PROBABILITY

Ever wondered if it'll rain tomorrow or if you'll win a game? That's probability! It's a math way to guess how likely things are to happen.

A SAMPLE SPACES

> **Definition Outcome**
>
> An **outcome** is one possible result of a random experiment.

Ex: What are the outcomes when you flip a coin?

Answer: The outcomes are Heads (H)= and Tails (T)= .

Ex: What are the outcomes when you roll a six-sided die?

Answer: The outcomes are 1= ,2= ,3= ,4= ,5= ,and 6= .

> **Definition Sample Space**
>
> The **sample space** is the set of all possible outcomes of a random experiment.

Ex: What's the sample space when you flip a coin?

Answer: The sample space is {Heads, Tails} = { , }, or just {H, T} for short.

**Ex:** What's the sample space when you roll a six-sided die?

Answer: The sample space is {1, 2, 3, 4, 5, 6} = { , , , , , }.

B EVENTS

> **Definition Event**
>
> An **event** is a set of outcomes from the outcomes of the sample space. We write it E.

Ex: In the experiment of rolling a die, find E the event of rolling an even number.

Answer: Among the outcomes of the sample space {1, 2, 3, 4, 5, 6} = { , , , , , } ,the event of rolling an even number is $E = \{2, 4, 6\} = \{$, , $\}$.

C MULTI-STEP RANDOM EXPERIMENTS

Discover: A **multi-step random experiment** is an experiment that involves a sequence of actions, where each action (or step) has its own set of possible outcomes. For example, tossing two coins is a multi-step experiment because it consists of two separate coin tosses:

- The first coin toss (step 1) can result in Heads (H) or Tails (T).

- The second coin toss (step 2) can also result in Heads (H) or Tails (T).

The overall outcome of the experiment is found by combining the results of each step. For instance, the outcome HT means the first coin landed on Heads and the second on Tails.

Using different representations—such as grids, tables, tree diagrams, or simply listing the outcomes—helps us organize and visualize all the possible combinations that result from multiple steps.

> **Method Representations of Multi-Step Random Experiments**
>
> When an experiment involves more than one step, we can represent the sample space (the set of all possible outcomes) in several ways:
>
> - using a **grid** (to visually map combinations along two axes),
>
> - using a **table** (to organize outcomes in rows and columns),
>
> - using a **tree diagram** (to show each sequential step), or

- by **listing** all possible outcomes.

Ex: For the random experiment of tossing two coins, display the sample space by:

1. using a grid

2. using a table

3. using a tree diagram

4. listing all possible outcomes

Answer:

1. **Grid:**

2. **Table:**

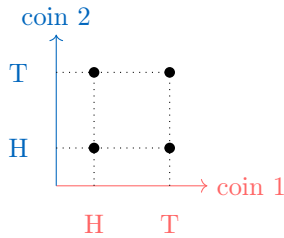| coin 2 coin 1	H	T
H	HH	HT
T	TH	TT
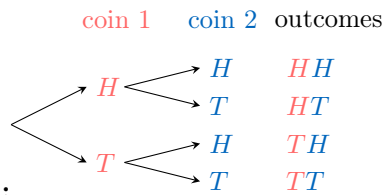
3. **Tree diagram:**

4. **List:**$\{HH, HT, TH, TT\}$

D COMPLEMENTARY EVENTS

Ever wonder what happens if you look for everything **except** a certain event? That's where the complementary event comes in! It's just everything in the sample space that isn't in your event. We usually write it as E' ("E-prime").

Definition **Complementary Event**

The **complementary event** of an event E is all the outcomes in the sample space that are **not** in E. We write it E'.

Ex: In the experiment of rolling a die, let E the event of rolling an even number. Find E'.

Answer: The sample space is $\{1, 2, 3, 4, 5, 6\}$ = {⚀,⚁,⚂,⚃,⚄,⚅} and $E = \{2, 4, 6\}$ = {⚁,⚃,⚅}.

So E' is all the other numbers: $\{1, 3, 5\}$ = {⚀,⚂,⚄}. These are the odd numbers.

E PROBABILITY

Discover: When you flip a coin, there are two possible outcomes: heads or tails. The chance of getting heads is the same as getting tails—it's 1 out of 2! In math, we write:

$$P(\text{"Getting Heads"}) = \tfrac{1}{2}$$

The probability   of getting heads   is equal   1 chance out of 2
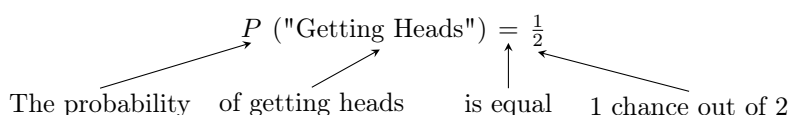
This means heads will happen about half the time!

