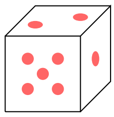
# PROBABILITY

## A SAMPLE SPACES

### A.1 FINDING THE SAMPLE SPACES

**MCQ 1:** A fair six-sided die is rolled once.



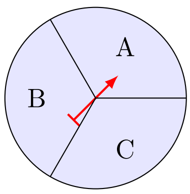
Find the sample space.

- ☐ $\{1, 2, 3, 4, 5\}$
- ☐ $\{1, 2, 3, 4, 5, 6, 7\}$
- ☒ $\{1, 2, 3, 4, 5, 6\}$

*Answer:*

- The sample space is all possible outcomes.
- When rolling a fair six-sided die, the possible outcomes are the numbers on the die's faces.
- So, the sample space is $\{1, 2, 3, 4, 5, 6\}$.

**MCQ 2:** You spin the arrow on the spinner below.



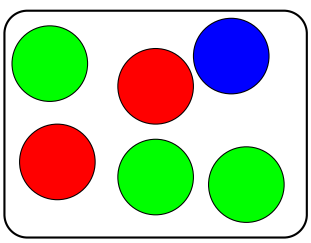

Find the sample space.

- ☒ $\{A, B, C\}$
- ☐ $\{A, B\}$
- ☐ $\{A, C\}$

*Answer:*

- The sample space is all possible outcomes of spinning the arrow.
- Here, the possible outcomes are $A$, $B$, and $C$.
- So, the sample space is $\{A, B, C\}$.

**MCQ 3:** A ball is chosen randomly from a bag containing 2 red balls, 1 blue ball, and 3 green balls.




Find the sample space.

- ☒ {Red, Blue, Green}
- ☐ {2 Red, 1 Blue, 3 Green}
- ☐ {Red, Red, Blue, Green, Green, Green}

*Answer:*

- When choosing a ball randomly from the bag containing 2 red balls, 1 blue ball, and 3 green balls, the balls are identical in color, so we do not distinguish between them based on quantity.
- So, the sample space (all possible outcomes) is: {Red, Blue, Green}

**MCQ 4:** A letter is chosen randomly from the word BANANA. Find all possible outcomes for the chosen letter.

- ☒ {B, N, A}
- ☐ {B, A, N, A, N, A}
- ☐ {A, B, N, A, B, N}

*Answer:*

- When choosing a letter randomly from the word "BANANA", the possible outcomes are the distinct letters in the word.
- So, the sample space (all possible outcomes) is: {B, A, N} or {B, N, A}. The order in which the letters are listed does not matter.

**MCQ 5:** A couple is expecting a baby. What is the sample space for this random experiment?

- ☒ {boy, girl}
- ☐ {boy}
- ☐ {girl}

*Answer:*

- The sample space is all possible outcomes for the sex of the baby.
- The possible outcomes are "boy" or "girl".
- So, the sample space is {boy, girl}.

## B EVENTS

### B.1 FINDING EVENTS FOR DIE-ROLLING EVENTS

**MCQ 6:** If you roll a die, what is the set of outcomes for the event "getting a 3"?

- ☐ $\{1, 3, 5\}$
- ☐ $\{2, 3, 4\}$
- ☐ $\{1, 2, 3\}$

☒ $\{3\}$

*Answer:* The set of outcomes for the event "getting a 3" is $\{3\}$.

**MCQ 7:** If you roll a die, what is the set of outcomes for the event "getting a 5 or 6"?

☒ $\{5, 6\}$

☐ $\{4, 5, 6\}$

☐ $\{1, 2, 3\}$

☐ $\{3, 4, 5\}$

*Answer:* The set of outcomes for the event "getting a 5 or 6" is $\{5, 6\}$.

**MCQ 8:** If you roll a die, what is the set of outcomes for the event "getting a number greater than or equal to 4"?

☐ $\{1, 2, 3\}$

☒ $\{4, 5, 6\}$

☐ $\{3, 4, 5\}$

☐ $\{2, 3, 4\}$

*Answer:* The set of outcomes for the event "getting a number greater than or equal to 4" is $\{4, 5, 6\}$.

**MCQ 9:** If you roll a die, what is the set of outcomes for the event "even number"?

☐ $\{1, 3, 5\}$

☒ $\{2, 4, 6\}$

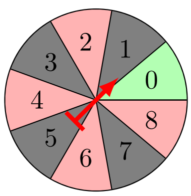
☐ $\{1, 2, 3, 4, 5, 6\}$

☐ $\{2, 3, 4, 5\}$

*Answer:* The set of outcomes for the event "even number" is $\{2, 4, 6\}$.

### B.2 FINDING EVENTS IN A CASINO SPINNER

**MCQ 10:** If you spin the spinner below, what is the set of outcomes for the event "getting a 2"?




☒ $\{2\}$

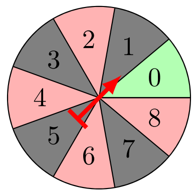
☐ $\{1, 2, 3\}$

☐ $\{2, 4, 6\}$

☐ $\{0, 1, 2\}$

*Answer:* The set of outcomes for the event "getting a 2" is $\{2\}$.

**MCQ 11:** If you spin the spinner below, what is the set of outcomes for the event "red"?




☐ $\{1, 3, 5, 7\}$

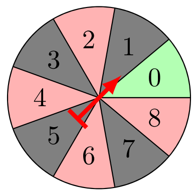
☐ $\{0\}$

☒ $\{2, 4, 6, 8\}$

☐ $\{1, 2, 3, 4\}$

*Answer:* The set of outcomes for the event "red" is $\{2, 4, 6, 8\}$.

