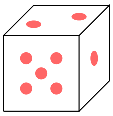
PROBABILITY

A ALGEBRA OF EVENTS

A.1 SAMPLE SPACE

A.1.1 FINDING THE SAMPLE SPACES

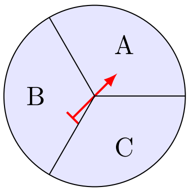
MCQ 1: A fair six-sided die is rolled once.



Find the sample space.

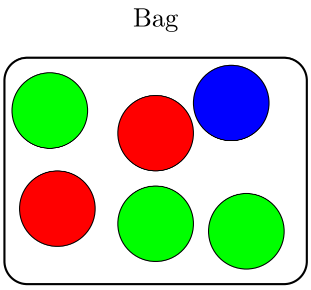
- ☐ $\{1, 2, 3, 4, 5\}$
- ☐ $\{1, 2, 3, 4, 5, 6, 7\}$
- ☐ $\{1, 2, 3, 4, 5, 6\}$

MCQ 2: Find the sample space that the spinner can land on:



- ☐ $\{A, B, C\}$
- ☐ $\{A, B\}$
- ☐ $\{A, C\}$

MCQ 3: A ball is chosen randomly from a bag containing 2 red balls, 1 blue ball, and 3 green balls.



Find the sample space.

- ☐ {Red, Blue, Green}
- ☐ {2 Red, 1 Blue, 3 Green}
- ☐ {Red, Red, Blue, Green, Green, Green}

MCQ 4: A letter is chosen randomly from the word BANANA. Find all possible outcomes for the chosen letter.

- ☐ {B, N, A}
- ☐ {B, A, N, A, N, A}
- ☐ {A, B, N, A, B, N}

A.2 EVENTS

A.2.1 FINDING THE EVENTS

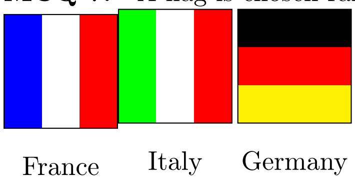
MCQ 5: A letter is chosen randomly from the word ORANGE. Find the event where the chosen letter is a vowel.

- ☐ {O, R, A, N, G, E}
- ☐ {O, A, E}
- ☐ {R, G, N}
- ☐ {A, G, E}

MCQ 6: A fair six-sided dice is rolled once. Find the event where the outcome is an even number.

- ☐ {1, 3, 5}
- ☐ {2, 4, 6 }
- ☐ {1, 2, 3, 4, 5, 6}
- ☐ {2, 3, 4, 5}

MCQ 7: A flag is chosen randomly from:



Find the event where the outcome is a flag with blue in them.

- ☐ {France }
- ☐ {Italy, France}
- ☐ {Italy, France, Germany}

MCQ 8: A flag is chosen randomly from:

Find the event where the outcome is a flag with red in them.

- ☐ {France, Japan}
- ☐ {Italy, France}
- ☐ {Italy, France, Germany, Japan}

MCQ 9: A flag is chosen randomly from:

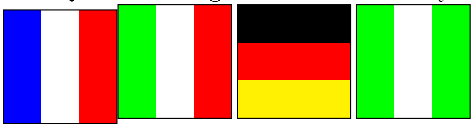
Find the event where the outcome is a flag with green in them.

- ☐ {France, Nigeria}
- ☐ {Italy, Nigeria}
- ☐ {Italy, France, Germany}

A.3 COMPLEMENTARY EVENT

A.3.1 FINDING THE COMPLEMENTARY EVENTS

MCQ 10: A flag is chosen randomly from the following:

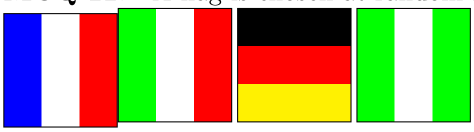


France Italy Germany Nigeria

Let E be the event where the selected flag contains green.
Find the complement of event E, denoted as E'.

- ☐ $E' = \{\text{France, Germany}\}$
- ☐ $E' = \{\text{Italy, Nigeria}\}$
- ☐ $E' = \{\text{Italy, France, Germany}\}$

MCQ 11: A flag is chosen at random from the following set:



France Italy Germany Nigeria

Let E be the event where the chosen flag contains the color red.
Find the complement of event E, denoted E'.

