LOGARITHMS

A DEFINITION

A.1 EVALUATING LOGARITHMS

Ex 1: Evaluate $\log 100 =$ ☐.

Ex 2: Evaluate $\log 0.1 =$ ☐.

Ex 3: Evaluate $\log\left(\frac{1}{100}\right) =$ ☐.

Ex 4: Evaluate $\log\sqrt{10} =$ ☐.

Ex 5: Evaluate $\log 1 =$ ☐.

A.2 EVALUATING USING A CALCULATOR

Ex 6: Evaluate (round to 2 decimal places).

$$\log(2) \approx \square$$

Ex 7: Evaluate (round to 2 decimal places).

$$\log(0.2) \approx \square$$

Ex 8: Evaluate (round to 2 decimal places).

$$\log(2 \times 10^9) \approx \square$$

A.3 SOLVING EXPONENTIAL EQUATIONS USING LOGARITHMS

Ex 9: Find x such that $8 = 10^x$.

$x \approx$ ☐ (rounded to 3 decimal places)

Ex 10: Find x such that $0.4 = 10^x$.

$x \approx$ ☐ (rounded to 3 decimal places)

Ex 11: Find x such that $250 = 10^x$.

$x \approx$ ☐ (rounded to 3 decimal places)

A.4 SOLVING FOR x WHEN $\log(x)$ IS GIVEN

Ex 12: Find x such that $\log(x) = 3$.

$$x = \square$$

Ex 13: Find x such that $\log(x) = -1$.

$$x = \square$$

Ex 14: Find x such that $\log(x) = 0$.

$$x = \square$$

Ex 15: Find x such that $\log(x) = 7$.

$$x = \square$$

B LAWS OF LOGARITHMS

B.1 WRITING AS A SINGLE LOGARITHM: LEVEL 1

Ex 16: Write as a single logarithm

$$\log(5) + \log(3) = \square$$

Ex 17: Write as a single logarithm in the form $\log k$:

$$\log(15) - \log(5) = \square$$

Ex 18: Write as a single logarithm in the form $\log k$:

$$\log(4) + \log\left(\frac{1}{2}\right) = \square$$

Ex 19: Write as a single logarithm in the form $\log k$:

$$\log(18) - \log(3) = \square$$

B.2 WRITING AS A SINGLE LOGARITHM: LEVEL 2

Ex 20: Write as a single logarithm in the form $\log k$:

$$\log(8) + 1 = \square$$

Ex 21: Write as a single logarithm in the form $\log k$:

$$\log(3) + 2 = \square$$

Ex 22: Write as a single logarithm in the form $\log k$:

$$2 - \log(25) = \square$$

Ex 23: Write as a single logarithm in the form $\log k$:

$$\log(200) - 2 = \square$$

B.3 WRITING AS A SINGLE LOGARITHM: LEVEL 3

Ex 24: Write as a single logarithm in the form $\log k$:

$$2\log(3) + 1 = \square$$

Ex 25: Write as a single logarithm in the form $\log k$:

$$3\log(2) - \log(4) = \square$$

Ex 26: Write as a single logarithm in the form $\log k$:

$$2\log(20) - 2 = \square$$

Ex 27: Write as a single logarithm in the form $\log k$:

$$2\log(30) - 1 = \square$$

C USING LOGARITHMS TO SOLVE EXPONENTIAL EQUATIONS

C.1 SOLVING EXPONENTIAL EQUATIONS: LEVEL 1

Ex 28: Solve $2^x = 7$ (give your answer to 3 decimal places).

$x = \square$

Ex 29: Solve $3^x = 15$ (give your answer to 3 decimal places).

$x = \square$

Ex 30: Solve $5^x = 100$ (give your answer to 3 decimal places).

$x = \square$

Ex 31: Solve $6^x = 80$ (give your answer to 3 decimal places).

$x = \square$

C.2 SOLVING EXPONENTIAL EQUATIONS: LEVEL 2

Ex 32: Solve $5 \cdot 2^x = 7$ (give your answer to 3 decimal places).

$x = \square$

Ex 33: Solve $-2^x = -10$ (give your answer to 3 decimal places).

$x = \square$

Ex 34: Solve $4 \cdot 3^x = 60$ (give your answer to 3 decimal places).

$x = \square$

Ex 35: Solve $-2 \cdot (0.5)^x = -4$ (give your answer to 3 decimal places).

$x = \square$

D APPLICATIONS OF LOGARITHMS

D.1 APPLYING OF LOGARITHMS IN SCIENCE

Ex 36: The pH scale in chemistry is $\text{pH} = -\log_{10}[H^+]$ where $[H^+]$ is the hydrogen ion concentration in moles per litre. The pH of a solution is 3.2. Find the hydrogen ion concentration $[H^+]$ (give your answer in scientific notation with 3 significant digits).

$\square \times \square$ mol/L

Ex 37: The Richter scale measures earthquake intensity using the formula $M = \log_{10}\left(\frac{I}{I_0}\right)$, where M is the magnitude, I is the intensity of the earthquake, and I_0 is the intensity of a standard earthquake.
An earthquake has a magnitude of 4.5 on the Richter scale. Find the intensity ratio $\frac{I}{I_0}$ (give your answer in scientific notation with 3 significant digits).

$\frac{I}{I_0} = \square \times \square$

Ex 38: The intensity of sound is measured in decibels (dB) using the formula $L = 10\log_{10}\left(\frac{I}{I_0}\right)$, where L is the sound level in decibels, I is the intensity of the sound, and I_0 is the reference intensity (threshold of human hearing).
A sound has a level of 75 decibels. Find the intensity ratio $\frac{I}{I_0}$ (give your answer in scientific notation with 3 significant digits).

$\frac{I}{I_0} = \square \times \square$

E GRAPHS OF LOGARITHMIC FUNCTIONS

E.1 FINDING DOMAINS

MCQ 39: Find the domain of the function $f : x \mapsto \log(x - 4)$.

- ☐ $\mathbb{R}$
- ☐ $[-4, +\infty)$
- ☐ $(4, +\infty)$
- ☐ $(-\infty, 4)$

MCQ 40: Find the domain of the function $f : x \mapsto \log(2 - x)$.

☐ $\mathbb{R}$

☐ $[-2, +\infty)$

☐ $(2, +\infty)$

☐ $(-\infty, 2)$

MCQ 41: Find the domain of the function $f : x \mapsto \log(2x - 6)$.

☐ $\mathbb{R}$

☐ $[3, +\infty)$

☐ $(3, +\infty)$

☐ $(-\infty, 3)$

MCQ 42: Find the domain of the function $f : x \mapsto \log(9 - 3x)$.

☐ $\mathbb{R}$

☐ $[3, +\infty)$

☐ $(3, +\infty)$

☐ $(-\infty, 3)$

E.2 CALCULATING $f(x)$

Ex 43: For $f : x \mapsto 3\log(x)$, find in simplest form:

1. $f(1) =$ ☐
2. $f(10) =$ ☐

Ex 44: For $f : x \mapsto \frac{1}{1+\log(x)}$, find in simplest form:

1. $f(1) =$ ☐
2. $f(10) =$ ☐

Ex 45: For $f : x \mapsto x\log(x + 1)$, find in simplest form:

1. $f(0) =$ ☐
2. $f(1) =$ ☐