PROBABILITY

Ever wondered if it'll rain tomorrow or if you'll win a game? That's probability! It's a math way to guess how likely things are to happen.

A SAMPLE SPACES

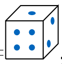
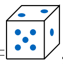
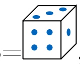
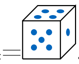
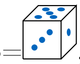
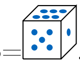
Definition Outcome

An **outcome** is one possible result of a random experiment.

Ex: What are the outcomes when you flip a coin? 

Answer: The outcomes are Heads (H)= and Tails (T)=.

Ex: What are the outcomes when you roll a six-sided die?

Answer: The outcomes are 1=,2=,3=,4=,5=,and 6=.

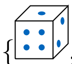
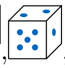
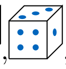
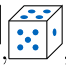
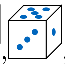
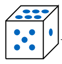
Definition Sample Space

The **sample space** is the set of all possible outcomes of a random experiment.

Ex: What's the sample space when you flip a coin?

Answer: The sample space is {Heads, Tails} = {,}, or just {H, T} for short.

Ex: What's the sample space when you roll a six-sided die?

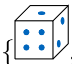
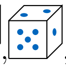
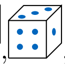
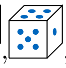
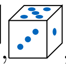
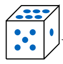
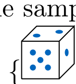
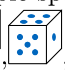
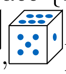
Answer: The sample space is {1, 2, 3, 4, 5, 6} = {,,,,, }.

B EVENTS

Definition Event

An **event** is a set of outcomes from the outcomes of the sample space. We write it E.

Ex: In the experiment of rolling a die, find E the event of rolling an even number.

Answer: Among the outcomes of the sample space {1, 2, 3, 4, 5, 6} = {,,,,, } ,the event of rolling an even number is $E = \{2, 4, 6\}$ = {,,}.

C MULTI-STEP RANDOM EXPERIMENTS

Method Representations of Multi-Step Random Experiments

When an experiment involves more than one step, we can represent the sample space (the set of all possible outcomes) in several ways:

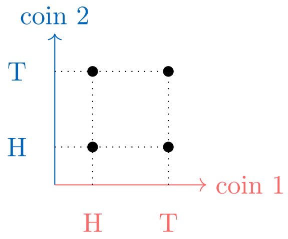
- using a **grid** (to visually map combinations along two axes),
- using a **table** (to organize outcomes in rows and columns),
- using a **tree diagram** (to show each sequential step), or
- by **listing** all possible outcomes.

Ex: For the random experiment of tossing two coins, display the sample space by:

1. using a grid
2. using a table
3. using a tree diagram
4. listing all possible outcomes

Answer:

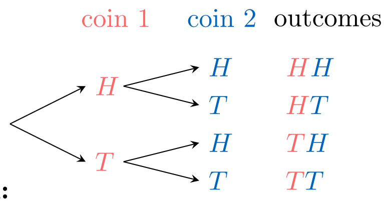
1. **Grid:**



2. **Table:**

coin 2 / coin 1	H	T
H	HH	HT
T	TH	TT

3. **Tree diagram:**



4. **List:** $\{HH, HT, TH, TT\}$

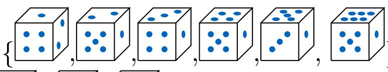
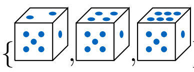
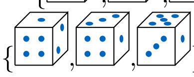
D COMPLEMENTARY EVENTS

Ever wonder what happens if you look for everything **except** a certain event? That's where the complementary event comes in! It's just everything in the sample space that isn't in your event. We usually write it as E' ("E-prime").

Definition Complementary Event

The **complementary event** of an event E is all the outcomes in the sample space that are **not** in E. We write it E'.