- ☐ $E' = \{\text{France, Germany}\}$
- ☐ $E' = \{\text{Nigeria}\}$
- ☐ $E' = \{\text{Italy, France, Germany}\}$

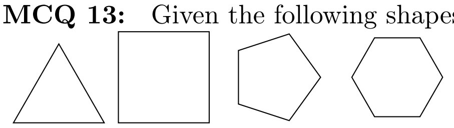
MCQ 12: A child's name is chosen randomly from the following list:

- Emily (girl's name)
- James (boy's name)
- Ava (girl's name)
- Sophia (girl's name)

Let E be the event where the selected name is a boy's name.
Find the complement of event E, denoted as E'.

- ☐ $E' = \{\text{Emily, Ava, Sophia}\}$
- ☐ $E' = \{\text{James}\}$
- ☐ $E' = \{\text{James, Ava}\}$

MCQ 13: Given the following shapes:

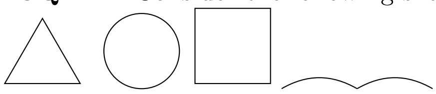


Triangle Square Pentagon Hexagon

Let E be the event where a polygon with an even number of sides is chosen.
Find the complement of event E, denoted as E'.

- ☐ $E' = \{\text{Square, Hexagon}\}$
- ☐ $E' = \{\text{Triangle, Pentagon}\}$
- ☐ $E' = \{\text{Triangle, Square, Pentagon, Hexagon}\}$

MCQ 14: Consider the following shapes:



Triangle Circle Square Curve

Let E be the event where the shape is a polygon.
Find the complement of event E, denoted as E'.

- ☐ $E' = \{\text{Triangle, Square}\}$
- ☐ $E' = \{\text{Triangle, Circle, Square, Curve}\}$
- ☐ $E' = \{\text{Circle, Curve}\}$

A.4 MULTI-STEP RANDOM EXPERIMENTS

A.4.1 FINDING OUTCOME IN A TABLE

MCQ 15: The table below shows the possible outcomes for the sexes of two children, first and second, where each can be a Boy (B) or a Girl (G).

first child \ second child	B	G
B	BB	?
G	GB	GG

Find the missing outcome.

- ☐ BB
- ☐ BG
- ☐ GB

MCQ 16: The table below shows the possible outcomes when selecting two letters at random from the word "MAT" **with replacement** (after choosing a letter, it is put back before the next selection).

letter 1 \ letter 2	M	A	T
M	MM	MA	MT
A	AM	AA	AT
T	TM	?	TT

Find the missing outcome.

- ☐ TT
- ☐ TA
- ☐ AT

MCQ 17: The table below shows the possible outcomes when selecting two letters at random from the word "CODE" **with replacement** (after choosing a letter, it is put back before the next selection).

letter 1 \ letter 2	C	O	D	E
C	CC	CO	CD	CE
O	OC	OO	OD	OE
D	DC	?	DD	DE
E	EC	EO	ED	EE

Find the missing outcome.

- ☐ DO
- ☐ OD
- ☐ DC

MCQ 18: The table below shows the possible outcomes when selecting two letters at random from the word "NODE" **without replacement** (after choosing a letter, it is not put back before the next selection). An "X" means no outcome is possible.

letter 1 \ letter 2	N	O	D	E
N	**X**	?	ND	NE
O	ON	**X**	OD	OE
D	DN	DO	**X**	DE
E	EN	EO	ED	**X**

Find the missing outcome.

- ☐ NN
- ☐ NO
- ☐ ON

MCQ 19: The table below shows the possible outcomes when a coach selects two players at random from four players (A, B, C, D) **without replacement** (after choosing a player, they are not put back before the next selection). An "X" means no outcome is possible.

Player 1 \ Player 2	A	B	C	D
A	**X**	?	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Find the missing outcome.

- ☐ AB
- ☐ BA
- ☐ CA

A.4.2 COUNTING THE NUMBER OF POSSIBLE OUTCOMES IN A TABLE

Ex 20: The table below shows the possible outcomes for the sexes of two children, first and second, where each can be a Boy (B) or a Girl (G).

first child \ second child	B	G
B	BB	BG
G	GB	GG

Count the number of possible outcomes.

☐ possibles outcomes.

Ex 21: There are four players: A, B, C, and D. For position 1, only players A and B are eligible. For position 2, only players C and D are eligible. The table below shows the possible selections for the two positions.

position 1 \ position 2	C	D
A	AC	AD
B	BC	BD

Count the number of possible outcomes.