**MCQ 12:** If you spin the spinner below, what is the set of outcomes for the event "getting an odd number"?




☐ $\{0, 1, 3\}$

☐ $\{2, 4, 6, 8\}$

☐ $\{1, 2, 3, 4\}$

☒ $\{1, 3, 5, 7\}$

*Answer:* The set of outcomes for the event "getting an odd number" is $\{1, 3, 5, 7\}$.

## C MULTI-STEP RANDOM EXPERIMENTS

### C.1 FINDING OUTCOME IN A TABLE

**MCQ 13:** The table below shows the possible outcomes for the sexes of two children, first and second, where each can be a Boy (B) or a Girl (G).

| second child / first child | $B$ | $G$ |
|---|---|---|
| $B$ | $BB$ | ? |
| $G$ | $GB$ | $GG$ |

Find the missing outcome.

☐ $BB$

☒ $BG$

☐ $GB$

*Answer:*

- The first child is represented by the row ("first child"), and the second child by the column ("second child").
- The missing outcome is $BG$.

**MCQ 14:** The table below shows the possible outcomes when selecting two letters at random from the word "MAT" **with replacement** (after choosing a letter, it is put back before the next selection).

| letter 1 \ letter 2 | $M$ | $A$ | $T$ |
|---|---|---|---|
| $M$ | $MM$ | $MA$ | $MT$ |
| $A$ | $AM$ | $AA$ | $AT$ |
| $T$ | $TM$ | ? | $TT$ |

Find the missing outcome.

☐ $TT$

☒ $TA$

☐ $AT$

*Answer:*

- The first letter is "Letter 1" (row), and the second letter is "Letter 2" (column).
- The missing outcome is $TA$.

**MCQ 15:** The table below shows the possible outcomes when selecting two letters at random from the word "CODE" **with replacement** (after choosing a letter, it is put back before the next selection).

| letter 1 \ letter 2 | $C$ | $O$ | $D$ | $E$ |
|---|---|---|---|---|
| $C$ | $CC$ | $CO$ | $CD$ | $CE$ |
| $O$ | $OC$ | $OO$ | $OD$ | $OE$ |
| $D$ | $DC$ | ? | $DD$ | $DE$ |
| $E$ | $EC$ | $EO$ | $ED$ | $EE$ |

Find the missing outcome.

☒ $DO$

☐ $OD$

☐ $DC$

*Answer:*

- The first letter is "Letter 1" (row), and the second letter is "Letter 2" (column).
- The missing outcome is $DO$.

**MCQ 16:** The table below shows the possible outcomes when selecting two letters at random from the word "NODE" **without replacement** (after choosing a letter, it is not put back before the next selection). An "X" means no outcome is possible.

| letter 1 \ letter 2 | $N$ | $O$ | $D$ | $E$ |
|---|---|---|---|---|
| $N$ | **X** | ? | $ND$ | $NE$ |
| $O$ | $ON$ | **X** | $OD$ | $OE$ |
| $D$ | $DN$ | $DO$ | **X** | $DE$ |
| $E$ | $EN$ | $EO$ | $ED$ | **X** |

Find the missing outcome.

☐ $NN$

☒ $NO$

☐ $ON$

*Answer:*

- The first letter is "Letter 1" (row), and the second letter is "Letter 2" (column).
- The missing outcome is $NO$.

**MCQ 17:** The table below shows the possible outcomes when a coach selects two players at random from four players (A, B, C, D) **without replacement** (after choosing a player, they are not put back before the next selection). An "X" means no outcome is possible.

| Player 1 \ Player 2 | $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|---|
| $A$ | **X** | ? | $AC$ | $AD$ |
| $B$ | $BA$ | **X** | $BC$ | $BD$ |
| $C$ | $CA$ | $CB$ | **X** | $CD$ |
| $D$ | $DA$ | $DB$ | $DC$ | **X** |

Find the missing outcome.

☒ $AB$

☐ $BA$

☐ $CA$

*Answer:*

- The first player is "Player 1" (row), and the second player is "Player 2" (column).
- The missing outcome is $AB$.

## C.2 COUNTING THE NUMBER OF POSSIBLE OUTCOMES IN A TABLE

**Ex 18:** The table below shows the possible outcomes for the sexes of two children, first and second, where each can be a Boy (B) or a Girl (G).

| first child \ second child | $B$ | $G$ |
|---|---|---|
| $B$ | $BB$ | $BG$ |
| $G$ | $GB$ | $GG$ |

Count the number of possible outcomes.

[4] possible outcomes.

*Answer:*

- The sample space (the possible outcomes) is $\{BB, BG, GB, GG\}$.
- The number of possible outcomes is 4.

**Ex 19:** There are four players: A, B, C, and D. For position 1, only players A and B are eligible. For position 2, only players C and D are eligible. The table below shows the possible selections for the two positions.

| position 1 \ position 2 | $C$ | $D$ |
|---|---|---|
| $A$ | $AC$ | $AD$ |
| $B$ | $BC$ | $BD$ |

Count the number of possible outcomes.

4 possible outcomes.

*Answer:*

- The sample space (the possible outcomes) is $\{AC, AD, BC, BD\}$.
- The number of possible outcomes is 4.

**Ex 20:** There are four players: A, B, C, and D. A coach selects two players at random without replacement. The table below shows the possible selections for the two positions. An "X" means no outcome is possible.

| Player 1 \ Player 2 | $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|---|
| $A$ | **X** | $AB$ | $AC$ | $AD$ |
| $B$ | $BA$ | **X** | $BC$ | $BD$ |
| $C$ | $CA$ | $CB$ | **X** | $CD$ |
| $D$ | $DA$ | $DB$ | $DC$ | **X** |

Count the number of possible outcomes.

12 possible outcomes.

*Answer:*

- The sample space (the possible outcomes) is $\{AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC\}$.
- The number of possible outcomes is 12.

