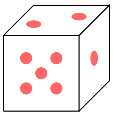
PROBABILITY

A ALGEBRA OF EVENTS

A.1 SAMPLE SPACE

A.1.1 FINDING THE SAMPLE SPACES

MCQ 1: A fair six-sided die is rolled once.



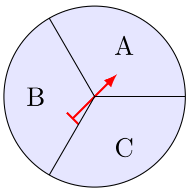
Find the sample space.

- ☐ $\{1, 2, 3, 4, 5\}$
- ☐ $\{1, 2, 3, 4, 5, 6, 7\}$
- ☒ $\{1, 2, 3, 4, 5, 6\}$

Answer:

- The sample space is all possible outcomes.
- When rolling a fair six-sided die, the possible outcomes are the numbers on the die's faces.
- So, the sample space is $\{1, 2, 3, 4, 5, 6\}$.

MCQ 2: Find the sample space that the spinner can land on:



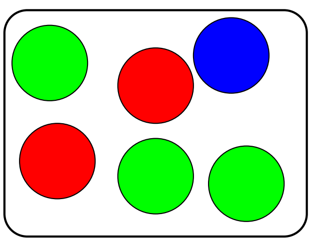
- ☒ $\{A, B, C\}$
- ☐ $\{A, B\}$
- ☐ $\{A, C\}$

Answer:

- The sample space is all possible outcomes.
- The spinner has three distinct regions labeled A, B, and C.
- So, the sample space is: $\{A, B, C\}$.

MCQ 3: A ball is chosen randomly from a bag containing 2 red balls, 1 blue ball, and 3 green balls.

Bag



Find the sample space.

- ☒ {Red, Blue, Green}
- ☐ {2 Red, 1 Blue, 3 Green}
- ☐ {Red, Red, Blue, Green, Green, Green}

Answer:

- When choosing a ball randomly from the bag containing 2 red balls, 1 blue ball, and 3 green balls, the balls are identical in color, so we do not distinguish between them based on quantity.
- So, the sample space (all possible outcomes) is: {Red, Blue, Green}

MCQ 4: A letter is chosen randomly from the word BANANA. Find all possible outcomes for the chosen letter.

- ☒ {B, N, A}
- ☐ {B, A, N, A, N, A}
- ☐ {A, B, N, A, B, N}

Answer:

- When choosing a letter randomly from the word "BANANA", the possible outcomes are the distinct letters in the word.
- So, the sample space (all possible outcomes) is: {B, A, N} or {B, N, A}. The order in which the letters are listed does not matter.

A.2 EVENTS

A.2.1 FINDING THE EVENTS

MCQ 5: A letter is chosen randomly from the word ORANGE. Find the event where the chosen letter is a vowel.

- ☐ {O, R, A, N, G, E}
- ☒ {O, A, E}
- ☐ {R, G, N}
- ☐ {A, G, E}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- When choosing a letter randomly from the word "ORANGE", the event where the chosen letter is a vowel consists of the vowels in the word.
- So, the event is: {O, A, E}.

MCQ 6: A fair six-sided dice is rolled once.
Find the event where the outcome is an even number.

- ☐ {1, 3, 5}

☒ {2, 4, 6 }

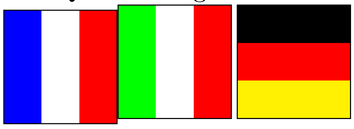
☐ {1, 2, 3, 4, 5, 6}

☐ {2, 3, 4, 5}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- When rolling a fair six-sided dice, the event where the outcome is an even number consists of the even numbers on the dice.
- So, the event is: {2, 4, 6}.

MCQ 7: A flag is chosen randomly from:



France Italy Germany

Find the event where the outcome is a flag with blue in them.

☒ {France }

☐ {Italy, France}

☐ {Italy, France, Germany}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- Among the given flags, only the French flag has blue in it.
- So, the correct answer is: {France}.

MCQ 8: A flag is chosen randomly from:

France Italy Germany Japan

Find the event where the outcome is a flag with red in them.

☐ {France, Japan}

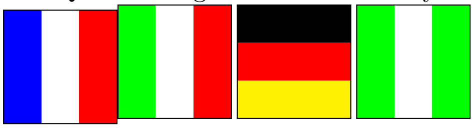
☐ {Italy, France}

☒ {Italy, France, Germany, Japan}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- Among the given flags, France, Italy, Germany, and Japan have red in them.
- So, the correct answer is: {Italy, France, Germany, Japan}.

MCQ 9: A flag is chosen randomly from:



France Italy Germany Nigeria

Find the event where the outcome is a flag with green in them.

☐ {France, Nigeria}

☒ {Italy, Nigeria}

☐ {Italy, France, Germany}

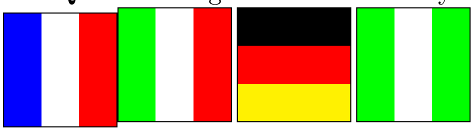
Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- Among the given flags, Italy and Nigeria have green in them.
- So, the correct answer is: {Italy, Nigeria}.

A.3 COMPLEMENTARY EVENT

A.3.1 FINDING THE COMPLEMENTARY EVENTS

MCQ 10: A flag is chosen randomly from the following:



France Italy Germany Nigeria

Let E be the event where the selected flag contains green. Find the complement of event E, denoted as E'.

☒ E' = {France, Germany}

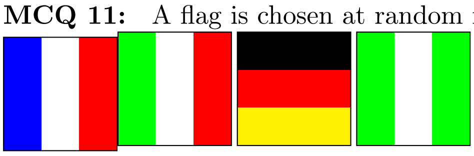
☐ E' = {Italy, Nigeria}

☐ E' = {Italy, France, Germany}

Answer:

- The event E includes flags with the color green: {Italy, Nigeria}.
- The complement event E' consists of flags that do not have green.
- So, the complement event E' = {France, Germany}.

MCQ 11: A flag is chosen at random from the following set:



France Italy Germany Nigeria

Let E be the event where the chosen flag contains the color red. Find the complement of event E, denoted E'.

☐ E' = {France, Germany}

☒ E' = {Nigeria}

☐ E' = {Italy, France, Germany}

Answer:

- The event E consists of flags that contain the color red: {France, Italy, Germany}.
- The complement of event E, denoted E', consists of flags that do not contain the color red.
- Since Nigeria is the only flag without the color red, E' = {Nigeria}.
- So, the correct answer is E' = {Nigeria}.

MCQ 12: A child's name is chosen randomly from the following list:

- Emily (girl's name)
- James (boy's name)
- Ava (girl's name)
- Sophia (girl's name)

Let E be the event where the selected name is a boy's name.
Find the complement of event E, denoted as E'.

☒ E' = {Emily, Ava, Sophia}

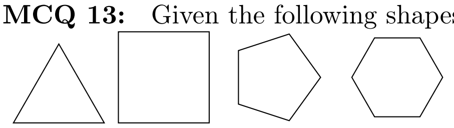
☐ E' = {James}

☐ E' = {James, Ava}

Answer:

- The event E includes boy's names: {James}.
- The complement event E' consists of names that are not boy's names (i.e., girl's names).
- So, the complement event E' = {Emily, Ava, Sophia}.

MCQ 13: Given the following shapes:



Triangle Square Pentagon Hexagon

Let E be the event where a polygon with an even number of sides is chosen.
Find the complement of event E, denoted as E'.