☐ possible outcomes.

Ex 22: There are four players: A, B, C, and D. A coach selects two players at random without replacement. The table below shows the possible selections for the two positions. An "X" means no outcome is possible.

Player 1 \ Player 2	A	B	C	D
A	**X**	AB	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Count the number of possible outcomes.

☐ possible outcomes.

Ex 23: There are three students: X, Y, and Z. A teacher selects one student each day, on Monday and Tuesday, to recite a poem. The selection is made without replacement, meaning the same student cannot be chosen both days. The table below shows the possible selections for the two days.

Monday \ Tuesday	X	Y	Z
X	**X**	XY	XZ
Y	YX	**X**	YZ
Z	ZX	ZY	**X**

Count the number of possible outcomes.

☐ possible outcomes.

A.4.3 COUNTING THE NUMBER OF POSSIBLE OUTCOMES FOR AN EVENT

Ex 24: There are four players: A, B, C, and D. A coach selects two players at random without replacement. The table below shows the possible selections for the two positions. An "X" means no outcome is possible.

Player 1 \ Player 2	A	B	C	D
A	**X**	AB	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Count the number of outcomes for the event that player A is selected.

☐ outcomes.

Ex 25: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "double digit" (both dice show the same number).

[ ] outcomes.

Ex 26: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "at least one 6" (at least one die shows a 6).

[ ] outcomes.

Ex 27: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "the sum of the dice is equal to 11."

[ ] outcomes.

Ex 28: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "the sum of the dice is equal to 7."

[ ] outcomes.

Ex 29: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

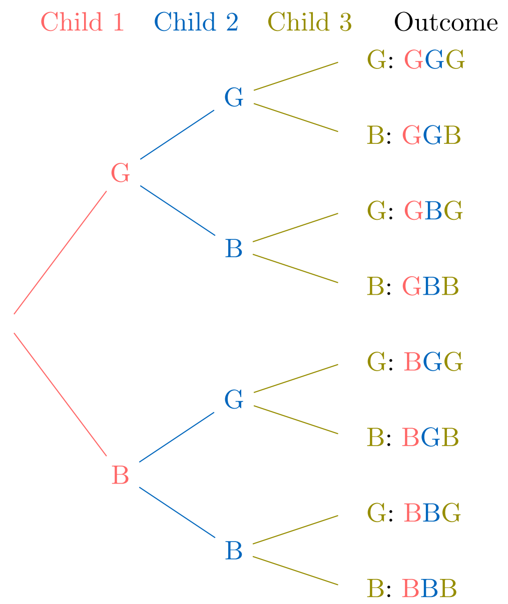
blue die red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "a result inferior or equal to 3" (the sum of the dice is less than or equal to 3).

[ ] outcomes.

A.4.4 COUNTING THE NUMBER OF POSSIBLE OUTCOMES IN AN TREE DIAGRAM

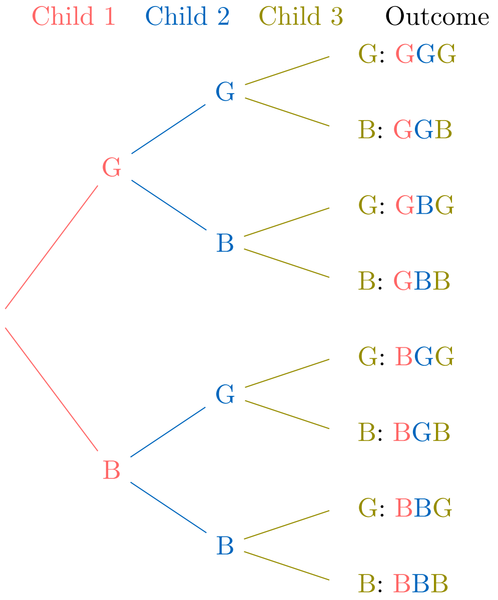
Ex 30: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



Count the number of possible outcomes for the event where the first child is a boy.

