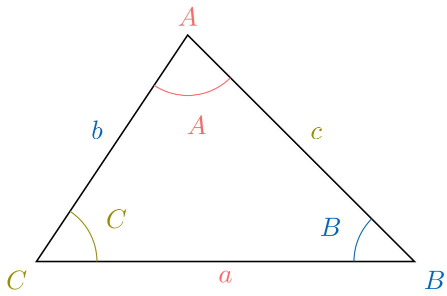
NON-RIGHT-ANGLED TRIANGLE TRIGONOMETRY

In previous chapters, we explored trigonometry in right-angled triangles using sine, cosine, and tangent. However, many real-world problems involve triangles that are not right-angled. To solve these, we use the Law of Sines and the Law of Cosines, which apply to any triangle.

Convention for Labelling Triangles:

- For triangle ABC, the angles at vertices A, B, and C are labelled A, B, and C respectively.
- The sides opposite these angles are labelled a, b, and c respectively.



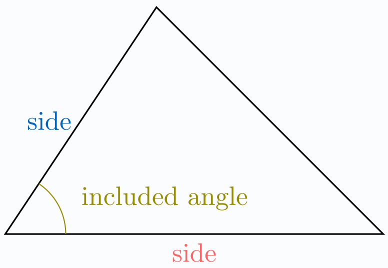
A AREA OF A TRIANGLE USING TWO SIDES AND THE INCLUDED ANGLE

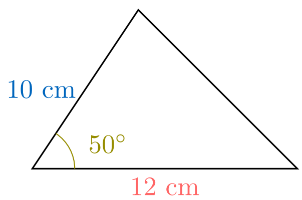
When two sides and the included angle of a triangle are known, the area can be calculated using the formula involving the sine of the angle. This is particularly useful in non-right-angled triangles.

Proposition Area Formula

The area of a triangle is:

$$\text{Area} = \frac{1}{2} \times \text{side} \times \text{side} \times \sin(\text{included angle})$$



Ex: For the triangle 

, find the area.

Answer:

$$\text{Area} = \frac{1}{2} \times 10 \times 12 \times \sin(50^\circ)$$
$$\approx 46\,\text{cm}^2$$

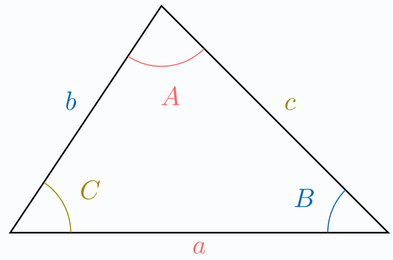
B LAW OF SINES

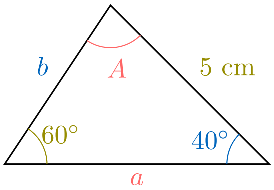
The Law of Sines relates the sides of a triangle to the sines of its angles. It is useful when we know two angles and one side (AAS or ASA) or one angle and two sides (SSA, which may have ambiguous cases).

Proposition Law of Sines

For a triangle with sides a, b, c opposite angles A, B, C:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Ex: For the triangle 

, find side a.

Answer: First, find angle A: $A = 180° - 40° - 60° = 80°$.
Using the Law of Sines:

$$\frac{a}{\sin 60°} = \frac{5}{\sin 40°}$$
$$a = 5 \cdot \frac{\sin 60°}{\sin 40°}$$
$$a \approx 6.73\,\text{cm}$$

C LAW OF COSINES

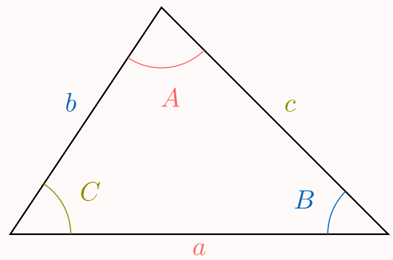
The Law of Cosines is a generalization of the Pythagorean theorem for non-right-angled triangles. It is useful when we know three sides (SSS) or two sides and the included angle (SAS).

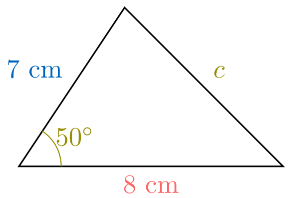
Definition Law of Cosines

For a triangle with sides a, b, c opposite angles A, B, C:

$$c^2 = a^2 + b^2 - 2ab\cos C \text{ and } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Similarly for the other sides.



Ex: For the triangle 

, find side c.

Answer: Using the Law of Cosines:

$$c^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 50°$$
$$c^2 \approx 41.0$$
$$c \approx \sqrt{41}$$
$$c \approx 6.4\,\text{cm}$$

D SOLVING REAL-WORLD PROBLEMS USING SINE AND COSINE LAWS

The Laws of Sines and Cosines are powerful tools for solving a wide range of problems involving non-right-angled triangles, especially in real-world contexts. To solve these problems effectively, follow the structured steps below:

Method **Solving Real-World Problems Using Sine and Cosine Laws**

- **Draw a clear diagram** representing the situation described in the problem.
- **Label the known and unknown** sides and angles in the triangle.
- **Identify the configuration** (e.g., two sides and included angle for Cosine Law, or two angles and a side for Sine Law).
- **Choose the appropriate law** (Sine Law for ratios of sides to sines of angles, Cosine Law for relating sides and cosine of an angle).
- **Write and solve the equation** to find the unknown value.