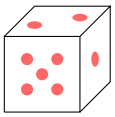
PROBABILITY

A ALGEBRA OF EVENTS

A.1 SAMPLE SPACE

A.1.1 FINDING THE SAMPLE SPACES

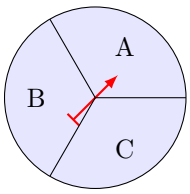
MCQ 1: A fair six-sided die is rolled once.



Find the sample space.

- ☐ $\{1, 2, 3, 4, 5\}$
- ☐ $\{1, 2, 3, 4, 5, 6, 7\}$
- ☐ $\{1, 2, 3, 4, 5, 6\}$

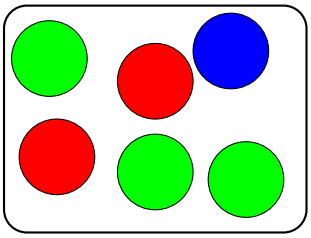
MCQ 2: Find the sample space that the spinner can land on:



- ☐ $\{A, B, C\}$
- ☐ $\{A, B\}$
- ☐ $\{A, C\}$

MCQ 3: A ball is chosen randomly from a bag containing 2 red balls, 1 blue ball, and 3 green balls.

Bag



Find the sample space.

- ☐ {Red, Blue, Green}
- ☐ {2 Red, 1 Blue, 3 Green}
- ☐ {Red, Red, Blue, Green, Green, Green}

MCQ 4: A letter is chosen randomly from the word BANANA. Find all possible outcomes for the chosen letter.

- ☐ {B, N, A}
- ☐ {B, A, N, A, N, A}
- ☐ {A, B, N, A, B, N}

A.2 EVENTS

A.2.1 FINDING THE EVENTS

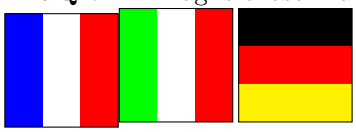
MCQ 5: A letter is chosen randomly from the word ORANGE. Find the event where the chosen letter is a vowel.

- ☐ {O, R, A, N, G, E}
- ☐ {O, A, E}
- ☐ {R, G, N}
- ☐ {A, G, E}

MCQ 6: A fair six-sided dice is rolled once. Find the event where the outcome is an even number.

- ☐ {1, 3, 5}
- ☐ {2, 4, 6 }
- ☐ {1, 2, 3, 4, 5, 6}
- ☐ {2, 3, 4, 5}

MCQ 7: A flag is chosen randomly from:

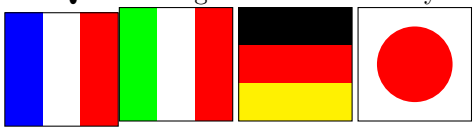


France Italy Germany

Find the event where the outcome is a flag with blue in them.

- ☐ {France }
- ☐ {Italy, France}
- ☐ {Italy, France, Germany}

MCQ 8: A flag is chosen randomly from:



France Italy Germany Japan

Find the event where the outcome is a flag with red in them.

- ☐ {France, Japan}
- ☐ {Italy, France}
- ☐ {Italy, France, Germany, Japan}

MCQ 9: A flag is chosen randomly from:

France Italy Germany Nigeria

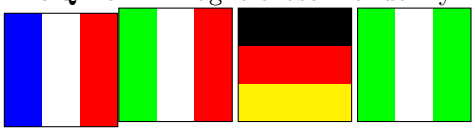
Find the event where the outcome is a flag with green in them.

- ☐ {France, Nigeria}
- ☐ {Italy, Nigeria}
- ☐ {Italy, France, Germany}

A.3 COMPLEMENTARY EVENT

A.3.1 FINDING THE COMPLEMENTARY EVENTS

MCQ 10: A flag is chosen randomly from the following:

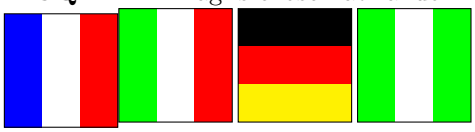


France Italy Germany Nigeria

Let E be the event where the selected flag contains green.
Find the complement of event E, denoted as E'.

- ☐ $E' = \{\text{France, Germany}\}$
- ☐ $E' = \{\text{Italy, Nigeria}\}$
- ☐ $E' = \{\text{Italy, France, Germany}\}$

MCQ 11: A flag is chosen at random from the following set:



France Italy Germany Nigeria

Let E be the event where the chosen flag contains the color red.
Find the complement of event E, denoted E'.