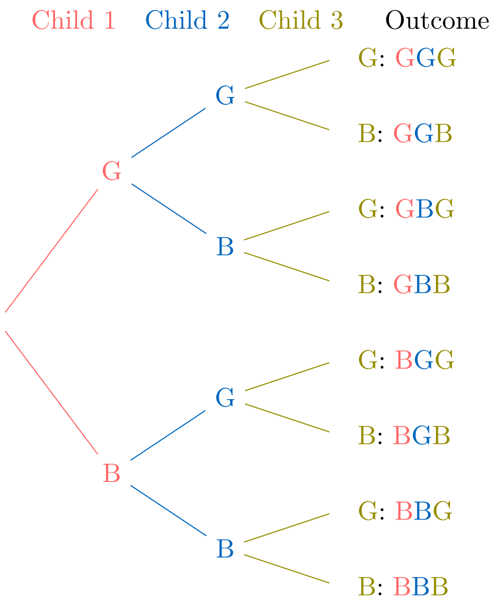
[ ]

Ex 31: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



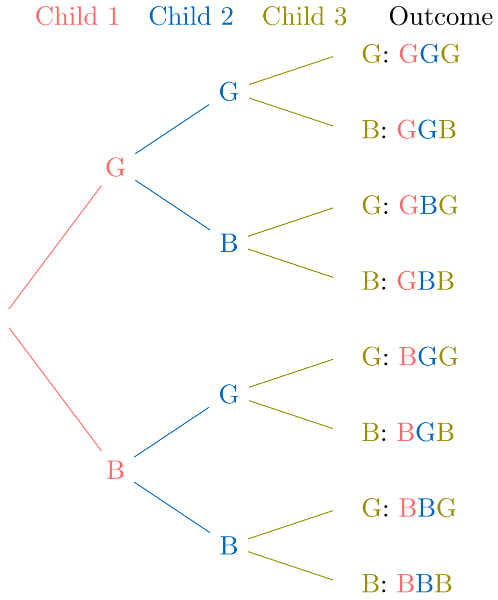
Count the number of possible outcomes for the event where there are exactly two girls.

Ex 32: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below shows all 8 possible gender outcomes for the three children.



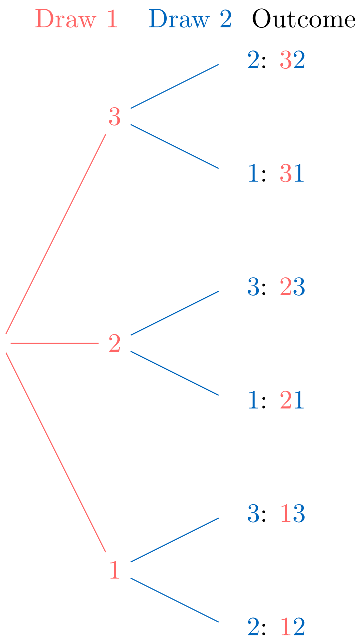
Count the number of possible outcomes for the event where there are at least two girls.

Ex 33: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



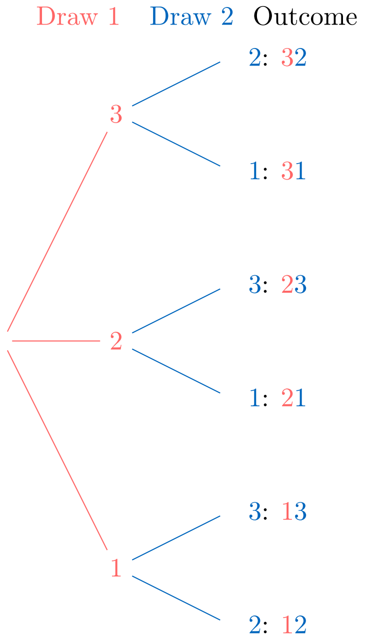
Count the number of possible outcomes for the event where the family has mixed-sex children (at least one boy and one girl).

Ex 34: Tickets numbered 1, 2, and 3 are placed in a bag. One ticket is drawn and set aside, then a second ticket is drawn. The tree diagram below shows all possible outcomes for these two selections.



Count the number of possible outcomes in the sample space for the two ticket selections.

Ex 35: Tickets numbered 1, 2, and 3 are placed in a bag. One ticket is drawn and set aside, then a second ticket is drawn. The tree diagram below displays all possible outcomes for these two selections.



Count the number of possible outcomes for the event where the ticket numbered 1 is drawn (either as the first or second ticket).

B PROBABILITY

B.1 DEFINITION

B.1.1 DETERMINING THE PROBABILITY

MCQ 36: Keziah eats rice often. Let E be the event that Keziah eats rice this week. Find $P(E)$, the probability that Keziah eats rice this week.