☐ E' = {Square, Hexagon}

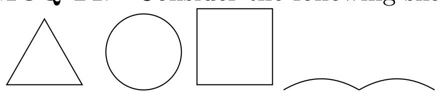
☒ E' = {Triangle, Pentagon}

☐ E' = {Triangle, Square, Pentagon, Hexagon}

Answer:

- Triangle: three sides. Square: four sides. Pentagon: five sides. Hexagon: six sides.
- The event E includes polygons with an even number of sides: {Square, Hexagon}.
- The complementary event E' consists of polygons that do not have an even number of sides (have an odd number of sides).
- Therefore, the complementary event E' = {Triangle, Pentagon}.

MCQ 14: Consider the following shapes:



Triangle Circle Square Curve

Let E be the event where the shape is a polygon.
Find the complement of event E, denoted as E'.

☐ E' = {Triangle, Square}

☐ E' = {Triangle, Circle, Square, Curve}

☒ E' = {Circle, Curve}

Answer:

- The event E includes shapes that are polygons: {Triangle, Square}.
- The complementary event E' consists of shapes that are not polygons.
- Therefore, the complementary event E' = {Circle, Curve}.

A.4 MULTI-STEP RANDOM EXPERIMENTS

A.4.1 FINDING OUTCOME IN A TABLE

MCQ 15: The table below shows the possible outcomes for the sexes of two children, first and second, where each can be a Boy (B) or a Girl (G).

second child / first child	B	G
B	BB	?
G	GB	GG

Find the missing outcome.

☐ BB

☒ BG

☐ GB

Answer:

- The first child is represented by the row ("first child"), and the second child by the column ("second child").
- The missing outcome is BG.

MCQ 16: The table below shows the possible outcomes when selecting two letters at random from the word "MAT" **with replacement** (after choosing a letter, it is put back before the next selection).

letter 2 / letter 1	M	A	T
M	MM	MA	MT
A	AM	AA	AT
T	TM	?	TT

Find the missing outcome.

☐ TT

☒ TA

☐ AT

Answer:

- The first letter is "Letter 1" (row), and the second letter is "Letter 2" (column).
- The missing outcome is TA.

MCQ 17: The table below shows the possible outcomes when selecting two letters at random from the word "CODE" **with replacement** (after choosing a letter, it is put back before the next selection).

letter 2 letter 1	C	O	D	E
C	CC	CO	CD	CE
O	OC	OO	OD	OE
D	DC	?	DD	DE
E	EC	EO	ED	EE

Find the missing outcome.

☒ DO

☐ OD

☐ DC

Answer:

- The first letter is "Letter 1" (row), and the second letter is "Letter 2" (column).
- The missing outcome is DO.

MCQ 18: The table below shows the possible outcomes when selecting two letters at random from the word "NODE" **without replacement** (after choosing a letter, it is not put back before the next selection). An "X" means no outcome is possible.

letter 2 letter 1	N	O	D	E
N	**X**	?	ND	NE
O	ON	**X**	OD	OE
D	DN	DO	**X**	DE
E	EN	EO	ED	**X**

Find the missing outcome.

☐ NN

☒ NO

☐ ON

Answer:

- The first letter is "Letter 1" (row), and the second letter is "Letter 2" (column).
- The missing outcome is NO.

MCQ 19: The table below shows the possible outcomes when a coach selects two players at random from four players (A, B, C, D) **without replacement** (after choosing a player, they are not put back before the next selection). An "X" means no outcome is possible.

Player 2 Player 1	A	B	C	D
A	**X**	?	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Find the missing outcome.

☒ AB

☐ BA

☐ CA

Answer:

- The first player is "Player 1" (row), and the second player is "Player 2" (column).
- The missing outcome is AB.

A.4.2 COUNTING THE NUMBER OF POSSIBLE OUTCOMES IN A TABLE

Ex 20: The table below shows the possible outcomes for the sexes of two children, first and second, where each can be a Boy (B) or a Girl (G).

second child first child	B	G
B	BB	BG
G	GB	GG

Count the number of possible outcomes.

[4] possibles outcomes.

Answer:

- The sample space (the possible outcomes) is $\{GG, GB, BP, FF\}$.
- The number of possible outcomes is 4.

Ex 21: There are four players: A, B, C, and D. For position 1, only players A and B are eligible. For position 2, only players C and D are eligible. The table below shows the possible selections for the two positions.

position 2 position 1	C	D
A	AC	AD
B	BC	BD

Count the number of possible outcomes.

[4] possible outcomes.

Answer:

- The sample space (the possible outcomes) is $\{AC, AD, BC, BD\}$.
- The number of possible outcomes is 4.

Ex 22: There are four players: A, B, C, and D. A coach selects two players at random without replacement. The table below shows the possible selections for the two positions. An "X" means no outcome is possible.

Player 2 / Player 1	A	B	C	D
A	**X**	AB	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Count the number of possible outcomes.

12 possible outcomes.

Answer:

- The sample space (the possible outcomes) is $\{AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC\}$.
- The number of possible outcomes is 12.

Ex 23: There are three students: X, Y, and Z. A teacher selects one student each day, on Monday and Tuesday, to recite a poem. The selection is made without replacement, meaning the same student cannot be chosen both days. The table below shows the possible selections for the two days.

Tuesday / Monday	X	Y	Z
X	**X**	XY	XZ
Y	YX	**X**	YZ
Z	ZX	ZY	**X**

Count the number of possible outcomes.

6 possible outcomes.

Answer:

- The sample space (the possible outcomes) is $\{XY, XZ, YX, YZ, ZX, ZY\}$.
- The number of possible outcomes is 6.

A.4.3 COUNTING THE NUMBER OF POSSIBLE OUTCOMES FOR AN EVENT

Ex 24: There are four players: A, B, C, and D. A coach selects two players at random without replacement. The table below shows the possible selections for the two positions. An "X" means no outcome is possible.

Player 2 / Player 1	A	B	C	D
A	**X**	AB	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Count the number of outcomes for the event that player A is selected.

6 outcomes.

Answer:

- The event that player A is selected includes all outcomes where A is either Player 1 or Player 2.
- From the table:
  - When A is Player 1 (row A): AB, AC, AD (3 outcomes).
  - When A is Player 2 (column A): BA, CA, DA (3 outcomes).
- Total outcomes: $\{AB, AC, AD, BA, CA, DA\}$, which is 6 outcomes.

Ex 25: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "double digit" (both dice show the same number).

6 outcomes.

Answer:

- The event "double digit" includes all outcomes where the red die and blue die show the same number.
- From the table:
  - Possible doubles: 11, 22, 33, 44, 55, 66 (6 outcomes).
- Total outcomes: $\{11, 22, 33, 44, 55, 66\}$, which is 6 outcomes.

Ex 26: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "at least one 6" (at least one die shows a 6).

11 outcomes.

Answer:

- The event "at least one 6" includes all outcomes where at least one of the red die or blue die shows a 6.
- From the table:
  - Red die = 6 (row 6): 61, 62, 63, 64, 65, 66 (6 outcomes).
  - Blue die = 6 (column 6), excluding (6,6) already counted: 16, 26, 36, 46, 56 (5 outcomes).
- Total outcomes: $\{16, 26, 36, 46, 56, 61, 62, 63, 64, 65, 66\}$, which is 11 outcomes.

Ex 27: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "the sum of the dice is equal to 11."

$\boxed{2}$ outcomes.

Answer:

- The event "the sum of the dice is equal to 11" includes all outcomes where the red die and blue die sum to 11.
- From the table:
  - Possible pairs: 56 (5 + 6 = 11), 65 (6 + 5 = 11) (2 outcomes).
- Total outcomes: $\{56, 65\}$, which is 2 outcomes.

Ex 28: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "the sum of the dice is equal to 7."

$\boxed{6}$ outcomes.

