PROBABILITY

A SAMPLE SPACE

A.1 FINDING THE SAMPLE SPACES

MCQ 1: A fair six-sided die is rolled once.

Find the sample space.

 \Box {1, 2, 3, 4, 5}

 \Box {1, 2, 3, 4, 5, 6, 7}

 $\boxtimes \ \{1,2,3,4,5,6\}$

Answer:

- The sample space is all possible outcomes.
- When rolling a fair six-sided die, the possible outcomes are the numbers on the die's faces.
- So, the sample space is $\{1, 2, 3, 4, 5, 6\}$.





- $\boxtimes \{A, B, C\}$
- $\Box \{A, B\}$
- $\Box \{A, C\}$

Answer:

- The sample space is all possible outcomes.
- The spinner has three distinct regions labeled A, B, and C.
- So, the sample space is: $\{A, B, C\}$.

MCQ 3: A ball is chosen randomly from a bag containing 2 red balls, 1 blue ball, and 3 green balls.



Find the sample space.

 \boxtimes {Red, Blue, Green}

- \square {2 Red, 1 Blue, 3 Green}
- \square {Red, Red, Blue, Green, Green, Green}

Answer:

- When choosing a ball randomly from the bag containing 2 red balls, 1 blue ball, and 3 green balls, the balls are identical in color, so we do not distinguish between them based on quantity.
- So, the sample space (all possible outcomes) is: {Red, Blue, Green}

MCQ 4: A letter is chosen randomly from the word BANANA. Find all possible outcomes for the chosen letter.

- $\boxtimes \{B, N, A\}$
- $\Box \ \{B, A, N, A, N, A\}$
- $\Box \{A, B, N, A, B, N\}$

Answer:

- When choosing a letter randomly from the word "BANANA", the possible outcomes are the distinct letters in the word.
- So, the sample space (all possible outcomes) is: {B, A, N} or {B, N, A}. The order in which the letters are listed does not matter.

B EVENTS

B.1 FINDING THE EVENTS

MCQ 5: A letter is chosen randomly from the word ORANGE. Find the event where the chosen letter is a vowel.

- $\Box \ \{O,\,R,\,A,\,N,\,G,\,E\}$
- $\boxtimes \{O, A, E\}$
- \Box {R, G, N}
- \Box {A, G, E}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- When choosing a letter randomly from the word "ORANGE", the event where the chosen letter is a vowel consists of the vowels in the word.
- So, the event is: $\{O, A, E\}$.

MCQ 6: A fair six-sided dice is rolled once. Find the event where the outcome is an even number.

- $\Box \{1, 3, 5\}$
- $\boxtimes \{2, 4, 6\}$
- \Box {1, 2, 3, 4, 5, 6}

 \Box {2, 3, 4, 5}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- When rolling a fair six-sided dice, the event where the outcome is an even number consists of the even numbers on the dice.
- So, the event is: $\{2, 4, 6\}$.





France Italy Germany

Find the event where the outcome is a flag with blue in them.

- \boxtimes {France }
- \Box {Italy, France}
- \Box {Italy, France, Germany}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- Among the given flags, only the French flag has blue in it.
- So, the correct answer is: {France}.

MCQ 8: A flag is chosen randomly from:



France Italy Germany Japan

Find the event where the outcome is a flag with red in them.

- \Box {France, Japan}
- \Box {Italy, France}
- \boxtimes {Italy, France, Germany, Japan}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- Among the given flags, France, Italy, Germany, and Japan have red in them.
- So, the correct answer is: {Italy, France, Germany, Japan}.

MCQ 9: A flag is chosen randomly from:



Find the event where the outcome is a flag with green in them.

- \Box {France, Nigeria}
- \boxtimes {Italy, Nigeria}
- \Box {Italy, France, Germany}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- Among the given flags, Italy and Nigeria have green in them.
- So, the correct answer is: {Italy, Nigeria}.

C COMPLEMENTARY EVENT

C.1 FINDING THE COMPLEMENTARY EVENTS

MCQ 10: A flag is chosen randomly from the following:



France Italy Germany Nigeria

Let E be the event where the selected flag contains green. Find the complement of event E, denoted as E'.

- $\boxtimes E' = \{ France, Germany \}$
- $\Box E' = \{$ Italy, Nigeria $\}$
- $\Box E' = \{ \text{Italy, France, Germany} \}$

Answer:

- The event E includes flags with the color green: {Italy, Nigeria}.
- The complement event E^\prime consists of flags that do not have green.
- So, the complement event $E' = \{$ France, Germany $\}$.