**Ex 21:** There are three students: X, Y, and Z. A teacher selects one student each day, on Monday and Tuesday, to recite a poem. The selection is made without replacement, meaning the same student cannot be chosen both days. The table below shows the possible selections for the two days.

| Monday \ Tuesday | $X$ | $Y$ | $Z$ |
|---|---|---|---|
| $X$ | **X** | $XY$ | $XZ$ |
| $Y$ | $YX$ | **X** | $YZ$ |
| $Z$ | $ZX$ | $ZY$ | **X** |

Count the number of possible outcomes.

6 possible outcomes.

*Answer:*

- The sample space (the possible outcomes) is $\{XY, XZ, YX, YZ, ZX, ZY\}$.
- The number of possible outcomes is 6.

## C.3 COUNTING THE NUMBER OF POSSIBLE OUTCOMES FOR AN EVENT

**Ex 22:** There are four players: A, B, C, and D. A coach selects two players at random without replacement. The table below shows the possible selections for the two positions. An "X" means no outcome is possible.

| Player 1 \ Player 2 | $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|---|
| $A$ | **X** | $AB$ | $AC$ | $AD$ |
| $B$ | $BA$ | **X** | $BC$ | $BD$ |
| $C$ | $CA$ | $CB$ | **X** | $CD$ |
| $D$ | $DA$ | $DB$ | $DC$ | **X** |

Count the number of outcomes for the event that player A is selected.

6 outcomes.

*Answer:*

- The event "player A is selected" includes all outcomes where A is either Player 1 or Player 2.
- From the table:
  - When A is Player 1 (row A): $AB$, $AC$, $AD$ (3 outcomes).
  - When A is Player 2 (column A): $BA$, $CA$, $DA$ (3 outcomes).
- Total outcomes: $\{AB, AC, AD, BA, CA, DA\}$, which is 6 outcomes.

**Ex 23:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| red die \ blue die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Count the number of outcomes for the event "double" (both dice show the same number).

6 outcomes.

*Answer:*

- The event "double" includes all outcomes where the red die and blue die show the same number.
- From the table:
  - Doubles: 11, 22, 33, 44, 55, 66 (6 outcomes).
- Total outcomes: 6.

**Ex 24:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| red die \ blue die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Count the number of outcomes for the event "at least one 6" (at least one die shows a 6).

11 outcomes.

*Answer:*

- The event "at least one 6" includes all outcomes where at least one of the dice shows a 6.
- From the table:
  - Red die = 6 (row 6): 6 outcomes.
  - Blue die = 6 (column 6), excluding (6,6) already counted: 5 more outcomes.
- Total outcomes: 11.

**Ex 25:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| blue die / red die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Count the number of outcomes for the event "the sum of the dice is equal to 11."

2 outcomes.

*Answer:*

- The event "the sum of the dice is 11" includes all outcomes where the red die and blue die sum to 11.
- From the table:
  - Possible pairs: 56 (5+6=11), 65 (6+5=11).
- Total outcomes: 2.

**Ex 26:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| blue die / red die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Count the number of outcomes for the event "the sum of the dice is equal to 7."

6 outcomes.

*Answer:*

- The event "the sum of the dice is 7" includes all outcomes where the red die and blue die sum to 7.
- From the table:
  - Possible pairs: 16 (1+6), 25 (2+5), 34 (3+4), 43 (4+3), 52 (5+2), 61 (6+1).
- Total outcomes: 6.

**Ex 27:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| blue die / red die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Count the number of outcomes for the event "the sum of the dice is less than or equal to 3."

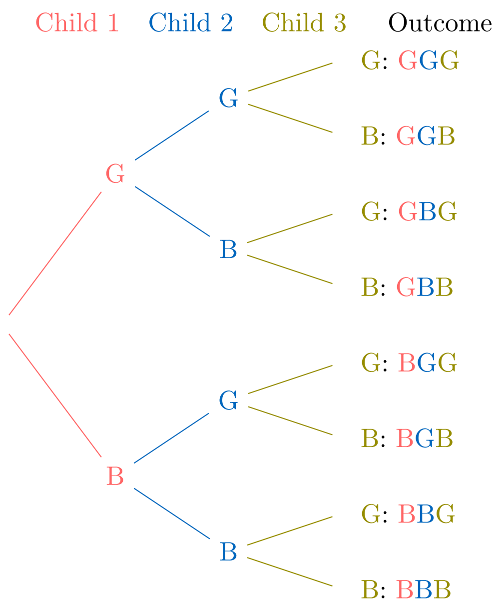
3 outcomes.

*Answer:*

- The event "the sum of the dice is less than or equal to 3" includes all outcomes where the sum is 2 or 3.
- From the table:
  - Possible pairs: 11 (1+1=2), 12 (1+2=3), 21 (2+1=3).
- Total outcomes: 3.

## C.4 COUNTING THE NUMBER OF POSSIBLE OUTCOMES IN A TREE DIAGRAM

**Ex 28:** Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible sex outcomes for the three children.




Count the number of possible outcomes for the event where the first child is a boy.

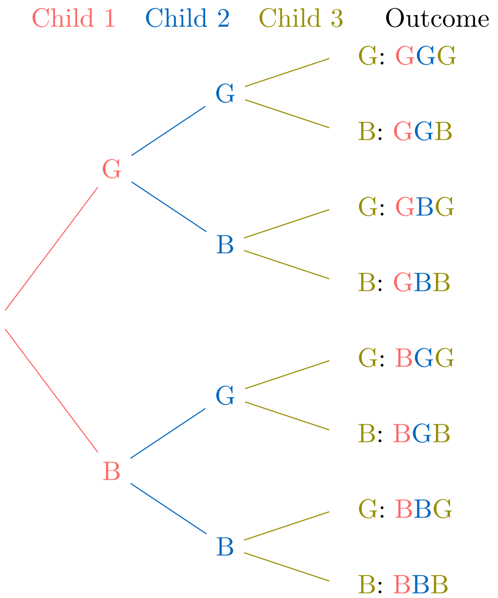
4

*Answer:*

- The event where the first child is a boy includes all outcomes starting with B, represented as $B{*}{*}$.