Answer:

- The event "the sum of the dice is equal to 7" includes all outcomes where the red die and blue die sum to 7.
- From the table:
  - Possible pairs: 16 (1 + 6 = 7), 25 (2 + 5 = 7), 34 (3 + 4 = 7), 43 (4 + 3 = 7), 52 (5 + 2 = 7), 61 (6 + 1 = 7) (6 outcomes).
- Total outcomes: $\{16, 25, 34, 43, 52, 61\}$, which is 6 outcomes.

Ex 29: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "a result inferior or equal to 3" (the sum of the dice is less than or equal to 3).

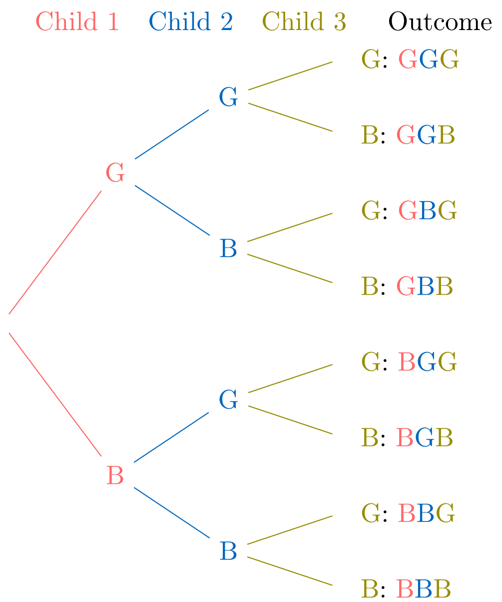
$\boxed{3}$ outcomes.

Answer:

- The event "a result inferior or equal to 3" includes all outcomes where the sum of the red die and blue die is less than or equal to 3.
- From the table:
  - Possible pairs: 11 (1 + 1 = 2), 12 (1 + 2 = 3), 21 (2 + 1 = 3) (3 outcomes).
- Total outcomes: $\{11, 12, 21\}$, which is 3 outcomes.

A.4.4 COUNTING THE NUMBER OF POSSIBLE OUTCOMES IN AN TREE DIAGRAM

Ex 30: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



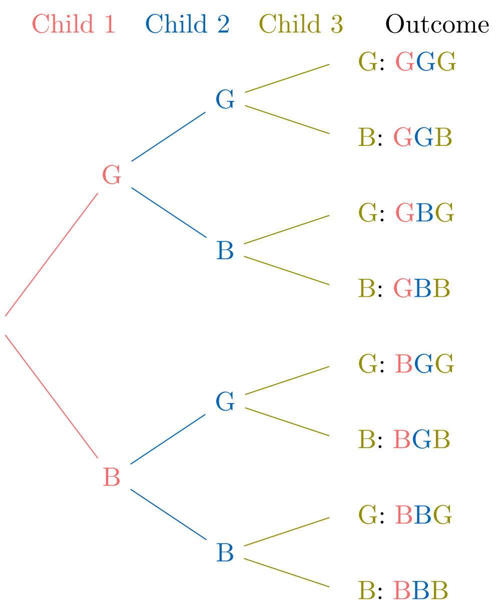
Count the number of possible outcomes for the event where the first child is a boy.

4

Answer:

- The event where the first child is a boy includes all outcomes starting with B, represented as $B**$.
- These outcomes are: BBB, BBG, BGB, BGG.
- The number of possible outcomes is 4.

Ex 31: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



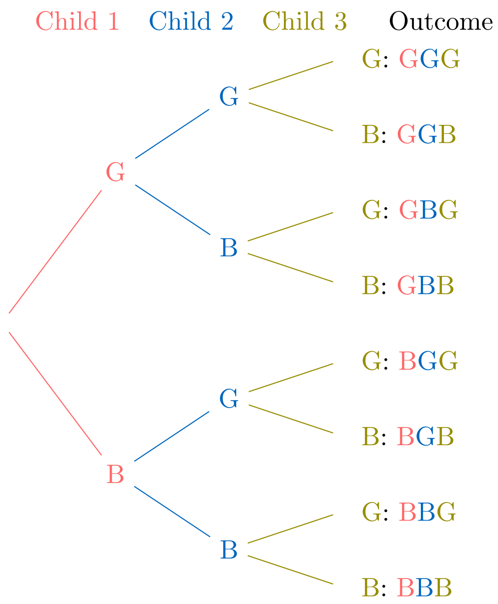
Count the number of possible outcomes for the event where there are exactly two girls.

3

Answer:

- The event where there are exactly two girls includes all outcomes with exactly two G's and one B.
- These outcomes are: BGG, GBG, GGB.
- The number of possible outcomes is 3.

Ex 32: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below shows all 8 possible gender outcomes for the three children.



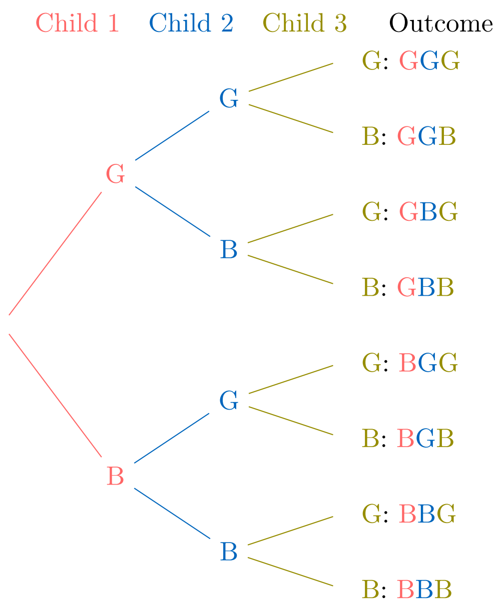
Count the number of possible outcomes for the event where there are at least two girls.

4

Answer:

- The event where there are at least two girls includes all outcomes with two or three girls.
- These outcomes are: BGG, GBG, GGB, GGG.
- The number of possible outcomes is 4.

Ex 33: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



Count the number of possible outcomes for the event where the family has mixed-sex children (at least one boy and one girl).

6

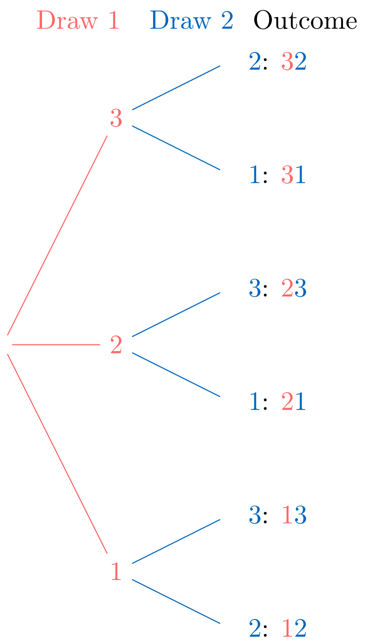
Answer:

- The event where the family has mixed-sex children includes all outcomes with at least one boy (B) and one girl (G), excluding all-boys (BBB) and all-girls (GGG).

- These outcomes are: BBG, BGB, GBB, GGB, GBG, BGG.

- The number of possible outcomes is 6.

Ex 34: Tickets numbered 1, 2, and 3 are placed in a bag. One ticket is drawn and set aside, then a second ticket is drawn. The tree diagram below shows all possible outcomes for these two selections.



Count the number of possible outcomes in the sample space for the two ticket selections.