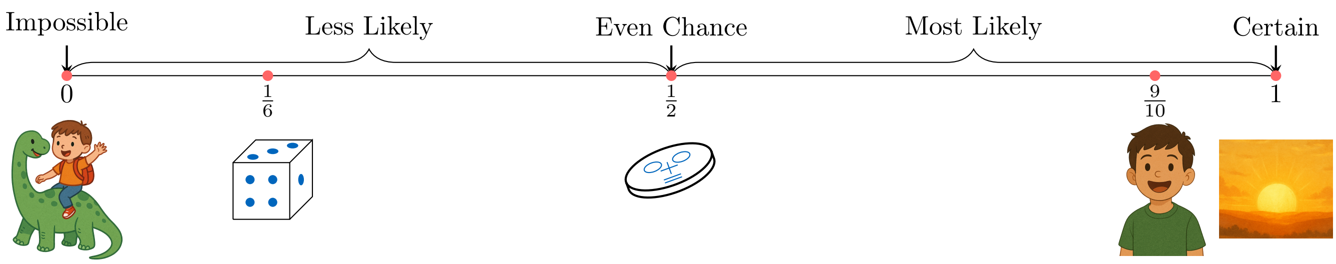
Ex: In the experiment of rolling a die, let E the event of rolling an even number. Find E'.

Answer: The sample space is $\{1, 2, 3, 4, 5, 6\}$ =  and $E = \{2, 4, 6\}$ = .
So E' is all the other numbers: $\{1, 3, 5\}$ = . These are the odd numbers.

E PROBABILITY

Definition Probability

The **probability** of an event E, written $P(E)$, is a number that tells us how likely the event is to happen. It's always between 0 (impossible) and 1 (certain).



Ex: The probability of an event "even chance" can be represented as:

- **Fraction**: $\frac{1}{2}$
- **Decimal**: To convert the fraction to a decimal, divide the numerator by the denominator: $1 \div 2 = 0.5$.
- **Percentage**: To convert the decimal to a percentage, multiply by 100%: $0.5 \times 100\% = 50\%$.

F EQUALLY LIKELY

Have you ever flipped a fair coin or rolled a fair die? In these experiments, each outcome is just as likely as the others. We call these equally likely outcomes.

Definition Equally Likely

When all outcomes are **equally likely**, the probability of an event E is:

$$P(E) = \frac{\textbf{number of outcomes in the event}}{\textbf{number of outcomes in the sample space}}$$

Ex: What's the probability of rolling an even number with a fair six-sided die?

Answer:

- Sample space $= \{1, 2, 3, 4, 5, 6\}$ (6 outcomes).
- $E = \{2, 4, 6\}$ (3 outcomes).
- $$\begin{aligned} P(E) &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

So, there's a $\frac{1}{2}$ chance (or 50%) of rolling an even number!

G COMPLEMENT RULE

Ever wondered how to quickly find the chance that something **doesn't** happen? There's a shortcut for that! It's called the complement rule.

Proposition Complement Rule

For any event E and its complementary event E', the probabilities always add up to 1:

$$P(E) + P(E') = 1 \quad \text{or} \quad P(E') = 1 - P(E).$$

Ex: Farid has a 0.8 (80%) chance of finishing his homework on time tonight (event E). What's the chance he **doesn't** finish on time?

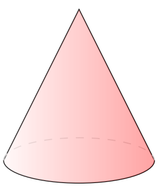
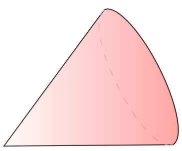
Answer: The complementary event E' is "Farid does **not** finish his homework on time." By the complement rule:

$$\begin{aligned} P(E') &= 1 - P(E) \\ &= 1 - 0.8 \\ &= 0.2 \end{aligned}$$

So, there's a 0.2 (or 20%) chance he doesn't finish on time!

H EXPERIMENTAL PROBABILITY

Isaac wishes to determine how a cone lands when tossed—base down or point down? The possible outcomes are as follows:

- Base down: 
- Point down: 

Due to the cone's shape potentially favoring one outcome, Isaac cannot predict the probabilities. He conducts 50 trials and records the results:

- Base down: 30 times.
- Point down: 20 times.

The chance of landing base down is approximately 30 times over 50 times. So, he estimates:

- $P(\text{"base down"}) = \frac{30}{50} = 0.6$ (60%).
- $P(\text{"point down"}) = \frac{20}{50} = 0.4$ (40%).

The more trials he conducts, the closer his estimates approach the true probabilities.

Theorem Law of Large Numbers

The probability of an event E can be estimated using the formula:

$$P(E) \approx \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

Here, "trials" refer to the number of times the experiment is repeated.