France Italy Germany Nigeria

Let E be the event where the chosen flag contains the color red. Find the complement of event E, denoted E'.

- $\square E' = \{ France, Germany \}$
- $\boxtimes E' = \{ Nigeria \}$
- $\Box E' = \{$ Italy, France, Germany $\}$

Answer:

- The event E consists of flags that contain the color red: {France, Italy, Germany}.
- The complement of event E, denoted E', consists of flags that do not contain the color red.



- Since Nigeria is the only flag without the color red, $E' = \{Nigeria\}.$
- So, the correct answer is $E' = {\text{Nigeria}}.$

MCQ 12: A child's name is chosen randomly from the following list:

- Emily (girl's name)
- James (boy's name)
- Ava (girl's name)
- Sophia (girl's name)

Let E be the event where the selected name is a boy's name. Find the complement of event E, denoted as E'.

- $\boxtimes E' = \{\text{Emily, Ava, Sophia}\}\$
- $\Box E' = {\text{James}}$
- $\Box E' = \{ James, Ava \}$

Answer:

- The event E includes boy's names: {James}.
- The complement event E' consists of names that are not boy's names (i.e., girl's names).
- So, the complement event $E' = \{\text{Emily, Ava, Sophia}\}.$

MCQ 13: Given the following shapes:



Triangle Square Pentagon Hexagon

Let ${\cal E}$ be the event where a polygon with an even number of sides is chosen.

Find the complement of event E, denoted as E'.

 $\Box E' = \{ Square, Hexagon \}$

 $\boxtimes E' = \{\text{Triangle, Pentagon}\}$

 $\Box E' = \{$ Triangle, Square, Pentagon, Hexagon $\}$

Answer:

- Triangle: three sides. Square: four sides. Pentagon: five sides. Hexagon: six sides.
- The event E includes polygons with an even number of sides: {Square, Hexagon}.
- The complementary event E' consists of polygons that do not have an even number of sides (have an odd number of sides).
- Therefore, the complementary event $E' = {\text{Triangle, Pentagon}}.$





Triangle Circle Square Curve

Let E be the event where the shape is a polygon. Find the complement of event E, denoted as E'. $\square E' = \{\text{Triangle, Square}\}$

- $\Box E' = \{ \text{Triangle, Circle, Square, Curve} \}$
- $\boxtimes E' = \{ \text{Circle, Curve} \}$

Answer:

- The event E includes shapes that are polygons: {Triangle, Square}.
- The complementary event E' consists of shapes that are not polygons.
- Therefore, the complementary event $E' = \{$ Circle, Curve $\}$.

D PROBABILITY

D.1 DETERMINING THE PROBABILITY

MCQ 15: Keziah eats rice often. Let E be the event that Keziah eats rice this week. Find P(E), the probability that Keziah eats rice this week.

$$\square P(E) = 1\%$$
$$\square P(E) = 50\%$$

$$\boxtimes P(E) = 99\%$$

Answer:

- Since he eats rice often, it's very likely.
- So, the probability that Keziah eats rice this week is P(E) = 99%.

MCQ 16: Emily drinks water every day. Let E be the event that Emily drinks water tomorrow. Find P(E), the probability that Emily drinks water tomorrow.

- $\Box P(E) = 50\%$
- $\Box P(E) = 90\%$
- $\boxtimes P(E) = 100\%$

Answer:

- She drinks water every day, so it's certain.
- So, the probability that Emily drinks water tomorrow is P(E) = 100%.

MCQ 17: It almost never snows in July in the Sahara Desert. Let E be the event that it snows this July in the Sahara Desert. Find P(E), the probability that it snows this July.

- $\boxtimes P(E) = 0.01\%$ $\Box P(E) = 5\%$
- $\Box P(E) = 99.9\%$

Answer:

- It's extremely rare, so very unlikely.
- So, the probability that it snows this July is P(E) = 0.01%.

(°<u>+</u>°)

MCQ 18: Samuel loves playing basketball. Let E be the event that Samuel plays basketball this weekend. Find P(E), the probability that Samuel plays this weekend.

- $\Box P(E) = 5\%$
- $\Box P(E) = 20\%$
- $\boxtimes P(E) = 90\%$

Answer:

- He loves it, so it's very likely.
- So, the probability that Samuel plays this weekend is P(E) = 90%.

