LINEAR FUNCTIONS

Many situations in the real world involve a constant rate of change. For example, the total cost of items at a store increases by the same amount for each item you add, or the distance you travel increases steadily when you move at a constant speed. These relationships can be modeled using a very common type of function: the **linear function**.
A linear function is a function whose graph is a straight line and whose rate of change is constant.

A DEFINITION

Definition Linear Function

A **linear function** is a function that can be written in the form:

$$f(x) = ax + b$$

where a and b are constants (real numbers) and x is the input variable.

- The constant a is called the **slope** or **gradient**. It represents the constant rate of change of the function: how much $f(x)$ changes when x increases by 1.
- The constant b is called the **y-intercept**. It represents the initial value of the function, i.e., the value of $f(0)$ when $x = 0$.

Ex: Access to a swimming pool costs a fixed entrance fee of \$12, plus \$5 for each hour you stay. Let $P(x)$ be the total price for x hours at the pool. Write a formula for $P(x)$ as a linear function of x.

Answer: The total price, $P(x)$, is the sum of the fixed entrance fee and the variable cost, which depends on the number of hours, x.
We can write this as an equation:

$$P(x) = (\text{Fixed fee}) + (\text{Cost per hour}) \times (\text{Number of hours})$$

Substituting the values from the problem:

$$\begin{aligned} P(x) &= \underbrace{12}_{\text{Fixed fee}} + \underbrace{5 \times x}_{\text{Cost for } x \text{ hours}} \\ &= 12 + 5x \end{aligned}$$

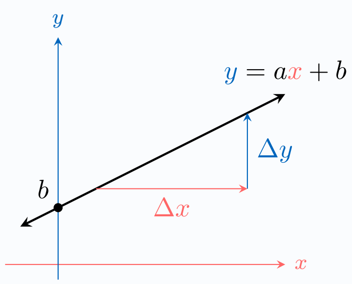
Therefore, the formula is $\boldsymbol{P(x) = 5x + 12}$. This is a linear function with slope 5 and y-intercept 12.

B GRAPH OF A LINEAR FUNCTION

Proposition Graph

The graph of a linear function $f(x) = ax + b$ is a straight line with equation $y = ax + b$.

- The line crosses the y-axis at the point $(0, b)$. This point is the y-intercept.
- The slope a is the ratio of the vertical change (**rise**, $\boldsymbol{\Delta y}$) to the horizontal change (**run**, $\boldsymbol{\Delta x}$) between any two points on the line:
$$a = \frac{\Delta y}{\Delta x}.$$
For two points (x_1, y_1) and (x_2, y_2) on the line (with $x_1 \neq x_2$), we can write
$$a = \frac{y_2 - y_1}{x_2 - x_1}.$$



Ex: Draw the graph of the function $f(x) = 0.5x + 1$.

Answer: The function is $f(x) = \frac{1}{2}x + 1$.

1. **Identify the y-intercept:** The y-intercept is $b = 1$. So, we plot our first point at $(0, 1)$.

2. **Use the slope:** The slope is $a = \frac{1}{2}$. This means that

$$a = \frac{\Delta y}{\Delta x} = \frac{1}{2},$$

   so for every "run" of 2 units to the right, we "rise" 1 unit up. From $(0, 1)$, we move 2 units right and 1 unit up to find our next point at $(2, 2)$.

3. **Draw the line:** Draw a straight line through these two points and extend it in both directions.