- These outcomes are: BBB, BBG, BGB, BGG.
- The number of possible outcomes is 4.

**Ex 29:** Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible sex outcomes for the three children.



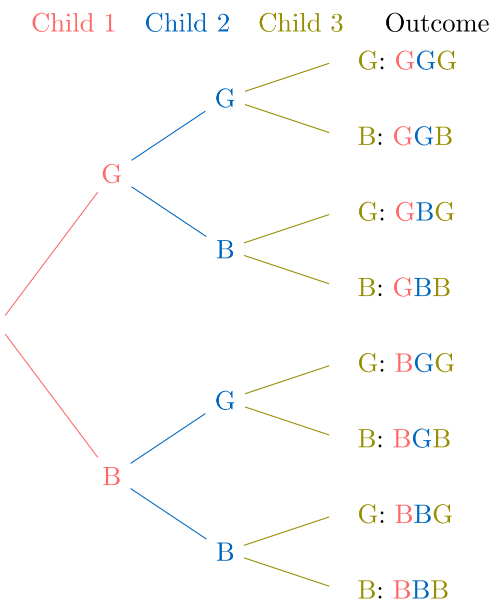

Count the number of possible outcomes for the event where there are exactly two girls.

3

*Answer:*

- The event where there are exactly two girls includes all outcomes with exactly two G's and one B.
- These outcomes are: BGG, GBG, GGB.
- The number of possible outcomes is 3.

**Ex 30:** Parents have three children, each either a boy (B) or a girl (G). The tree diagram below shows all 8 possible sex outcomes for the three children.



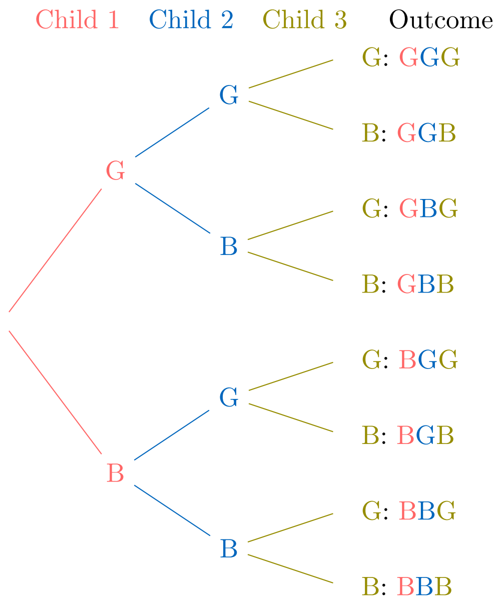

Count the number of possible outcomes for the event where there are at least two girls.

4

*Answer:*

- The event where there are at least two girls includes all outcomes with two or three girls.
- These outcomes are: BGG, GBG, GGB, GGG.
- The number of possible outcomes is 4.

**Ex 31:** Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible sex outcomes for the three children.




Count the number of possible outcomes for the event where the family has mixed-sex children (at least one boy and one girl).

6

*Answer:*

- The event where the family has mixed-sex children includes all outcomes with at least one boy (B) and one girl (G), excluding all-boys (BBB) and all-girls (GGG).
- These outcomes are: BBG, BGB, BGG, GGB, GBG, GBB.
- The number of possible outcomes is 6.

# D COMPLEMENTARY EVENTS

## D.1 FINDING THE COMPLEMENTARY EVENTS

**MCQ 32:** If you roll a die, what is the set of outcomes for the event "not getting a 6"?

- ☐ $\{2, 3, 4\}$
- ☐ $\{1, 2, 3, 4, 5, 6\}$
- ☒ $\{1, 2, 3, 4, 5\}$
- ☐ $\{1, 3, 5\}$

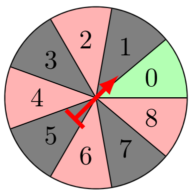
*Answer:* The set of outcomes for the event "not getting a 6" is $\{1, 2, 3, 4, 5\}$.

**MCQ 33:** If you roll a die, what is the set of outcomes for the event "not getting an odd number"?

- ☒ $\{2, 4, 6\}$
- ☐ $\{1, 2, 3, 4, 5, 6\}$
- ☐ $\{1, 2, 3\}$
- ☐ $\{1, 3, 5\}$

*Answer:* The set of outcomes for the event "not getting an odd number" is $\{2, 4, 6\}$.

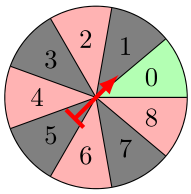
**MCQ 34:** If you spin the spinner below, what is the set of outcomes for the event "not getting a 4"?




- ☐ $\{1, 2, 3, 4\}$
- ☒ $\{0, 1, 2, 3, 5, 6, 7, 8\}$
- ☐ $\{2, 4, 6, 8\}$
- ☐ $\{4, 5, 6\}$

*Answer:* The set of outcomes for the event "not getting a 4" is $\{0, 1, 2, 3, 5, 6, 7, 8\}$.

**MCQ 35:** If you spin the spinner below, what is the set of outcomes for the event "not getting red"?




- ☒ $\{0, 1, 3, 5, 7\}$
- ☐ $\{2, 4, 6, 8\}$
- ☐ $\{1, 2, 3, 4, 5, 6, 7, 8\}$
- ☐ $\{0\}$

*Answer:* The set of outcomes for the event "not getting red" is $\{0, 1, 3, 5, 7\}$.