Definition Probability

The **probability** of an event E, written $P(E)$, is a number that tells us how likely the event is to happen. It's always between 0 (impossible) and 1 (certain).

Impossible   Less Likely   Even Chance   Most Likely   Certain

0 $\frac{1}{6}$ $\frac{1}{2}$ $\frac{9}{10}$ 1

Ex: The probability of an event "even chance" can be represented as:

- **Fraction**: $\frac{1}{2}$

- **Decimal**: To convert the fraction to a decimal, divide the numerator by the denominator: $1 \div 2 = 0.5$.
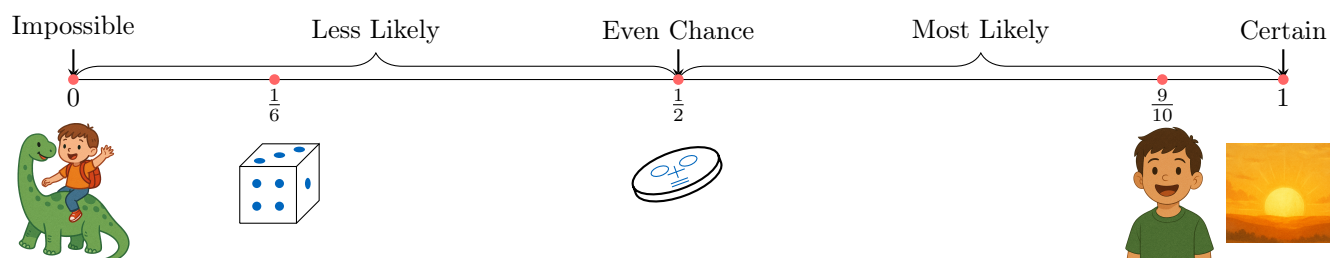
- **Percentage**: To convert the decimal to a percentage, multiply by 100%: $0.5 \times 100\% = 50\%$.

F EQUALLY LIKELY

Have you ever flipped a fair coin or rolled a fair die? In these experiments, each outcome is just as likely as the others. We call these equally likely outcomes.

Definition Equally Likely

When all outcomes are **equally likely**, the probability of an event E is:

$$P(E) = \frac{\textbf{number of outcomes in the event}}{\textbf{number of outcomes in the sample space}}$$

Ex: What's the probability of rolling an even number with a fair six-sided die?

Answer:

- Sample space $= \{1, 2, 3, 4, 5, 6\}$ (6 outcomes).

- $E = \{2, 4, 6\}$ (3 outcomes).

-

$$P(E) = \frac{3}{6}$$
$$= \frac{1}{2}$$

So, there's a $\frac{1}{2}$ chance (or 50%) of rolling an even number!

G COMPLEMENT RULE

Ever wondered how to quickly find the chance that something **doesn't** happen? There's a shortcut for that! It's called the complement rule.

Proposition Complement Rule

For any event E and its complementary event E', the probabilities always add up to 1:

$$P(E) + P(E') = 1 \quad \text{or} \quad P(E') = 1 - P(E).$$

Ex: Farid has a 0.8 (80%) chance of finishing his homework on time tonight (event E). What's the chance he **doesn't** finish on time?
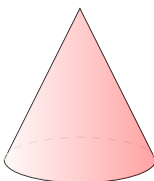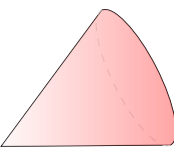
Answer: The complementary event E' is "Farid does **not** finish his homework on time." By the complement rule:

$$P(E') = 1 - P(E)$$
$$= 1 - 0.8$$
$$= 0.2$$

So, there's a 0.2 (or 20%) chance he doesn't finish on time!

H EXPERIMENTAL PROBABILITY

Isaac wishes to determine how a cone lands when tossed—base down or point down? The possible outcomes are as follows:

- Base down:

- Point down:

Due to the cone's shape potentially favoring one outcome, Isaac cannot predict the probabilities. He conducts 50 trials and records the results:

- Base down: 30 times.

- Point down: 20 times.

The chance of landing base down is approximately 30 times over 50 times. So, he estimates:

- $P(\text{"base down"}) = \frac{30}{50} = 0.6$ (60%).

- $P(\text{"point down"}) = \frac{20}{50} = 0.4$ (40%).

The more trials he conducts, the closer his estimates approach the true probabilities.

Theorem Law of Large Numbers

The probability of an event E can be estimated using the formula:

$$P(E) \approx \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

Here, "trials" refer to the number of times the experiment is repeated.