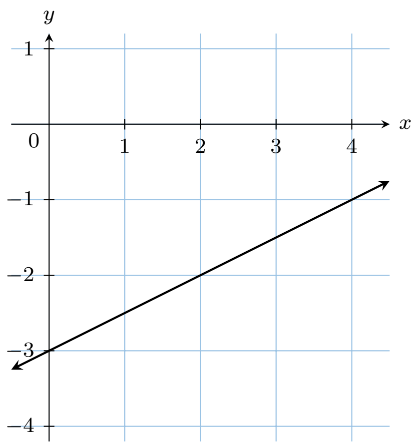
Answer:



- The graph is a line with a slope $a = \frac{-2}{1} = -2$ and a y-intercept $b = 3$. Therefore, the function is:

$$f(x) = -2x + 3$$

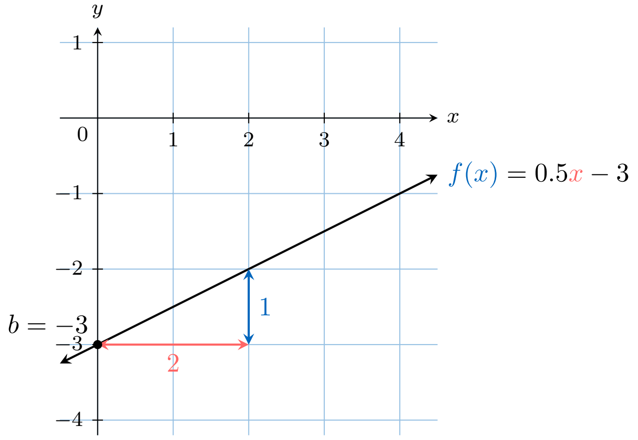
Ex 29: The graph of the function is shown below:



Find the function:

$$f(x) = \boxed{0.5x - 3}$$

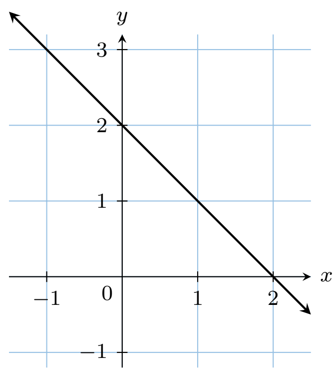
Answer:

- 

- The graph is a line with a slope $a = \frac{1}{2} = 0.5$ and a y-intercept $b = -3$. Therefore, the function is:

$$f(x) = 0.5x - 3$$

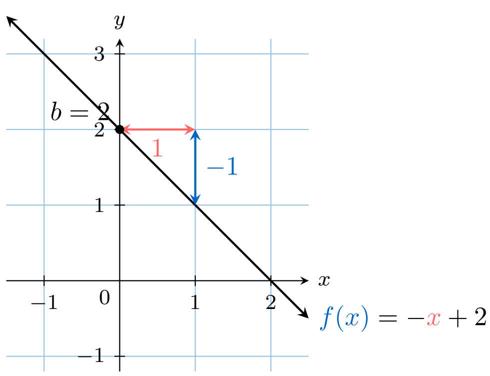
Ex 30: The graph of the function is shown below:



Find the function:

$$f(x) = \boxed{-x + 2}$$

Answer:

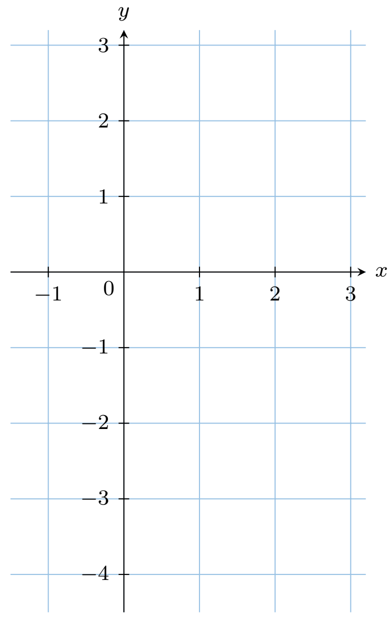
- 

- The graph is a line with a slope $a = \frac{-1}{1} = -1$ and a y-intercept $b = 2$. Therefore, the function is:

$$f(x) = -x + 2$$

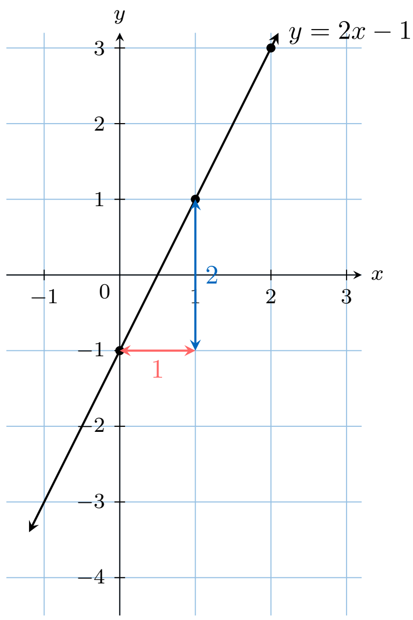
B.2 PLOTTING LINES FROM LINEAR FUNCTION

Ex 31: Plot the graph of the function $f(x) = 2x - 1$:

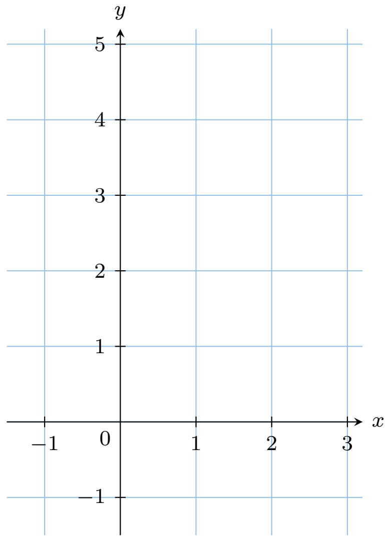


Answer:

- **y-Intercept and Slope Method:**
  - The y-intercept is -1, so plot the point $(0, -1)$.
  - The slope is 2. From $(0, -1)$, move 1 unit right ($\Delta x = 1$), then 2 units up ($\Delta y = 2$), to reach $(1, 1)$.
- **Two Points Method:**
  - Choose two values for x (e.g., $x = 0$ and $x = 2$).
  - When $x = 0$, $f(0) = 2 \times 0 - 1 = -1 \to$ point $(0, -1)$.
  - When $x = 2$, $f(2) = 2 \times 2 - 1 = 4 - 1 = 3 \to$ point $(2, 3)$.

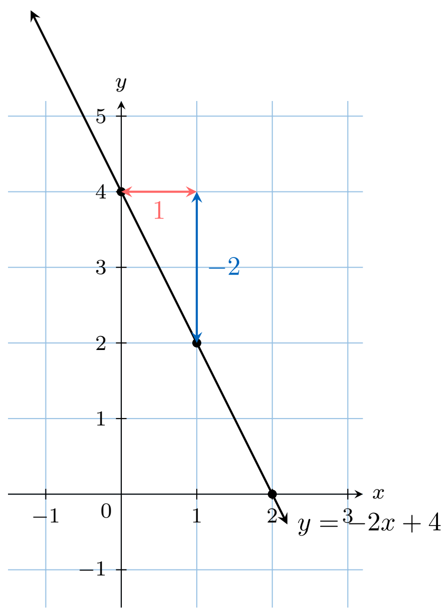


Ex 32: Plot the graph of the function $f(x) = -2x + 4$:

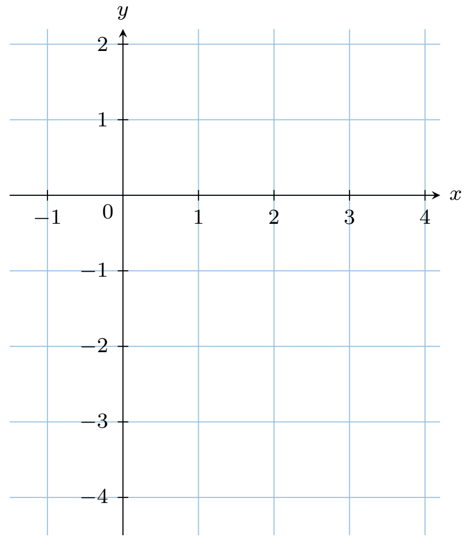


Answer:

- **y-Intercept and Slope Method:**
  - The y-intercept is 4, so plot the point $(0, 4)$.
  - The slope is -2. From $(0, 4)$, move 1 unit right ($\Delta x = 1$), then 2 units down ($\Delta y = -2$), to reach $(1, 2)$.
- **Two Points Method:**
  - Choose two values for x (e.g., $x = 0$ and $x = 2$).
  - When $x = 0$, $f(0) = -2 \times 0 + 4 = 4 \to$ point $(0, 4)$.
  - When $x = 2$, $f(2) = -2 \times 2 + 4 = -4 + 4 = 0 \to$ point $(2, 0)$.



Ex 33: Plot the graph of the function $f(x) = \frac{1}{2}x - 2$:



Answer:

- **y-Intercept and Slope Method:**
  - The y-intercept is -2, so plot the point $(0, -2)$.
  - The slope is $\frac{1}{2}$. From $(0, -2)$, move 2 units right ($\Delta x = 2$), then 1 unit up ($\Delta y = 1$), to reach $(2, -1)$.
- **Two Points Method:**
  - Choose two values for x (e.g., $x = 0$ and $x = 4$).
  - When $x = 0$, $f(0) = \frac{1}{2} \times 0 - 2 = -2 \to$ point $(0, -2)$.
  - When $x = 4$, $f(4) = \frac{1}{2} \times 4 - 2 = 2 - 2 = 0 \to$ point $(4, 0)$.