# E PROBABILITY

## E.1 DESCRIBING PROBABILITIES WITH WORDS

**MCQ 36:** The probability of winning a game is $\frac{1}{10}$. Find the word to describe this probability.

- ☐ Impossible
- ☒ Less Likely
- ☐ Even Chance
- ☐ Most Likely
- ☐ Certain

*Answer:* The correct answer is "Less Likely." The probability of winning is $\frac{1}{10}$, which means you have the chance to win 1 game out of 10 games played. So, it's Less Likely.

**MCQ 37:** The probability of winning a game is $\frac{4}{5}$. Find the word to describe this probability.

- ☐ Impossible
- ☐ Less Likely
- ☐ Even Chance
- ☒ Most Likely
- ☐ Certain

*Answer:* The correct answer is "Most Likely." The probability of winning is $\frac{4}{5}$, which means you have the chance to win 4 games out of 5 games played. So, it's Most Likely.

**MCQ 38:** The probability of winning a game is $\frac{1}{2}$. Find the word to describe this probability.

- ☐ Impossible
- ☐ Less Likely
- ☒ Even Chance
- ☐ Most Likely
- ☐ Certain

*Answer:* The correct answer is "Even Chance." The probability of winning is $\frac{1}{2}$, which means you have the chance to win 1 game out of 2 games played. So, it's an Even Chance.

**MCQ 39:** The probability of winning a game is 0. Find the word to describe this probability.

- ☒ Impossible
- ☐ Less Likely
- ☐ Even Chance
- ☐ Most Likely
- ☐ Certain

*Answer:* The correct answer is "Impossible." The probability of winning is 0, which means you have no chance to win the game. So, it's Impossible.

**MCQ 40:** The probability of winning a game is 1. Find the word to describe this probability.

- ☐ Impossible
- ☐ Less Likely
- ☐ Even Chance
- ☐ Most Likely
- ☒ Certain

*Answer:* The correct answer is "Certain." The probability of winning is 1, which means you will definitely win the game. So, it's Certain.

## E.2 MAKING DECISIONS USING PROBABILITIES

**MCQ 41:** Louis advises you to play because the probability of winning this game is $\frac{3}{4}$. Do you follow his advice?

☒ Yes

☐ No

*Answer:* The correct answer is "Yes." The probability of winning is $\frac{3}{4}$, which means you have the chance to win 3 games out of 4 games played. So it is most likely. Therefore, it's a good idea to follow Louis's advice and play.

**MCQ 42:** Louis advises you to play because the probability of winning this game is $\frac{1}{4}$. Do you follow his advice?

☐ Yes

☒ No

*Answer:* The correct answer is "No." The probability of winning is $\frac{1}{4}$, which means you have the chance to win 1 game out of 4 games played. So it is less likely. Therefore, it's not a good idea to follow Louis's advice and play.

**MCQ 43:** The probability of succeeding a penalty is $\frac{1}{2}$ for Louis and $\frac{3}{4}$ for Hugo. Which player do you choose to take the penalty?

☐ Louis

☒ Hugo

*Answer:* The correct answer is "Hugo." The probability of succeeding for Louis is $\frac{1}{2}$, which means he has an even chance to succeed. For Hugo, it's $\frac{3}{4}$, which means he is most likely to succeed because he has the chance to succeed in 3 out of 4 penalties. So, Hugo is the better choice to take the penalty.

**MCQ 44:** The probability of succeeding a penalty is $\frac{1}{4}$ for Louis and $\frac{3}{5}$ for Hugo. Which player do you choose to take the penalty?

☐ Louis

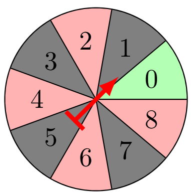
☒ Hugo

*Answer:* The correct answer is "Hugo." The probability of succeeding for Louis is $\frac{1}{4}$, which means he is less likely to succeed because he has the chance to succeed in 1 out of 4 penalties. For Hugo, it's $\frac{3}{5}$, which means he is most likely to succeed because he has the chance to succeed in 3 out of 5 penalties. So, Hugo is the better choice to take the penalty.

# F EQUALLY LIKELY

## F.1 FINDING PROBABILITIES IN A CASINO SPINNER

**Ex 45:** You spin the casino spinner shown below. Calculate the probability of the event "getting a 2".



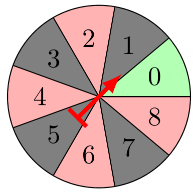

$$P(\text{"getting a 2"}) = \boxed{\frac{1}{9}}$$

*Answer:*

- The number of outcomes in the universe when spinning the casino spinner is 9, since there are 9 sections (0 to 8).
- The number of outcomes for the event "getting a 2" is 1, as there is one section labeled 2 on the spinner.
- Therefore, the probability of getting a 2 is given by:

$$\begin{aligned} P(\text{"getting a 2"}) &= \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the universe}} \\ &= \frac{1}{9} \end{aligned}$$

**Ex 46:** You spin the casino spinner shown below. Calculate the probability of the event "not getting a 4".



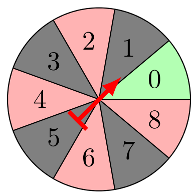

$$P(\text{"not getting a 4"}) = \boxed{\frac{8}{9}}$$

*Answer:*

- The number of outcomes in the universe when spinning the casino spinner is 9, since there are 9 sections (0 to 8).
- The number of outcomes for the event "not getting a 4" is 8, as there are eight sections that are not 4: 0, 1, 2, 3, 5, 6, 7, and 8.
- Therefore, the probability of not getting a 4 is given by:

$$\begin{aligned} P(\text{"not getting a 4"}) &= \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the universe}} \\ &= \frac{8}{9} \end{aligned}$$

**Ex 47:** You spin the casino spinner shown below. Calculate the probability of the event "red".




$$P(\text{"red"}) = \boxed{\frac{4}{9}}$$

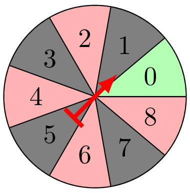
*Answer:*

- The number of outcomes in the universe when spinning the casino spinner is 9, since there are 9 sections (0 to 8).
- The number of outcomes for the event "red" is 4, as there are four red sections on the spinner: 2, 4, 6, and 8.