- ☐ $E' = \{\text{France, Germany}\}$
- ☐ $E' = \{\text{Nigeria}\}$
- ☐ $E' = \{\text{Italy, France, Germany}\}$

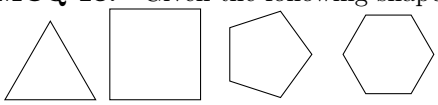
MCQ 12: A child's name is chosen randomly from the following list:

- Emily (girl's name)
- James (boy's name)
- Ava (girl's name)
- Sophia (girl's name)

Let E be the event where the selected name is a boy's name.
Find the complement of event E, denoted as E'.

- ☐ $E' = \{\text{Emily, Ava, Sophia}\}$
- ☐ $E' = \{\text{James}\}$
- ☐ $E' = \{\text{James, Ava}\}$

MCQ 13: Given the following shapes:

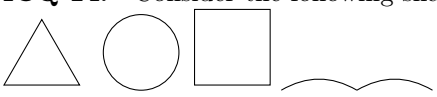


Triangle Square Pentagon Hexagon

Let E be the event where a polygon with an even number of sides is chosen.
Find the complement of event E, denoted as E'.

- ☐ $E' = \{\text{Square, Hexagon}\}$
- ☐ $E' = \{\text{Triangle, Pentagon}\}$
- ☐ $E' = \{\text{Triangle, Square, Pentagon, Hexagon}\}$

MCQ 14: Consider the following shapes:



Triangle Circle Square Curve

Let E be the event where the shape is a polygon.
Find the complement of event E, denoted as E'.

- ☐ $E' = \{\text{Triangle, Square}\}$
- ☐ $E' = \{\text{Triangle, Circle, Square, Curve}\}$
- ☐ $E' = \{\text{Circle, Curve}\}$

A.4 MULTI-STEP RANDOM EXPERIMENTS

A.4.1 FINDING OUTCOME IN A TABLE

MCQ 15: The table below shows the possible outcomes for the sexes of two children, first and second, where each can be a Boy (B) or a Girl (G).

second child / first child	B	G
B	BB	?
G	GB	GG

Find the missing outcome.

- ☐ BB
- ☐ BG
- ☐ GB

MCQ 16: The table below shows the possible outcomes when selecting two letters at random from the word "MAT" **with replacement** (after choosing a letter, it is put back before the next selection).

letter 2 / letter 1	M	A	T
M	MM	MA	MT
A	AM	AA	AT
T	TM	?	TT

Find the missing outcome.

- ☐ TT
- ☐ TA
- ☐ AT

MCQ 17: The table below shows the possible outcomes when selecting two letters at random from the word "CODE" **with replacement** (after choosing a letter, it is put back before the next selection).

letter 2 / letter 1	C	O	D	E
C	CC	CO	CD	CE
O	OC	OO	OD	OE
D	DC	?	DD	DE
E	EC	EO	ED	EE

Find the missing outcome.

- ☐ DO
- ☐ OD
- ☐ DC

MCQ 18: The table below shows the possible outcomes when selecting two letters at random from the word "NODE" **without replacement** (after choosing a letter, it is not put back before the next selection). An "X" means no outcome is possible.

letter 1 \ letter 2	N	O	D	E
N	**X**	?	ND	NE
O	ON	**X**	OD	OE
D	DN	DO	**X**	DE
E	EN	EO	ED	**X**

Find the missing outcome.

- ☐ NN
- ☐ NO
- ☐ ON

MCQ 19: The table below shows the possible outcomes when a coach selects two players at random from four players (A, B, C, D) **without replacement** (after choosing a player, they are not put back before the next selection). An "X" means no outcome is possible.

Player 1 \ Player 2	A	B	C	D
A	**X**	?	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Find the missing outcome.

- ☐ AB
- ☐ BA
- ☐ CA

A.4.2 COUNTING THE NUMBER OF POSSIBLE OUTCOMES IN A TABLE

Ex 20: The table below shows the possible outcomes for the sexes of two children, first and second, where each can be a Boy (B) or a Girl (G).

first child \ second child	B	G
B	BB	BG
G	GB	GG

Count the number of possible outcomes.

☐ possibles outcomes.

Ex 21: There are four players: A, B, C, and D. For position 1, only players A and B are eligible. For position 2, only players C and D are eligible. The table below shows the possible selections for the two positions.

position 1 \ position 2	C	D
A	AC	AD
B	BC	BD

Count the number of possible outcomes.

☐ possible outcomes.

Ex 22: There are four players: A, B, C, and D. A coach selects two players at random without replacement. The table below shows the possible selections for the two positions. An "X" means no outcome is possible.

Player 1 \ Player 2	A	B	C	D
A	**X**	AB	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Count the number of possible outcomes.

☐ possible outcomes.

Ex 23: There are three students: X, Y, and Z. A teacher selects one student each day, on Monday and Tuesday, to recite a poem. The selection is made without replacement, meaning the same student cannot be chosen both days. The table below shows the possible selections for the two days.