6

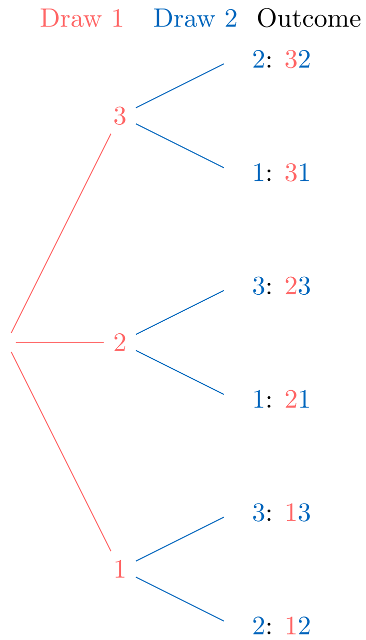
Answer:

- The sample space includes all possible pairs of tickets drawn, where the first number is the first ticket and the second is the ticket drawn afterward. Since a ticket cannot be drawn twice, pairs with repeated numbers (e.g., 11, 22, 33) are excluded.

- These outcomes are: 12, 13, 21, 23, 31, 32.

- The number of possible outcomes is 6.

Ex 35: Tickets numbered 1, 2, and 3 are placed in a bag. One ticket is drawn and set aside, then a second ticket is drawn. The tree diagram below displays all possible outcomes for these two selections.



Count the number of possible outcomes for the event where the ticket numbered 1 is drawn (either as the first or second ticket).

4

Answer:

- The event where ticket numbered 1 is drawn includes outcomes where 1 is either the first ticket ($1*$) or the second ticket ($*1$). Repeated numbers are impossible due to the selection process.

- These outcomes are: 12, 13, 21, 31.

- The number of possible outcomes is 4.

B PROBABILITY

B.1 DEFINITION

B.1.1 DETERMINING THE PROBABILITY

MCQ 36: Keziah eats rice often. Let E be the event that Keziah eats rice this week. Find $P(E)$, the probability that Keziah eats rice this week.

☐ $P(E) = 1\%$

☐ $P(E) = 50\%$

☒ $P(E) = 99\%$

Answer:

- Since he eats rice often, it's very likely.

- So, the probability that Keziah eats rice this week is $P(E) = 99\%$.

MCQ 37: Emily drinks water every day. Let E be the event that Emily drinks water tomorrow. Find $P(E)$, the probability that Emily drinks water tomorrow.

☐ $P(E) = 50\%$

☐ $P(E) = 90\%$

☒ $P(E) = 100\%$

Answer:

- She drinks water every day, so it's certain.
- So, the probability that Emily drinks water tomorrow is $P(E) = 100\%$.

MCQ 38: It almost never snows in July in the Sahara Desert. Let E be the event that it snows this July in the Sahara Desert. Find $P(E)$, the probability that it snows this July.

☒ $P(E) = 0.01\%$

☐ $P(E) = 5\%$

☐ $P(E) = 99.9\%$

Answer:

- It's extremely rare, so very unlikely.
- So, the probability that it snows this July is $P(E) = 0.01\%$.

MCQ 39: Samuel loves playing basketball. Let E be the event that Samuel plays basketball this weekend. Find $P(E)$, the probability that Samuel plays this weekend.

☐ $P(E) = 5\%$

☐ $P(E) = 20\%$

☒ $P(E) = 90\%$

Answer:

- He loves it, so it's very likely.
- So, the probability that Samuel plays this weekend is $P(E) = 90\%$.

MCQ 40: Benjamin rolls a die. Let E be the event that Benjamin rolls a number bigger than 7. Find $P(E)$, the probability that Benjamin rolls a number bigger than 7.

☒ $P(E) = 0\%$

☐ $P(E) = 50\%$

☐ $P(E) = 100\%$

Answer:

- A die only has numbers 1 to 6, so it's impossible.
- So, the probability that Benjamin rolls a number bigger than 7 is $P(E) = 0\%$.

B.2 EQUALLY LIKELY

B.2.1 DETERMINING THE PROBABILITY

Ex 41: A ball is chosen randomly from a bag containing 2 red balls, 3 blue balls.
Find the probability that we choose a red ball.

$$P(\text{"choosing a red ball"}) = \frac{\boxed{2}}{\boxed{5}}$$

Answer:

- To find the probability of choosing a red ball, divide the number of red balls by the total number of balls.
- $P(\text{"choosing a red ball"}) = \dfrac{\text{number of red balls}}{\text{total number of balls}} = \dfrac{2}{5}$

Ex 42: A card is drawn at random from a standard deck of 52 playing cards. Determine the probability of drawing an Ace and express your answer as a simplified fraction.

$$P(\text{"drawing an Ace"}) = \frac{\boxed{1}}{\boxed{13}}$$

Answer:

- To find the probability of drawing an Ace, divide the number of Aces by the total number of cards.
- There are 4 Aces in a standard deck of 52 playing cards.
- $P(\text{"drawing an Ace"}) = \dfrac{\text{number of Aces}}{\text{total number of cards}} = \dfrac{4}{52} = \dfrac{1 \times \cancel{4}}{13 \times \cancel{4}} = \dfrac{1}{13}$

Ex 43: A six-sided die is rolled once. Determine the probability of obtaining an even number.

$$P(\text{"rolling an even number"}) = \frac{\boxed{1}}{\boxed{2}}$$

Answer:

- To find the probability of rolling an even number, divide the number of even numbers by the total number of sides.
- There are 3 even numbers on a six-sided die (2, 4, and 6).
- $P(\text{rolling an even number}) = \dfrac{\text{number of even numbers}}{\text{total number of sides}} = \dfrac{3}{6} = \dfrac{1 \times \cancel{3}}{2 \times \cancel{3}} = \dfrac{1}{2}$

MCQ 44: A fruit is selected randomly from a basket containing 3 apples, 2 oranges, and 5 bananas.
Find the probability that the selected fruit is an orange (simplify the fraction).

$$P(\text{"selecting an orange"}) = \frac{\boxed{1}}{\boxed{5}}$$

Answer:

- To find the probability of selecting an orange, divide the number of oranges by the total number of fruits.
- There are 2 oranges out of 10 fruits in total.
- $$\begin{aligned} P(\text{"selecting an orange"}) &= \frac{\text{number of oranges}}{\text{total number of fruits}} \\ &= \frac{2}{10} \\ &= \frac{1 \times \cancel{2}}{5 \times \cancel{2}} \\ &= \frac{1}{5} \end{aligned}$$

B.2.2 DETERMINING THE PROBABILITY IN MULTI-STEP RANDOM EXPERIMENTS

Ex 45: A coach selects two players at random from a group of four players, labeled A, B, C, and D, without replacement (once a player is chosen, they are not available for the next selection). The table below shows all possible outcomes for selecting Player 1 and Player 2, where an "X" indicates an impossible outcome due to the same player being selected twice.

Player 1 \ Player 2	A	B	C	D
A	**X**	AB	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Calculate the probability that player C is selected as either Player 1 or Player 2, simplifying the fraction.