- ☐ $P(E) = 1\%$
- ☐ $P(E) = 50\%$
- ☐ $P(E) = 99\%$

MCQ 37: Emily drinks water every day. Let E be the event that Emily drinks water tomorrow. Find $P(E)$, the probability that Emily drinks water tomorrow.

- ☐ $P(E) = 50\%$
- ☐ $P(E) = 90\%$
- ☐ $P(E) = 100\%$

MCQ 38: It almost never snows in July in the Sahara Desert. Let E be the event that it snows this July in the Sahara Desert. Find $P(E)$, the probability that it snows this July.

- ☐ $P(E) = 0.01\%$
- ☐ $P(E) = 5\%$
- ☐ $P(E) = 99.9\%$

MCQ 39: Samuel loves playing basketball. Let E be the event that Samuel plays basketball this weekend. Find $P(E)$, the probability that Samuel plays this weekend.

- ☐ $P(E) = 5\%$
- ☐ $P(E) = 20\%$
- ☐ $P(E) = 90\%$

MCQ 40: Benjamin rolls a die. Let E be the event that Benjamin rolls a number bigger than 7. Find $P(E)$, the probability that Benjamin rolls a number bigger than 7.

- ☐ $P(E) = 0\%$
- ☐ $P(E) = 50\%$
- ☐ $P(E) = 100\%$

B.2 EQUALLY LIKELY

B.2.1 DETERMINING THE PROBABILITY

Ex 41: A ball is chosen randomly from a bag containing 2 red balls, 3 blue balls.
Find the probability that we choose a red ball.

$$P(\text{"choosing a red ball"}) = \frac{\square}{\square}$$

Ex 42: A card is drawn at random from a standard deck of 52 playing cards. Determine the probability of drawing an Ace and express your answer as a simplified fraction.

$$P(\text{"drawing an Ace"}) = \frac{\square}{\square}$$

Ex 43: A six-sided die is rolled once. Determine the probability of obtaining an even number.

$$P(\text{"rolling an even number"}) = \frac{\square}{\square}$$

MCQ 44: A fruit is selected randomly from a basket containing 3 apples, 2 oranges, and 5 bananas.
Find the probability that the selected fruit is an orange (simplify the fraction).

$$P(\text{"selecting an orange"}) = \frac{\square}{\square}$$

B.2.2 DETERMINING THE PROBABILITY IN MULTI-STEP RANDOM EXPERIMENTS

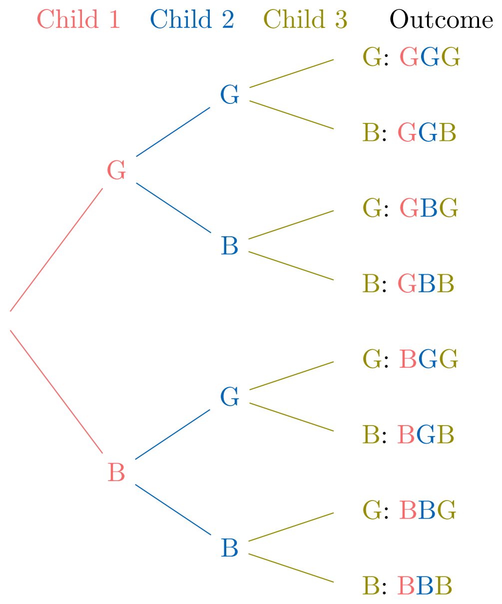
Ex 45: A coach selects two players at random from a group of four players, labeled A, B, C, and D, without replacement (once a player is chosen, they are not available for the next selection). The table below shows all possible outcomes for selecting Player 1 and Player 2, where an "X" indicates an impossible outcome due to the same player being selected twice.

Player 2 / Player 1	A	B	C	D
A	**X**	AB	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Calculate the probability that player C is selected as either Player 1 or Player 2, simplifying the fraction.

$$P(\text{"selecting player C"}) = \frac{\square}{\square}$$

Ex 46: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



Calculate the probability that the family has at least two girls, simplifying the fraction.

$$P(\text{"at least two girls"}) = \frac{\square}{\square}$$

Ex 47: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is exactly 7, simplifying the fraction.

$$P(\text{"sum is 7"}) = \frac{\square}{\square}$$

Ex 48: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is greater than or equal to 11, simplifying the fraction.