Monday \ Tuesday	X	Y	Z
X	**X**	XY	XZ
Y	YX	**X**	YZ
Z	ZX	ZY	**X**

Count the number of possible outcomes.

☐ possible outcomes.

A.4.3 COUNTING THE NUMBER OF POSSIBLE OUTCOMES FOR AN EVENT

Ex 24: There are four players: A, B, C, and D. A coach selects two players at random without replacement. The table below shows the possible selections for the two positions. An "X" means no outcome is possible.

Player 1 \ Player 2	A	B	C	D
A	**X**	AB	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Count the number of outcomes for the event that player A is selected.

☐ outcomes.

Ex 25: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "double digit" (both dice show the same number).

☐ outcomes.

Ex 26: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "at least one 6" (at least one die shows a 6).

☐ outcomes.

Ex 27: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "the sum of the dice is equal to 11."

☐ outcomes.

Ex 28: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "the sum of the dice is equal to 7."

☐ outcomes.

Ex 29: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

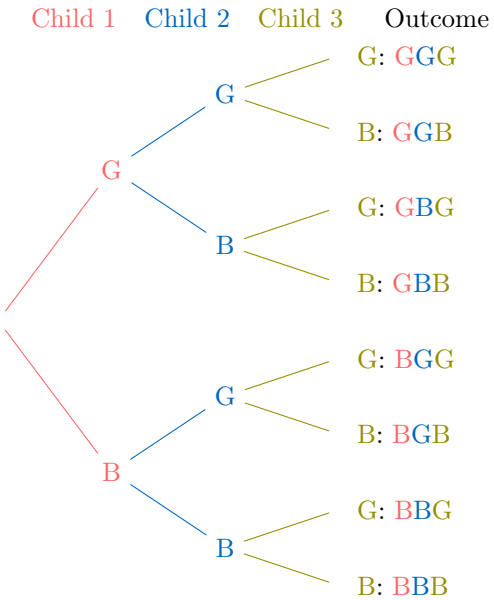
blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "a result inferior or equal to 3" (the sum of the dice is less than or equal to 3).

☐ outcomes.

A.4.4 COUNTING THE NUMBER OF POSSIBLE OUTCOMES IN AN TREE DIAGRAM

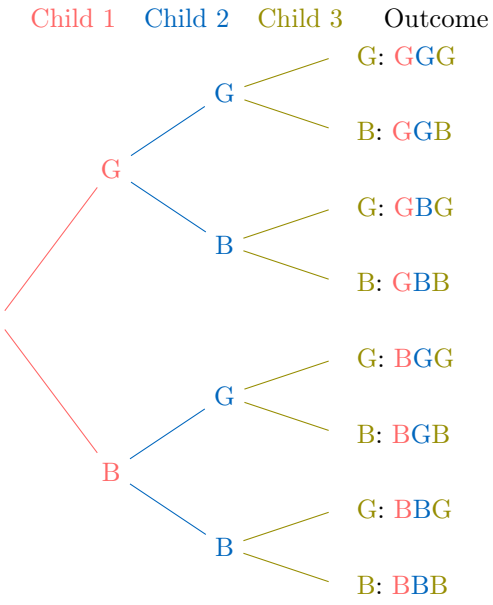
Ex 30: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



Count the number of possible outcomes for the event where the first child is a boy.

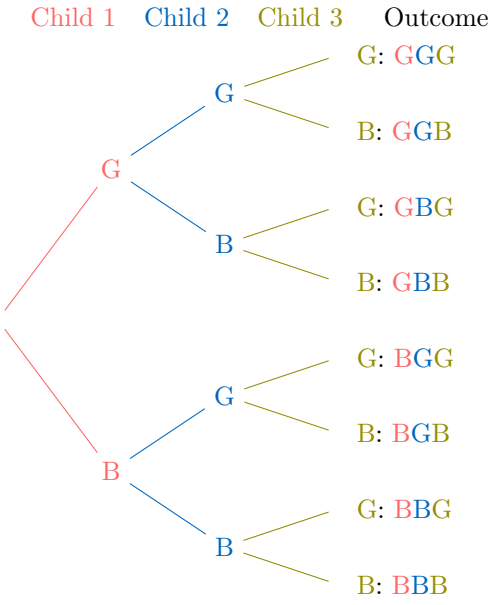
☐

Ex 31: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



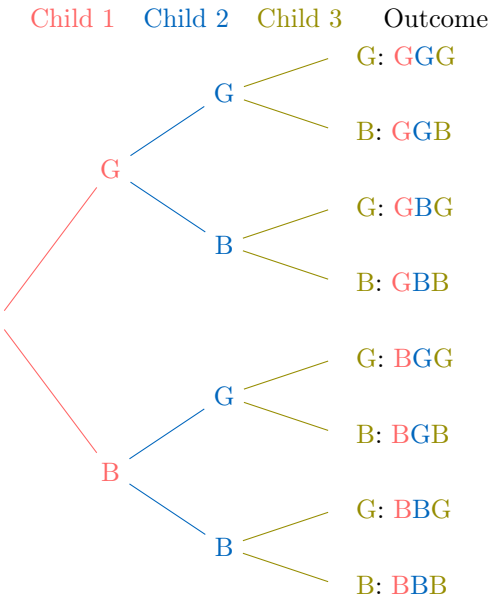
Count the number of possible outcomes for the event where there are exactly two girls.