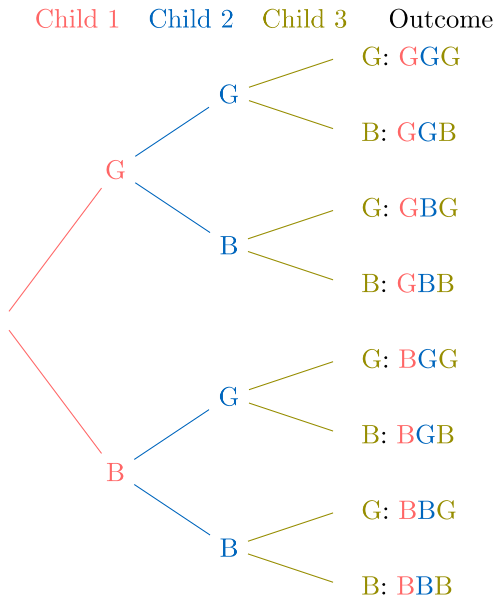
$$P(\text{"selecting player C"}) = \frac{\boxed{1}}{\boxed{2}}$$

Answer:

- The sample space consists of all possible pairs of players: AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC. This totals 12 outcomes.
- The event that player C is selected includes pairs where C is either Player 1 or Player 2: CA, CB, CD, AC, BC, DC. This totals 6 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"C is selected"}) &= \frac{\text{number of outcomes where C is selected}}{\text{number of possible outcomes}} \\ &= \frac{6}{12} \\ &= \frac{1 \times 6}{2 \times 6} \\ &= \frac{1}{2} \end{aligned}$$

Ex 46: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



Calculate the probability that the family has at least two girls, simplifying the fraction.

$$P(\text{"at least two girls"}) = \frac{\boxed{1}}{\boxed{2}}$$

Answer:

- The sample space consists of all possible gender outcomes for three children: BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG. This totals 8 outcomes.
- The event that the family has at least two girls includes outcomes with two or three girls: BGG, GBG, GGB, GGG. This totals 4 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"at least 2 girls"}) &= \frac{\text{Number of outcomes with at least 2 gir}}{\text{Total number of outcomes}} \\ &= \frac{4}{8} \\ &= \frac{1 \times 4}{2 \times 4} \\ &= \frac{1}{2} \end{aligned}$$

Ex 47: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

red die \ blue die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is exactly 7, simplifying the fraction.

$$P(\text{"sum is 7"}) = \boxed{\frac{1}{6}}$$

Answer:

- The sample space consists of all possible pairs of outcomes from rolling two six-sided dice: 36 outcomes (6 outcomes for Die 1 times 6 outcomes for Die 2).
- The event that the sum is exactly 7 includes pairs: 16 ($1+6=7$), 25 ($2+5=7$), 34 ($3+4=7$), 43 ($4+3=7$), 52 ($5+2=7$), 61 ($6+1=7$). This totals 6 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"sum is 7"}) &= \frac{\text{Number of outcomes with sum 7}}{\text{Total number of outcomes}} \\ &= \frac{6}{36} \\ &= \frac{1\times 6}{6\times 6} \\ &= \frac{1}{6} \end{aligned}$$

Ex 48: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is greater than or equal to 11, simplifying the fraction.

$$P(\text{"sum } \geq 11\text{"}) = \boxed{\frac{1}{12}}$$

Answer:

- The sample space consists of all possible pairs of outcomes from rolling two six-sided dice: 36 outcomes (6 outcomes for the red die times 6 outcomes for the blue die).
- The event that the sum is greater than or equal to 11 includes pairs: 56 ($5+6=11$), 65 ($6+5=11$), 66 ($6+6=12$). This totals 3 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"sum } \geq 11\text{"}) &= \frac{\text{Number of outcomes with sum } \geq 11}{\text{Total number of outcomes}} \\ &= \frac{3}{36} \\ &= \frac{1\times 3}{12\times 3} \\ &= \frac{1}{12} \end{aligned}$$

Ex 49: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is exactly 6 or 8, simplifying the fraction.

$$P(\text{"sum is 6 or 8"}) = \boxed{\frac{5}{18}}$$

Answer:

- The sample space consists of all possible pairs of outcomes from rolling two six-sided dice: 36 outcomes (6 outcomes for the red die times 6 outcomes for the blue die).
- The event that the sum is exactly 6 or 8 includes pairs: 15, 24, 33, 42, 51 (sum 6); 26, 35, 44, 53, 62 (sum 8). This totals 10 outcomes (5 for 6, 5 for 8).
- The probability is calculated as:

$$\begin{aligned} P(\text{"sum is 6 or 8"}) &= \frac{\text{Number of outcomes with sum 6 or 8}}{\text{Total number of outcomes}} \\ &= \frac{10}{36} \\ &= \frac{5\times 2}{18\times 2} \\ &= \frac{5}{18} \end{aligned}$$

B.3 COMPLEMENT RULE

B.3.1 APPLYING THE COMPLEMENT RULE

Ex 50: I toss a fair coin. The probability of getting heads is $\frac{1}{2}$. Find the probability of getting tails.

$$P(\text{"Getting tails"}) = \boxed{\frac{1}{2}}$$

Answer:

- The probability of getting heads is $\frac{1}{2}$.
- The event "Getting tails" is the complement of "Getting heads."
- Using the complement rule:

$$\begin{aligned} P(\text{"Getting tails"}) &= 1 - P(\text{"Getting heads"}) \\ &= 1 - \frac{1}{2} \\ &= \frac{2}{2} - \frac{1}{2} \\ &= \frac{1}{2} \end{aligned}$$

- So, the probability of getting tails is $\frac{1}{2} = 50\%$.

Ex 51: A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.
Find the probability that a student does not laugh at the joke.

$$P(\text{"Not laughing"}) = \boxed{30}\%$$

Answer:

- The probability that a student laughs at the joke is 70%.
- The event "Not laughing" is the complement event of "Laughing."
- Using the complement rule:

$$\begin{aligned} P(\text{"Not laughing"}) &= 1 - P(\text{"Laughing"}) \\ &= 1 - 70\% \\ &= 100\% - 70\% \\ &= 30\% \end{aligned}$$

- Therefore, the probability that a student does not laugh at the joke is 30%.

Ex 52: I randomly select a student in the class. The probability that a girl is selected is $\frac{9}{10}$.
Find the probability that a boy is selected.

$$P(\text{"Selecting a boy"}) = \frac{\boxed{1}}{\boxed{10}}$$

Answer:

- The probability that a girl is selected is $\frac{9}{10}$.
- The event "Selecting a boy" is the complement of "Selecting a girl."
- Using the complement rule:

$$\begin{aligned} P(\text{"Selecting a boy"}) &= 1 - P(\text{"Selecting a girl"}) \\ &= 1 - \frac{9}{10} \\ &= \frac{10}{10} - \frac{9}{10} \\ &= \frac{1}{10} \end{aligned}$$

- So, the probability that a boy is selected is $\frac{1}{10} = 10\%$.

Ex 53: The weather forecast predicts that there is a 70% chance of rain tomorrow.
Find the probability that it will not rain tomorrow.

$$P(\text{"No rain"}) = \boxed{30}\%$$

Answer:

- The probability that it will rain tomorrow is 70%.
- The event "No rain" is the complement of the event "Rain".
- Using the complement rule:

$$\begin{aligned} P(\text{"No rain"}) &= 1 - P(\text{"Rain"}) \\ &= 1 - 70\% \\ &= 100\% - 70\% \\ &= 30\% \end{aligned}$$

- Therefore, the probability that it will not rain tomorrow is 30%.

Ex 54: A survey shows that 70% of the students in a school love Math.
Find the probability that a randomly chosen student does not love Math.

$$P(\text{"Not loving Math"}) = \boxed{30}\%$$

Answer:

- The probability that a student loves Math is 70%.
- The event "Not loving Math" is the complement event of "Loving Math."
- Using the complement rule:

$$\begin{aligned} P(\text{"Not loving Math"}) &= 1 - P(\text{"Loving Math"}) \\ &= 1 - 70\% \\ &= 100\% - 70\% \\ &= 30\% \end{aligned}$$

- Therefore, the probability that a student does not love Math is 30%.

MCQ 55: A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.
Find the probability that a student does not laugh at the joke.