$$P(\text{"sum } \geq 11\text{"}) = \frac{\square}{\square}$$

Ex 49: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is exactly 6 or 8, simplifying the fraction.

$$P(\text{"sum is 6 or 8"}) = \frac{\square}{\square}$$

B.3 COMPLEMENT RULE

B.3.1 APPLYING THE COMPLEMENT RULE

Ex 50: I toss a fair coin. The probability of getting heads is $\frac{1}{2}$. Find the probability of getting tails.

$$P(\text{"Getting tails"}) = \frac{\square}{\square}$$

Ex 51: A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.
Find the probability that a student does not laugh at the joke.

$$P(\text{"Not laughing"}) = \square\,\%$$

Ex 52: I randomly select a student in the class. The probability that a girl is selected is $\frac{9}{10}$.
Find the probability that a boy is selected.

$$P(\text{"Selecting a boy"}) = \frac{\square}{\square}$$

Ex 53: The weather forecast predicts that there is a 70% chance of rain tomorrow.
Find the probability that it will not rain tomorrow.

$$P(\text{"No rain"}) = \square\,\%$$

Ex 54: A survey shows that 70% of the students in a school love Math.
Find the probability that a randomly chosen student does not love Math.

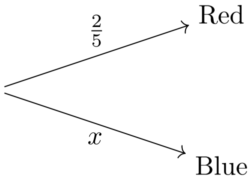
$$P(\text{"Not loving Math"}) = \square\,\%$$

MCQ 55: A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.
Find the probability that a student does not laugh at the joke.

- ☐ $P(\text{"Not laughing"}) = 30\%$
- ☐ $P(\text{"Not laughing"}) = 70\%$
- ☐ $P(\text{"Not laughing"}) = 50\%$

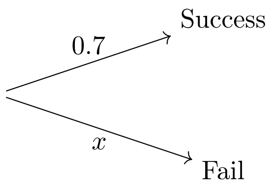
B.3.2 COMPLETING A PROBABILITY TREE DIAGRAM

Ex 56: From a bag containing red balls and blue balls, the probability of choosing a red ball is $\frac{2}{5}$. Find the probability x of choosing a blue ball.



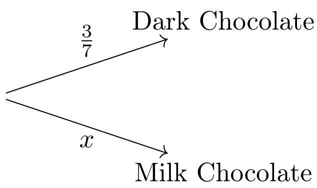
$x = \frac{\square}{\square}$

Ex 57: Jasper is playing basketball. The probability that he makes his first shot is 0.7. Find the probability x that he misses his first shot.



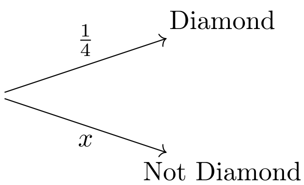
$x = \square$

Ex 58: In a box of assorted chocolates, the probability of picking a dark chocolate is $\frac{3}{7}$. Find the probability x of picking a milk chocolate.



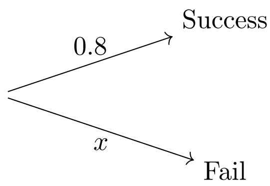
$x = \frac{\square}{\square}$

Ex 59: In a deck of cards, the probability of drawing a card from the suit of diamonds is $\frac{1}{4}$. Find the probability x of drawing a card that is not a diamond.



$x = \frac{\square}{\square}$

Ex 60: Emma is playing a video game. The probability that she completes a level is 0.8. Find the probability x that she fails to complete the level.

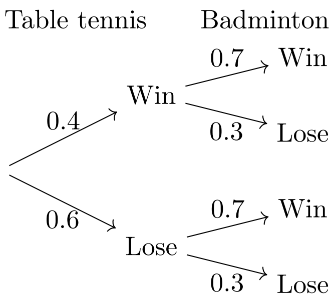


$x = \square$

B.4 PROBABILITY OF INDEPENDENT EVENTS

B.4.1 READING PROBABILITY TREE

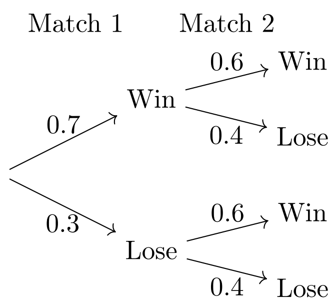
Ex 61: Niamh plays a game of table tennis on Saturday and a game of badminton on Sunday. The probability tree is represented:



Calculate the probability that Niamh wins both games.