- Therefore, the probability of landing on a red section is given by:

$$P(\text{"red"}) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the universe}} = \frac{4}{9}$$

**Ex 48:** You spin the casino spinner shown below. Calculate the probability of the event "getting an odd number".



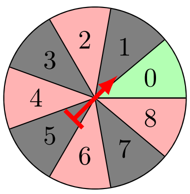

$$P(\text{"getting an odd number"}) = \boxed{\frac{4}{9}}$$

*Answer:*

- The number of outcomes in the universe when spinning the casino spinner is 9, since there are 9 sections (0 to 8).
- The number of outcomes for the event "getting an odd number" is 4, as there are four odd numbers on the spinner: 1, 3, 5, and 7.
- Therefore, the probability of getting an odd number is given by:

$$P(\text{"odd number"}) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the universe}} = \frac{4}{9}$$

**Ex 49:** You spin the casino spinner shown below. Calculate the probability of the event "not getting red".




$$P(\text{"not getting red"}) = \boxed{\frac{5}{9}}$$

*Answer:*

- The number of outcomes in the universe when spinning the casino spinner is 9, since there are 9 sections (0 to 8).
- The number of outcomes for the event "not getting red" is 5, as there are five sections that are not red: 0, 1, 3, 5, and 7.
- Therefore, the probability of not getting red is given by:

$$P(\text{"not getting red"}) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the universe}} = \frac{5}{9}$$

## F.2 FINDING PROBABILITIES IN A DICE EXPERIMENT

**Ex 50:** If you roll a die, what is the probability of the event "getting a 3"?

$$P(\text{"getting a 3"}) = \boxed{\frac{1}{6}}$$

*Answer:* There is 1 outcome (3) out of 6 possible outcomes, so the probability is $\frac{1}{6}$.

**Ex 51:** If you roll a die, what is the probability of the event "getting a 5 or 6"?

$$P(\text{"getting a 5 or 6"}) = \boxed{\frac{1}{3}}$$

*Answer:* There are 2 outcomes (5 or 6) out of 6 possible outcomes, so the probability is $\frac{2}{6} = \frac{1}{3}$.

**Ex 52:** If you roll a die, what is the probability of the event "getting a number greater than or equal to 4"?

$$P(\text{"number} \geq 4\text{"}) = \boxed{\frac{1}{2}}$$

*Answer:* There are 3 outcomes (4, 5, or 6) out of 6 possible outcomes, so the probability is $\frac{3}{6} = \frac{1}{2}$.

**Ex 53:** If you roll a die, what is the probability of the event "even number"?

$$P(\text{"even number"}) = \boxed{\frac{1}{2}}$$

*Answer:* There are 3 outcomes (2, 4, or 6) out of 6 possible outcomes, so the probability is $\frac{3}{6} = \frac{1}{2}$.

**Ex 54:** If you roll a die, what is the probability of the event "not getting a 6"?

$$P(\text{"not getting a 6"}) = \boxed{\frac{5}{6}}$$

*Answer:* There are 5 outcomes (1, 2, 3, 4, or 5) out of 6 possible outcomes, so the probability is $\frac{5}{6}$.

**Ex 55:** If you roll a die, what is the probability of the event "not getting an odd number"?

$$P(\text{"not getting an odd number"}) = \boxed{\frac{1}{2}}$$

*Answer:* There are 3 outcomes (2, 4, or 6) out of 6 possible outcomes, so the probability is $\frac{3}{6} = \frac{1}{2}$.

## F.3 CALCULATING THE PROBABILITY IN MULTI-STEP RANDOM EXPERIMENTS

**Ex 56:** A coach selects two players at random from a group of four players, labeled A, B, C, and D, without replacement (once a player is chosen, they are not available for the next selection). The table below shows all possible outcomes for selecting Player 1 and Player 2, where an "X" indicates an impossible outcome due to the same player being selected twice.

| Player 2 / Player 1 | $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|---|
| $A$ | **X** | $AB$ | $AC$ | $AD$ |
| $B$ | $BA$ | **X** | $BC$ | $BD$ |
| $C$ | $CA$ | $CB$ | **X** | $CD$ |
| $D$ | $DA$ | $DB$ | $DC$ | **X** |

Calculate the probability that player C is selected as either Player 1 or Player 2.

$$P(\text{"selecting player C"}) = \boxed{\frac{1}{2}}$$

*Answer:*

- The sample space consists of all possible pairs of players: AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC. This totals 12 outcomes.
- The event that player C is selected includes pairs where C is either Player 1 or Player 2: CA, CB, CD, AC, BC, DC. This totals 6 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"C is selected"}) &= \frac{\text{number of outcomes where C is selected}}{\text{number of possible outcomes}} \\ &= \frac{6}{12} \\ &= \frac{1}{2} \end{aligned}$$

**Ex 57:** Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.Calculate the probability that the family has at least two girls.