- ☒ $P(\text{"Not laughing"}) = 30\%$
- ☐ $P(\text{"Not laughing"}) = 70\%$
- ☐ $P(\text{"Not laughing"}) = 50\%$

Answer:

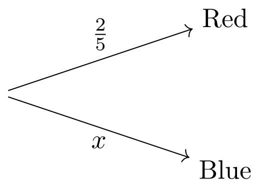
- The probability that a student laughs at the joke is 70%.
- The event "Not laughing" is the complement event of "Laughing".
- Using the complement rule:

$$\begin{aligned} P(\text{"Not laughing"}) &= 1 - P(\text{"Laughing"}) \\ &= 1 - 70\% \\ &= 100\% - 70\% \\ &= 30\% \end{aligned}$$

- So, the probability that a student does not laugh at the joke is 30%.

B.3.2 COMPLETING A PROBABILITY TREE DIAGRAM

Ex 56: From a bag containing red balls and blue balls, the probability of choosing a red ball is $\frac{2}{5}$. Find the probability x of choosing a blue ball.



$$x = \frac{\boxed{3}}{\boxed{5}}$$

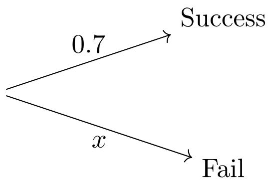
Answer:

- The probability of choosing a red ball from the bag is given as $\frac{2}{5}$.
- Since the only other option is choosing a blue ball, the probability of choosing a blue ball is calculated as follows:

$$\begin{aligned} P(\text{"Blue"}) &= 1 - P(\text{"Red"}) \\ &= 1 - \frac{2}{5} \\ &= \frac{5}{5} - \frac{2}{5} \\ &= \frac{3}{5} \end{aligned}$$

- So, the correct answer is $x = \frac{3}{5}$.

Ex 57: Jasper is playing basketball. The probability that he makes his first shot is 0.7. Find the probability x that he misses his first shot.



$$x = \boxed{0.3}$$

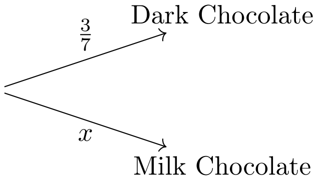
Answer: Let's figure this out step by step:

- The probability that Jasper makes his first shot is 0.7.
- Since the complement event is missing the shot, the probability x is calculated as follows:

$$\begin{aligned} P(\text{"Fail"}) &= 1 - P(\text{"Success"}) \\ &= 1 - 0.7 \\ &= 0.3 \end{aligned}$$

- So, the correct answer is $x = 0.3$.

Ex 58: In a box of assorted chocolates, the probability of picking a dark chocolate is $\frac{3}{7}$. Find the probability x of picking a milk chocolate.



$$x = \frac{\boxed{4}}{\boxed{7}}$$

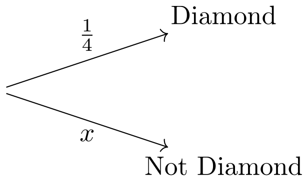
Answer:

- The probability of picking a dark chocolate from the box is $\frac{3}{7}$.
- Since the only other option is picking a milk chocolate, the probability is calculated as follows:

$$\begin{aligned} P(\text{"Milk chocolate"}) &= 1 - P(\text{"Dark chocolate"}) \\ &= 1 - \frac{3}{7} \\ &= \frac{7}{7} - \frac{3}{7} \\ &= \frac{4}{7} \end{aligned}$$

- So, the correct answer is $x = \frac{4}{7}$.

Ex 59: In a deck of cards, the probability of drawing a card from the suit of diamonds is $\frac{1}{4}$. Find the probability x of drawing a card that is not a diamond.



$$x = \frac{\boxed{3}}{\boxed{4}}$$

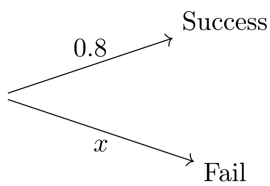
Answer:

- The probability of drawing a diamond from a deck of cards is $\frac{1}{4}$.
- Since the only other option is drawing a card that is not a diamond, the probability is calculated as follows:

$$\begin{aligned} P(\text{"Not diamond"}) &= 1 - P(\text{"Diamond"}) \\ &= 1 - \frac{1}{4} \\ &= \frac{4}{4} - \frac{1}{4} \\ &= \frac{3}{4} \end{aligned}$$

- So, the correct answer is $x = \frac{3}{4}$.

Ex 60: Emma is playing a video game. The probability that she completes a level is 0.8. Find the probability x that she fails to complete the level.



$$x = \boxed{0.2}$$

Answer: Let's figure this out step by step:

- The probability that Emma completes the level is 0.8.
- Since the complement event is failing to complete the level, the probability x is calculated as follows:

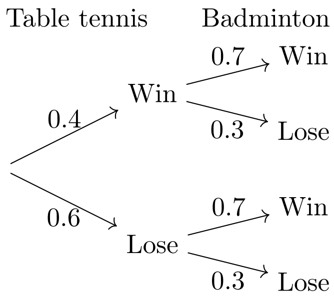
$$\begin{aligned} P(\text{"Fail"}) &= 1 - P(\text{"Success"}) \\ &= 1 - 0.8 \\ &= 0.2 \end{aligned}$$

- So, the correct answer is $x = 0.2$.

B.4 PROBABILITY OF INDEPENDENT EVENTS

B.4.1 READING PROBABILITY TREE

Ex 61: Niamh plays a game of table tennis on Saturday and a game of badminton on Sunday. The probability tree is represented:

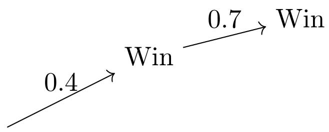


Calculate the probability that Niamh wins both games.

$$P(\text{"Win both"}) = \boxed{0.28}$$

Answer: Let's figure this out step by step:

- Since the table tennis game on Saturday and the badminton game on Sunday are independent events—winning one doesn't affect the other—we multiply the probabilities along the path where Niamh wins both.
- The probability of winning the table tennis game is 0.4 (from the first branch).
- The probability of winning the badminton game, given a win in table tennis, is 0.7 (from the second branch).
- Here's the relevant path on the tree diagram:

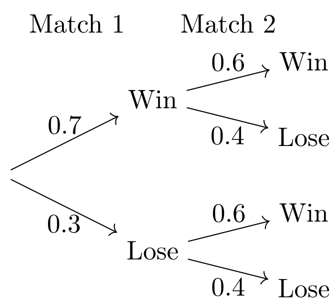


- Now, calculate the probability of winning both games:

$$\begin{aligned} P(\text{"Win both"}) &= P(\text{"Win table tennis"}) \times \\ &\quad P(\text{"Win badminton"}) \\ &= 0.4 \times 0.7 \\ &= 0.28 \end{aligned}$$

- So, the probability that Niamh wins both games is 0.28.

Ex 62: Sam is playing an online multiplayer game. The probability that Sam wins their first match is 0.7, and the probability that Sam wins their second match is 0.6.

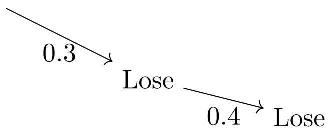


Calculate the probability that Sam loses both the first and second matches.

$$P(\text{"Both lose"}) = \boxed{0.12}$$

Answer: Let's figure this out step by step:

- Since losing the first match and losing the second match are independent events—one loss doesn't affect the other—we multiply their individual probabilities of losing to find the chance of both losing.
- The probability that Sam loses the first match is $1 - 0.7 = 0.3$.
- The probability that Sam loses the second match is $1 - 0.6 = 0.4$.
- Here's the relevant path on the tree diagram (following the



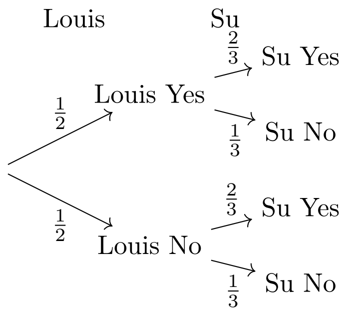
bottom branches):

- Now, calculate the probability that both lose:

$$\begin{aligned} P(\text{"Both lose"}) &= P(\text{"Lose first match"}) \times P(\text{"Lose second ma} \\ &= 0.3 \times 0.4 \\ &= 0.12 \end{aligned}$$

- So, the probability that Sam loses both the first and second matches is 0.12.