$P(\text{"Win both"}) = \square$

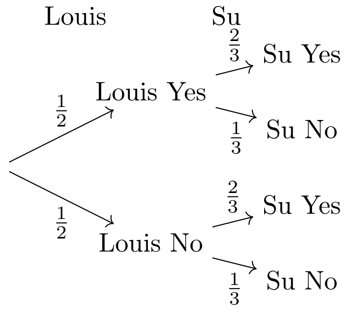
Ex 62: Sam is playing an online multiplayer game. The probability that Sam wins their first match is 0.7, and the probability that Sam wins their second match is 0.6.



Calculate the probability that Sam loses both the first and second matches.

$P(\text{"Both lose"}) = \square$

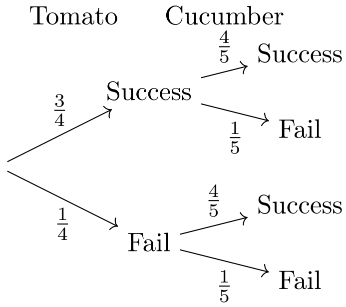
Ex 63: A party is happening this weekend! Louis might come with a probability of $\frac{1}{2}$, and Su might come with a probability of $\frac{2}{3}$.



Calculate the probability that both Louis and Su come to the party (simplify the fraction).

$$P(\text{"Both come"}) = \frac{\square}{\square}$$

Ex 64: Mia takes care of her garden. The probability that her tomato plants grow is $\frac{3}{4}$, and the probability that her cucumber plants grow is $\frac{4}{5}$.



Calculate the probability that both the tomato plants and the cucumber plants fail to grow (simplify the fraction).

$$P(\text{"Both fail"}) = \frac{\square}{\square}$$

B.4.2 FINDING THE PROBABILITY WITH INDEPENDENT EVENTS

Ex 65: Imagine you're at a carnival playing a game. You pick a ball from a bag containing 2 red balls and 3 blue balls, then roll a fair six-sided die. Find the probability of choosing a red ball **and** rolling a 6 (simplify the fraction).

$$P(\text{"Red" and "6"}) = \frac{\square}{\square}$$

Ex 66: Imagine you're at a carnival playing another game. You flip a fair coin and draw a card from a standard deck of 52 playing cards. Find the probability of getting heads **and** drawing an Ace (simplify the fraction).

$$P(\text{"Heads" and "Ace"}) = \frac{\square}{\square}$$

Ex 67: Imagine you're at a carnival playing a dice game. You roll a fair six-sided die two times in a row. Find the probability of getting a number greater than 4 (a 5 or 6) on both rolls (simplify the fraction).

$$P(\text{"Number} > 4\text{" and "Number} > 4\text{"}) = \frac{\square}{\square}$$

Ex 68: Sam is playing an online multiplayer game. The probability that Sam wins their first match is 0.7, and the probability that Sam wins their second match is 0.6.
Calculate the probability that Sam loses both the first and second matches.

$$P(\text{"Both lose"}) = \square$$

Ex 69: Mia takes care of her garden. The probability that her tomato plants grow is $\frac{3}{4}$, and the probability that her cucumber plants grow is $\frac{4}{5}$.
Calculate the probability that both the tomato plants and the cucumber plants fail to grow (simplify the fraction).

$$P(\text{"Both fail"}) = \frac{\square}{\square}$$

B.5 EXPERIMENTAL PROBABILITY

B.5.1 SOLVING REAL-WORLD PROBLEMS

Ex 70: During a week of basketball practice, Mia made 45 out of 60 free-throw attempts. Estimate the experimental probability that Mia will make her next free-throw attempt.

$$P(\text{"Making the next attempt"}) \approx \square \ \%$$

Ex 71: During a week, the school cafeteria recorded that out of 150 students, 120 chose a vegetarian meal. Estimate the probability that the next student will choose a vegetarian meal based on this experimental probability.

$$P(\text{"Choosing a Vegetarian meal"}) \approx \square \ \%$$

Ex 72: Over the course of a year, it rained on 120 days out of 300 recorded days. Estimate the experimental probability that it will rain.

$$P(\text{"Raining"}) \approx \square \ \%$$

Ex 73: A local bakery found that out of 200 customers, 150 ordered a croissant. Estimate the experimental probability that the next customer will order a croissant.

$$P(\text{"Ordering a croissant"}) \approx \square \ \%$$