MCQ 19: Benjamin rolls a die. Let E be the event that Benjamin rolls a number bigger than 7. Find P(E), the probability that Benjamin rolls a number bigger than 7.

 $\boxtimes P(E) = 0\%$

 $\Box P(E) = 50\%$

 $\Box P(E) = 100\%$

Answer:

- A die only has numbers 1 to 6, so it's impossible.
- So, the probability that Benjamin rolls a number bigger than 7 is P(E) = 0%.

E CALCULATE PROBABILITIES

E.1 DETERMINING THE PROBABILITY

Ex 20: A ball is chosen randomly from a bag containing 2 red balls, 3 blue balls.

Find the probability that we choose a red ball.

$$P("\text{choosing a red ball"}) = \frac{2}{5}$$

Answer:

• To find the probability of choosing a red ball, divide the number of red balls by the total number of balls.

•
$$P("choosing a red ball") = \frac{\text{number of red balls}}{\text{total number of balls}}$$

= $\frac{2}{5}$

Ex 21: A card is drawn at random from a standard deck of 52 playing cards. Determine the probability of drawing an Ace and express your answer as a simplified fraction.

$$P("$$
drawing an Ace" $) = \frac{1}{13}$

Answer:

• To find the probability of drawing an Ace, divide the number of Aces by the total number of cards.

• There are 4 Aces in a standard deck of 52 playing cards.

•
$$P("drawing an Ace") = \frac{\text{number of Aces}}{\text{total number of cards}}$$

= $\frac{4}{52}$
= $\frac{1 \times \cancel{4}}{13 \times \cancel{4}}$
= $\frac{1}{13}$

Ex 22: A six-sided die is rolled once. Determine the probability of obtaining an even number.

$$P("rolling an even number") = \frac{1}{2}$$

Answer:

- To find the probability of rolling an even number, divide the number of even numbers by the total number of sides.
- There are 3 even numbers on a six-sided die (2, 4, and 6).

•
$$P(\text{rolling an even number}) = \frac{\text{number of even numbers}}{\text{total number of sides}}$$

= $\frac{3}{6}$
= $\frac{1 \times \cancel{3}}{2 \times \cancel{3}}$
= $\frac{1}{2}$

MCQ 23: A fruit is selected randomly from a basket containing 3 apples, 2 oranges, and 5 bananas.

Find the probability that the selected fruit is an orange (simplify the fraction).

$$P("selecting an orange") = \frac{1}{5}$$

Answer:

- To find the probability of selecting an orange, divide the number of oranges by the total number of fruits.
- There are 2 oranges out of 10 fruits in total.

•
$$P("selecting an orange") = \frac{\text{number of oranges}}{\text{total number of fruits}}$$

= $\frac{2}{10}$
= $\frac{1 \times \cancel{2}}{5 \times \cancel{2}}$
= $\frac{1}{5}$

F COMPLEMENT RULE

F.1 APPLYING THE COMPLEMENT RULE

Ex 24: I toss a fair coin. The probability of getting heads is $\frac{1}{2}$. Find the probability of getting tails.

$$P("Getting tails") = \frac{1}{2}$$

(°±°)

- Answer:
 - The probability of getting heads is $\frac{1}{2}$.
 - The event "Getting tails" is the complement of "Getting heads."
 - Using the complement rule:

$$P("Getting tails") = 1 - P("Getting heads")$$
$$= 1 - \frac{1}{2}$$
$$= \frac{2}{2} - \frac{1}{2}$$
$$= \frac{1}{2}$$

• So, the probability of getting tails is $\frac{1}{2} = 50\%$.

Ex 25: A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.

Find the probability that a student does not laugh at the joke.

$$P("Not laughing") = 30\%$$

Answer:

- The probability that a student laughs at the joke is 70%.
- The event "Not laughing" is the complement event of "Laughing."
- Using the complement rule:

$$P("Not laughing") = 1 - P("Laughing")$$

= 1 - 70%
= 100% - 70%
= 30%

• Therefore, the probability that a student does not laugh at the joke is 30%.

Ex 26: I randomly select a student in the class. The probability that a girl is selected is $\frac{9}{10}$.