Ex 63: A party is happening this weekend! Louis might come with a probability of $\frac{1}{2}$, and Su might come with a probability of $\frac{2}{3}$.



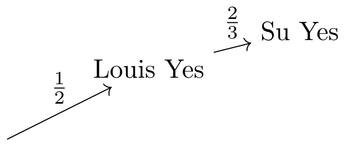
Calculate the probability that both Louis and Su come to the party (simplify the fraction).

$$P(\text{"Both come"}) = \frac{\boxed{1}}{\boxed{3}}$$

Answer: Let's figure this out step by step:

- Since Louis coming to the party and Su coming to the party are independent events—Louis's decision doesn't affect Su's—we multiply their individual probabilities to find the chance of both happening.
- The probability that Louis comes is $\frac{1}{2}$.
- The probability that Su comes is $\frac{2}{3}$.

- Here's the relevant path on the tree diagram:

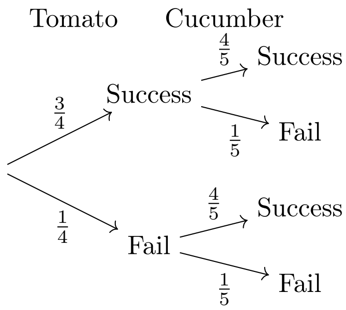


- Now, calculate the probability that both come:

$$\begin{aligned} P(\text{"Both come"}) &= P(\text{"Louis comes"}) \times P(\text{"Su comes"}) \\ &= \frac{1}{2} \times \frac{2}{3} \\ &= \frac{1 \times 2}{2 \times 3} \\ &= \frac{2}{6} \\ &= \frac{1}{3} \end{aligned}$$

- The fraction $\frac{2}{6}$ simplifies to $\frac{1}{3}$ by dividing both numerator and denominator by 2.
- So, the probability that both Louis and Su come to the party is $\frac{1}{3}$.

Ex 64: Mia takes care of her garden. The probability that her tomato plants grow is $\frac{3}{4}$, and the probability that her cucumber plants grow is $\frac{4}{5}$.

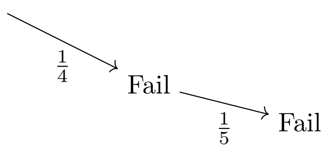


Calculate the probability that both the tomato plants and the cucumber plants fail to grow (simplify the fraction).

$$P(\text{"Both fail"}) = \frac{\boxed{1}}{\boxed{20}}$$

Answer: Let's figure this out step by step:

- Since the failure of the tomato plants and the failure of the cucumber plants are independent events—one's failure doesn't affect the other—we multiply their individual probabilities of failing to find the chance of both failing.
- The probability that the tomato plants fail is $1 - \frac{3}{4} = \frac{1}{4}$.
- The probability that the cucumber plants fail is $1 - \frac{4}{5} = \frac{1}{5}$.
- Here's the relevant path on the tree diagram (following the bottom branches):



- Now, calculate the probability that both fail:

$$\begin{aligned} P(\text{"Both fail"}) &= P(\text{"Tomato fail"}) \times P(\text{"Cucumber fail"}) \\ &= \frac{1}{4} \times \frac{1}{5} \\ &= \frac{1 \times 1}{4 \times 5} \\ &= \frac{1}{20} \end{aligned}$$

- So, the probability that both the tomato plants and the cucumber plants fail to grow is $\frac{1}{20}$.

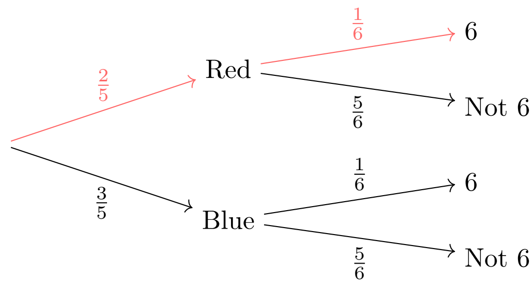
B.4.2 FINDING THE PROBABILITY WITH INDEPENDENT EVENTS

Ex 65: Imagine you're at a carnival playing a game. You pick a ball from a bag containing 2 red balls and 3 blue balls, then roll a fair six-sided die. Find the probability of choosing a red ball **and** rolling a 6 (simplify the fraction).

$$P(\text{"Red" and "6"}) = \frac{\boxed{1}}{\boxed{15}}$$

Answer: Let's figure this out step by step:

- Since picking a ball and rolling a die are independent events—one doesn't change the other—we multiply their individual probabilities to find the chance of both happening.
- The probability of choosing a red ball is $\frac{2}{5}$ because there are 2 red balls out of 5 total balls.
- The probability of rolling a 6 on a six-sided die is $\frac{1}{6}$ since there's only one 6 out of six possible outcomes.
- Here's the probability tree showing all possibilities:



- Now, calculate the probability of both events:

$$\begin{aligned} P(\text{"Red" and "6"}) &= P(\text{"Red"}) \times P(\text{"6"}) \\ &= \frac{2}{5} \times \frac{1}{6} \\ &= \frac{2}{30} \\ &= \frac{1 \times \cancel{2}}{15 \times \cancel{2}} \\ &= \frac{1}{15} \end{aligned}$$

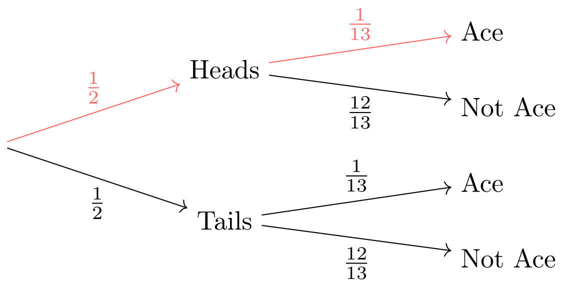
- So, the probability is $\frac{1}{15}$.

Ex 66: Imagine you're at a carnival playing another game. You flip a fair coin and draw a card from a standard deck of 52 playing cards. Find the probability of getting heads **and** drawing an Ace (simplify the fraction).

$$P(\text{"Heads" and "Ace"}) = \frac{\boxed{1}}{\boxed{26}}$$

Answer: Let's figure this out step by step:

- Since flipping a coin and drawing a card are independent events—one doesn't change the other—we multiply their individual probabilities to find the chance of both happening.
- The probability of getting heads is $\frac{1}{2}$ because a fair coin has two equally likely outcomes: heads or tails.
- The probability of drawing an Ace from a standard deck is $\frac{4}{52} = \frac{1}{13}$, since there are 4 Aces (one for each suit: hearts, diamonds, clubs, spades) in 52 cards.
- Here's the probability tree showing all possibilities:



- Now, calculate the probability of both events:

$$\begin{aligned} P(\text{"Heads" and "Ace"}) &= P(\text{"Heads"}) \times P(\text{"Ace"}) \\ &= \frac{1}{2} \times \frac{1}{13} \\ &= \frac{1}{26} \end{aligned}$$

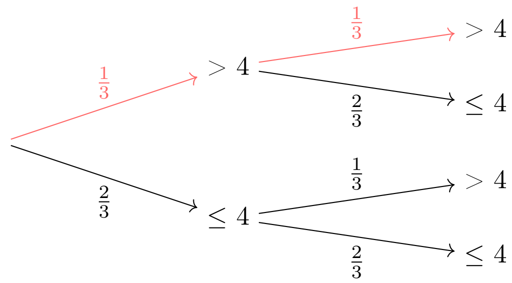
- The fraction $\frac{1}{26}$ is already in its simplest form, as 1 and 26 have no common factors other than 1.
- So, the probability is $\frac{1}{26}$.