$$P(\text{"at least two girls"}) = \boxed{\frac{1}{2}}$$

*Answer:*

- The sample space consists of all possible gender outcomes for three children: BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG. This totals 8 outcomes.
- The event that the family has at least two girls includes outcomes with two or three girls: BGG, GBG, GGB, GGG. This totals 4 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"at least 2 girls"}) &= \frac{\text{Number of outcomes with at least 2 girls}}{\text{Total number of outcomes}} \\ &= \frac{4}{8} \\ &= \frac{1}{2} \end{aligned}$$

**Ex 58:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| dé bleu / dé rouge | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Calculate the probability that the sum of the two dice is exactly 7.

$$P(\text{"sum is 7"}) = \boxed{\frac{1}{6}}$$

*Answer:*

- The sample space consists of all possible pairs of outcomes from rolling two six-sided dice: 36 outcomes (6 outcomes for Die 1 times 6 outcomes for Die 2).
- The event that the sum is exactly 7 includes pairs: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1). This totals 6 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"sum is 7"}) &= \frac{\text{Number of outcomes with sum 7}}{\text{Total number of outcomes}} \\ &= \frac{6}{36} \\ &= \frac{1}{6} \end{aligned}$$

**Ex 59:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| dé bleu / dé rouge | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Calculate the probability that the sum of the two dice is greater than or equal to 11.

$$P(\text{"sum } \geq 11\text{"}) = \boxed{\frac{1}{12}}$$

*Answer:*

- The sample space consists of all possible pairs of outcomes from rolling two six-sided dice: 36 outcomes (6 outcomes for the red die times 6 outcomes for the blue die).
- The event that the sum is greater than or equal to 11 includes pairs: (5,6), (6,5), (6,6). This totals 3 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"sum } \geq 11\text{"}) &= \frac{\text{Number of outcomes with sum } \geq 11}{\text{Total number of outcomes}} \\ &= \frac{3}{36} \\ &= \frac{1}{12} \end{aligned}$$

**Ex 60:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| dé bleu / dé rouge | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Calculate the probability that the sum of the two dice is exactly 6 or 8.

$$P(\text{"sum is 6 or 8"}) = \boxed{\frac{5}{18}}$$

*Answer:*

- The sample space consists of all possible pairs of outcomes from rolling two six-sided dice: 36 outcomes (6 outcomes for the red die times 6 outcomes for the blue die).
- The event that the sum is exactly 6 or 8 includes pairs: (1,5), (2,4), (3,3), (4,2), (5,1) for sum 6, and (2,6), (3,5), (4,4), (5,3), (6,2) for sum 8. This totals 10 outcomes.
- The probability is calculated as:

$$\begin{aligned}P(\text{"sum is 6 or 8"}) &= \frac{\text{Number of outcomes with sum 6 or 8}}{\text{Total number of outcomes}}\\ &= \frac{10}{36}\\ &= \frac{5}{18}\end{aligned}$$

# G COMPLEMENT RULE

## G.1 APPLYING THE COMPLEMENT RULE

**Ex 61:** I toss a fair coin. The probability of getting heads is $\frac{1}{2}$.
Find the probability of getting tails.

$$P(\text{"Getting tails"}) = \boxed{\frac{1}{2}}$$

*Answer:*

- The probability of getting heads is $\frac{1}{2}$.
- The event "Getting tails" is the complement of "Getting heads."
- Using the complement rule:

$$\begin{aligned}P(\text{"Getting tails"}) &= 1 - P(\text{"Getting heads"})\\ &= 1 - \frac{1}{2}\\ &= \frac{1}{2}\end{aligned}$$

- So, the probability of getting tails is $\frac{1}{2} = 50\%$.

**Ex 62:** A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.
Find the probability that a student does not laugh at the joke.

$$P(\text{"Not laughing"}) = \boxed{30}\%$$

*Answer:*

- The probability that a student laughs at the joke is 70%.
- The event "Not laughing" is the complement of "Laughing."
- Using the complement rule:

$$\begin{aligned}P(\text{"Not laughing"}) &= 1 - P(\text{"Laughing"})\\ &= 100\% - 70\%\\ &= 30\%\end{aligned}$$

- Therefore, the probability that a student does not laugh at the joke is 30%.

**Ex 63:** I randomly select a student in the class. The probability that a girl is selected is $\frac{9}{10}$.
Find the probability that a boy is selected.

$$P(\text{"Selecting a boy"}) = \boxed{\frac{1}{10}}$$

*Answer:*

- The probability that a girl is selected is $\frac{9}{10}$.
- The event "Selecting a boy" is the complement of "Selecting a girl."
- Using the complement rule:

$$\begin{aligned}P(\text{"Selecting a boy"}) &= 1 - P(\text{"Selecting a girl"})\\ &= 1 - \frac{9}{10}\\ &= \frac{1}{10}\end{aligned}$$

- So, the probability that a boy is selected is $\frac{1}{10} = 10\%$.

**Ex 64:** The weather forecast predicts that there is a 70% chance of rain tomorrow.
Find the probability that it will not rain tomorrow.

$$P(\text{"No rain"}) = \boxed{30}\%$$

*Answer:*

- The probability that it will rain tomorrow is 70%.
- The event "No rain" is the complement of "Rain".
- Using the complement rule:

$$\begin{aligned}P(\text{"No rain"}) &= 1 - P(\text{"Rain"})\\ &= 100\% - 70\%\\ &= 30\%\end{aligned}$$

- Therefore, the probability that it will not rain tomorrow is 30%.

**Ex 65:** In a loto game, the probability of winning is $\frac{1}{100}$.
Find the probability of losing.

$$P(\text{"Losing"}) = \boxed{\frac{99}{100}}$$

*Answer:*

- The probability of winning is $\frac{1}{100}$.
- The event "Losing" is the complement of "Winning."
- Using the complement rule:

$$\begin{aligned} P(\text{"Losing"}) &= 1 - P(\text{"Winning"}) \\ &= 1 - \frac{1}{100} \\ &= \frac{100}{100} - \frac{1}{100} \\ &= \frac{99}{100} \end{aligned}$$

- So, the probability of losing is $\frac{99}{100} = 99\%$.