Find the probability that a boy is selected.

$$P("\text{Selecting a boy"}) = \boxed{1}$$

Answer:

- The probability that a girl is selected is $\frac{9}{10}$.
- The event "Selecting a boy" is the complement of "Selecting a girl."
- Using the complement rule:

$$P("Selecting a boy") = 1 - P("Selecting a girl")$$
$$= 1 - \frac{9}{10}$$
$$= \frac{10}{10} - \frac{9}{10}$$
$$= \frac{1}{10}$$

• So, the probability that a boy is selected is $\frac{1}{10} = 10\%$.

Ex 27: The weather forecast predicts that there is a 70% chance of rain tomorrow.

Find the probability that it will not rain tomorrow.

$$P("\text{No rain"}) = \boxed{30}\%$$

Answer:

- The probability that it will rain tomorrow is 70%.
- The event "No rain" is the complement of the event "Rain".
- Using the complement rule:

$$P("No rain") = 1 - P("Rain")$$

= 1 - 70%
= 100% - 70%
= 30%

• Therefore, the probability that it will not rain tomorrow is 30%.

Ex 28: A survey shows that 70% of the students in a school love Math.

Find the probability that a randomly chosen student does not love Math.

$$P("Not loving Math") = 30\%$$

Answer:

- The probability that a student loves Math is 70%.
- The event "Not loving Math" is the complement event of "Loving Math."
- Using the complement rule:

$$P("Not loving Math") = 1 - P("Loving Math")$$

= 1 - 70%
= 100% - 70%
= 30%

• Therefore, the probability that a student does not love Math is 30%.

MCQ 29: A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.

Find the probability that a student does not laugh at the joke.

- $\boxtimes P("Not laughing") = 30\%$
- \square P("Not laughing") = 70%
- $\square P("Not laughing") = 50\%$

Answer:

- The probability that a student laughs at the joke is 70%.
- The event "Not laughing" is the complement event of "Laughing".



• Using the complement rule:

$$P("Not laughing") = 1 - P("Laughing")$$

= 1 - 70%
= 100% - 70%
= 30%

• So, the probability that a student does not laugh at the joke is 30%.

G EXPERIMENTAL PROBABILITY

G.1 SOLVING REAL-WORLD PROBLEMS

Ex 30: During a week of basketball practice, Mia made 45 out of 60 free-throw attempts. Estimate the experimental probability that Mia will make her next free-throw attempt.

$$P("Making the next attempt") \approx 75 \%$$

Answer:

- The experimental probability of Mia making a free-throw is the ratio of successful free-throws to the total number of attempts.
- $P("Making the next attempt") \approx \frac{Number of successes}{Total attempts}$ $\approx \frac{45}{60}$ ≈ 0.75 $\approx 75\%$
- So, the estimated probability that Mia will make her next free-throw attempt is 75%.

Ex 31: During a week, the school cafeteria recorded that out of 150 students, 120 chose a vegetarian meal. Estimate the probability that the next student will choose a vegetarian meal based on this experimental probability.

$$P($$
"Choosing a Vegetarian meal" $) \approx 80 \%$

Answer:

- The probability P("Vegetarian meal") is estimated by the ratio of times a vegetarian meal was chosen to the total number of trials.
- $P("Vegetarian meal") \approx \frac{Number of vegetarian meals}{Total number of students}$ $\approx \frac{120}{150}$ ≈ 0.8 $\approx 80\%$
- So, the estimated probability that the next student will choose a vegetarian meal is 80%.

Ex 32: Over the course of a year, it rained on 120 days out of 300 recorded days. Estimate the experimental probability that it will rain.

Answer:

• The experimental probability of raining is the ratio of rainy days to the total number of recorded days.

•
$$P("\text{Raining"}) \approx \frac{\text{Number of rainy days}}{\text{Total recorded days}}$$

 $\approx \frac{120}{300}$
 ≈ 0.4
 $\approx 40\%$

• So, the estimated probability that it will rain is 40%.

Ex 33: A local bakery found that out of 200 customers, 150 ordered a croissant. Estimate the experimental probability that the next customer will order a croissant.

$$P("Ordering a croissant") \approx \boxed{75}$$
 %

Answer:

• The experimental probability of a customer ordering a croissant is the ratio of customers who ordered a croissant to the total number of customers.

• $P("Ordering a croissant") \approx \frac{\text{Number of customers croissant}}{\text{Total number of customers}}$ $\approx \frac{150}{200}$ ≈ 0.75 $\approx 75\%$

• So, the estimated probability that the next customer will order a croissant is 75%.