Ex 67: Imagine you're at a carnival playing a dice game. You roll a fair six-sided die two times in a row. Find the probability of getting a number greater than 4 (a 5 or 6) on both rolls (simplify the fraction).

$$P(\text{"Number} > 4\text{" and "Number} > 4\text{"}) = \frac{\boxed{1}}{\boxed{9}}$$

Answer: Let's figure this out step by step:

- Since rolling the die the first time and rolling it the second time are independent events—each roll doesn't affect the other—we multiply their individual probabilities to find the chance of both happening.
- The probability of getting a number greater than 4 (a 5 or 6) on one roll is $\frac{2}{6} = \frac{1}{3}$, because there are 2 favorable outcomes (5 or 6) out of 6 possible outcomes (1, 2, 3, 4, 5, 6).
- Since it's the same die rolled twice, the probability is the same for the second roll: $\frac{1}{3}$.
- Here's the probability tree showing all possibilities:



- Now, calculate the probability of both events:

$$\begin{aligned} P(\text{"Nbr 1} > 4\text{" and "Nbr 2} > 4\text{"}) &= P(\text{"Nbr 1} > 4\text{"}) \times P(\text{"Nb} \\ &= \frac{1}{3} \times \frac{1}{3} \\ &= \frac{1}{9} \end{aligned}$$

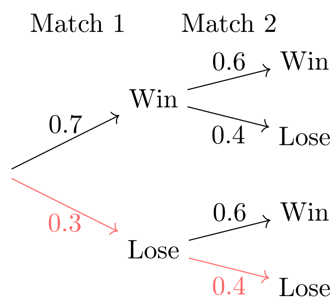
- The fraction $\frac{1}{9}$ is already in its simplest form, as 1 and 9 have no common factors other than 1.
- So, the probability is $\frac{1}{9}$.

Ex 68: Sam is playing an online multiplayer game. The probability that Sam wins their first match is 0.7, and the probability that Sam wins their second match is 0.6.
Calculate the probability that Sam loses both the first and second matches.

$$P(\text{"Both lose"}) = \boxed{0.12}$$

Answer: Let's figure this out step by step:

- Since losing the first match and losing the second match are independent events—one loss doesn't affect the other—we multiply their individual probabilities of losing to find the chance of both losing.
- The probability that Sam loses the first match is $1 - 0.7 = 0.3$.
- The probability that Sam loses the second match is $1 - 0.6 = 0.4$.
- The probability tree is:



- Now, calculate the probability that both lose:

$$\begin{aligned} P(\text{"Both lose"}) &= P(\text{"Lose first match"}) \times P(\text{"Lose second ma} \\ &= 0.3 \times 0.4 \\ &= 0.12 \end{aligned}$$

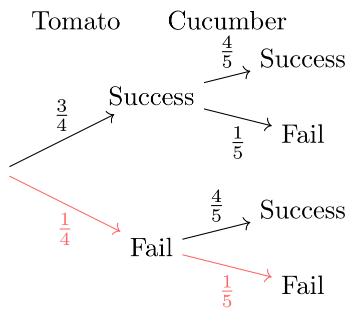
- So, the probability that Sam loses both the first and second matches is 0.12.

Ex 69: Mia takes care of her garden. The probability that her tomato plants grow is $\frac{3}{4}$, and the probability that her cucumber plants grow is $\frac{4}{5}$.
Calculate the probability that both the tomato plants and the cucumber plants fail to grow (simplify the fraction).

$$P(\text{"Both fail"}) = \frac{\boxed{1}}{\boxed{20}}$$

Answer: Let's figure this out step by step:

- Since the failure of the tomato plants and the failure of the cucumber plants are independent events—one's failure doesn't affect the other—we multiply their individual probabilities of failing to find the chance of both failing.
- The probability that the tomato plants fail is $1 - \frac{3}{4} = \frac{1}{4}$.
- The probability that the cucumber plants fail is $1 - \frac{4}{5} = \frac{1}{5}$.
- The probability tree is



- Now, calculate the probability that both fail:

$$\begin{aligned} P(\text{"Both fail"}) &= P(\text{"Tomato fail"}) \times P(\text{"Cucumber fail"}) \\ &= \frac{1}{4} \times \frac{1}{5} \\ &= \frac{1 \times 1}{4 \times 5} \\ &= \frac{1}{20} \end{aligned}$$

- So, the probability that both the tomato plants and the cucumber plants fail to grow is $\frac{1}{20}$.

B.5 EXPERIMENTAL PROBABILITY

B.5.1 SOLVING REAL-WORLD PROBLEMS

Ex 70: During a week of basketball practice, Mia made 45 out of 60 free-throw attempts. Estimate the experimental probability that Mia will make her next free-throw attempt.

$$P(\text{"Making the next attempt"}) \approx \boxed{75}\,\%$$

Answer:

- The experimental probability of Mia making a free-throw is the ratio of successful free-throws to the total number of attempts.
- $$\begin{aligned} P(\text{"Making the next attempt"}) &\approx \frac{\text{Number of successes}}{\text{Total attempts}} \\ &\approx \frac{45}{60} \\ &\approx 0.75 \\ &\approx 75\% \end{aligned}$$
- So, the estimated probability that Mia will make her next free-throw attempt is 75%.

Ex 71: During a week, the school cafeteria recorded that out of 150 students, 120 chose a vegetarian meal. Estimate the probability that the next student will choose a vegetarian meal based on this experimental probability.

$$P(\text{"Choosing a Vegetarian meal"}) \approx \boxed{80}\,\%$$

Answer:

- The probability $P(\text{"Vegetarian meal"})$ is estimated by the ratio of times a vegetarian meal was chosen to the total number of trials.
- $$\begin{aligned} P(\text{"Vegetarian meal"}) &\approx \frac{\text{Number of vegetarian meals}}{\text{Total number of students}} \\ &\approx \frac{120}{150} \\ &\approx 0.8 \\ &\approx 80\% \end{aligned}$$
- So, the estimated probability that the next student will choose a vegetarian meal is 80%.

Ex 72: Over the course of a year, it rained on 120 days out of 300 recorded days. Estimate the experimental probability that it will rain.

$$P(\text{"Raining"}) \approx \boxed{40}\,\%$$

Answer:

- The experimental probability of raining is the ratio of rainy days to the total number of recorded days.
- $$\begin{aligned} P(\text{"Raining"}) &\approx \frac{\text{Number of rainy days}}{\text{Total recorded days}} \\ &\approx \frac{120}{300} \\ &\approx 0.4 \\ &\approx 40\% \end{aligned}$$
- So, the estimated probability that it will rain is 40%.

Ex 73: A local bakery found that out of 200 customers, 150 ordered a croissant. Estimate the experimental probability that the next customer will order a croissant.

$$P(\text{"Ordering a croissant"}) \approx \boxed{75}\,\%$$

Answer:

- The experimental probability of a customer ordering a croissant is the ratio of customers who ordered a croissant to the total number of customers.

- $$\begin{aligned} P(\text{"Ordering a croissant"}) &\approx \frac{\text{Number of customers croissant}}{\text{Total number of customers}} \\ &\approx \frac{150}{200} \\ &\approx 0.75 \\ &\approx 75\% \end{aligned}$$

- So, the estimated probability that the next customer will order a croissant is 75%.