# H EXPERIMENTAL PROBABILITY

## H.1 CALCULATING EXPERIMENTAL PROBABILITIES IN PERCENTAGE FORM

**Ex 66:** During a classroom experiment, Ethan flips a coin 50 times and records that it lands on heads 30 times. Calculate the experimental probability that the coin lands on heads, and express the result in percentage form.

$$P(\text{"landing on heads"}) \approx \boxed{60}\ \%$$

*Answer:*

- The total number of trials in the experiment is 50, since Ethan flipped the coin 50 times.
- The number of successful outcomes for the event "landing on heads" is 30, as the coin landed on heads 30 times.
- Calculate the experimental probability:

$$\begin{aligned} P(\text{"landing on heads"}) &\approx \frac{\text{number of successful outcomes}}{\text{total number of trials}} \\ &\approx \frac{30}{50} \\ &\approx 30 \div 50 \\ &\approx 0.6 \\ &\approx 0.6 \times 100\% \\ &\approx 60\% \end{aligned}$$

**Ex 67:** During a week of basketball practice, Mia made 45 out of 60 free-throw attempts. Estimate the experimental probability that Mia will make her next free-throw attempt, and express the result in percentage form.

$$P(\text{"making the next attempt"}) \approx \boxed{75}\%$$

*Answer:*

- The total number of trials in the experiment is 60, since Mia made 60 free-throw attempts.
- The number of successful outcomes for the event "making the next attempt" is 45, as Mia successfully made 45 free-throws.
- Calculate the experimental probability:

$$\begin{aligned} P(\text{"making the next attempt"}) &\approx \frac{\text{number of successful outcom}}{\text{total number of trials}} \\ &= \frac{45}{60} \\ &= 45 \div 60 \\ &= 0.75 \\ &= 0.75 \times 100\% \\ &= 75\% \end{aligned}$$

**Ex 68:** During a week, the school cafeteria recorded that out of 150 students, 120 chose a vegetarian meal. Estimate the experimental probability that the next student will choose a vegetarian meal, and express the result in percentage form.

$$P(\text{choosing a vegetarian meal}) \approx \boxed{80}\%$$

*Answer:*

- The total number of trials in the experiment is 150, since 150 students were recorded.
- The number of successful outcomes for the event "choosing a vegetarian meal" is 120, as 120 students chose a vegetarian meal.
- Calculate the experimental probability:

$$\begin{aligned} P(\text{"vegetarian meal"}) &\approx \frac{\text{number of successful outcomes}}{\text{total number of trials}} \\ &= \frac{120}{150} \\ &= 120 \div 150 \\ &= 0.8 \\ &= 0.8 \times 100\% \\ &= 80\% \end{aligned}$$

**Ex 69:** Over the course of a year, it rained on 146 days out of 365 recorded days. Estimate the experimental probability that it will rain, and express the result in percentage form.

$$P(\text{"raining"}) \approx \boxed{40}\%$$

*Answer:*

- The total number of trials in the experiment is 365, since 365 days were recorded.
- The number of successful outcomes for the event "raining" is 146, as it rained on 146 days.

- Calculate the experimental probability:

$$\begin{aligned}P(\text{"raining"}) &\approx \frac{\text{number of successful outcomes}}{\text{total number of trials}}\\ &= \frac{146}{365}\\ &= 146 \div 365\\ &= 0.4\\ &= 0.4 \times 100\%\\ &= 40\%\end{aligned}$$

## H.2 CONDUCTING EXPERIMENTS TO ESTIMATE PROBABILITIES

**Ex 70:** In a experiment, you are asked to toss a fair coin at least 30. Follow these steps:

1. Note the number of times the coin lands on heads.
2. Note the total number of trials (tosses).
3. Calculate the experimental probability that the coin lands on heads, and express the result in decimal form.

*Answer:* To demonstrate the process, let's assume a sample result from the experiment: I conducted these experiments and noted each result using tally marks.

1. Number of heads = 𝍸𝍸𝍸𝍷𝍷𝍷
   Number of heads = 18
2. Number of trials = 𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸
   Number of trials = 40
3. Calculate the experimental probability that the coin lands on heads:

$$\begin{aligned}P(\text{"landing on heads"}) &\approx \frac{\text{number of successful outcomes}}{\text{total number of trials}}\\ &\approx \frac{18}{40}\\ &\approx 18 \div 40\\ &\approx 0.45\end{aligned}$$

This is a sample result; your actual probability will depend on your experiment's outcomes.

**Ex 71:** In a classroom experiment, you are asked of your friends at least 10 to choose randomly a single number from 1, 2, 3, 4, or 5. Follow these steps:

1. Note the number of times the answer is 5.
2. Note the total number of trials (friends asked).
3. Calculate the experimental probability that a friend chooses the number 5, and express the result in decimal form.

*Answer:* To demonstrate the process, let's assume a sample result from the experiment: I conducted this survey by asking 40 friends, and I noted each result using tally marks.

1. Number of times the answer is 5 = 𝍸𝍸𝍷𝍷
   Number of times the answer is 5 = 12
2. Number of trials = 𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸
   Number of trials = 40
3. Calculate the experimental probability that a friend chooses the number 5:

$$\begin{aligned}P(\text{"choosing the number 5"}) &\approx \frac{\text{number of successful outcomes}}{\text{total number of trials}}\\ &\approx \frac{12}{40}\\ &\approx 12 \div 40\\ &\approx 0.3\end{aligned}$$

This is a sample result; your actual probability will depend on your experiment's outcomes.