Ex 32: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below shows all 8 possible gender outcomes for the three children.



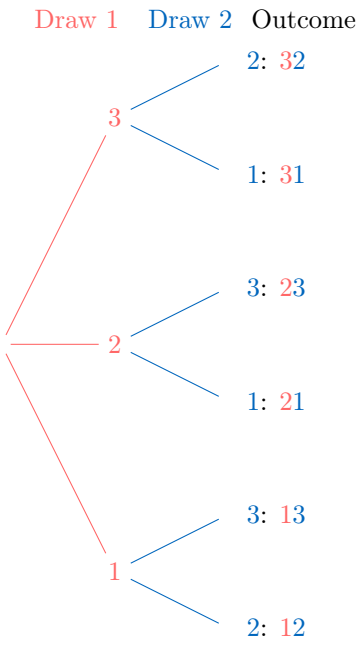
Count the number of possible outcomes for the event where there are at least two girls.

Ex 33: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



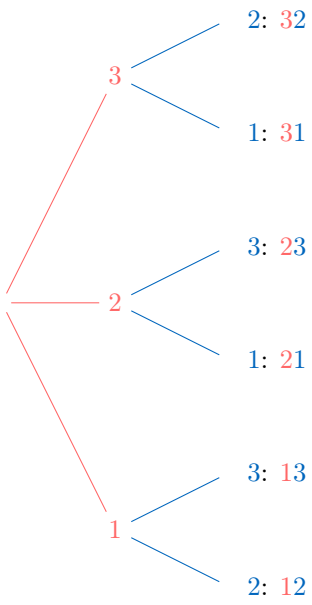
Count the number of possible outcomes for the event where the family has mixed-sex children (at least one boy and one girl).

Ex 34: Tickets numbered 1, 2, and 3 are placed in a bag. One ticket is drawn and set aside, then a second ticket is drawn. The tree diagram below shows all possible outcomes for these two selections.



Count the number of possible outcomes in the sample space for the two ticket selections.

Ex 35: Tickets numbered 1, 2, and 3 are placed in a bag. One ticket is drawn and set aside, then a second ticket is drawn. The tree diagram below displays all possible outcomes for these two selections.



Count the number of possible outcomes for the event where the ticket numbered 1 is drawn (either as the first or second ticket).

A.5 E OR F

A.5.1 FINDING THE UNION OF TWO EVENTS

MCQ 36: Let $E = \{A, C, D\}$ and $F = \{E, F, D\}$. Which option below correctly represents the event E or F?

Choose one true answer:

- ☐ E or $F = \{D\}$
- ☐ E or $F = \{A, C, D, E, F, H\}$
- ☐ E or $F = \{A, C, D, E, F\}$
- ☐ E or $F = \{A, C, E, F\}$

MCQ 37: A standard deck of 52 playing cards is used. Let event A be the event of drawing a red card, and let event B be the event of drawing a face card (Jack, Queen, or King). Which option below correctly represents the event A or B?

Choose one true answer:

- ☐ A or $B = \{$all red cards and all face cards$\}$
- ☐ A or $B = \{$all red face cards$\}$
- ☐ A or $B = \{$all red cards and all black face cards$\}$
- ☐ A or $B = \{$all cards$\}$

MCQ 38: A weather forecast for a week lists possible conditions each day. Let event $W = \{$Monday, Tuesday, Wednesday$\}$ be the event of it being windy, and let event $R = \{$Wednesday, Thursday, Friday$\}$ be the event of it raining. Which option below correctly represents the event W or R?

Choose one true answer:

- ☐ W or $R = \{$Monday$\}$
- ☐ W or $R = \{$Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday$\}$
- ☐ W or $R = \{$Tuesday, Wednesday$\}$
- ☐ W or $R = \{$Monday, Tuesday, Wednesday, Thursday, Friday$\}$

MCQ 39: A fruit basket contains various fruits. Let event $P = \{$cherry, peach, plum$\}$ be the event of selecting a pitted fruit, and let event $S = \{$mango, orange, peach$\}$ be the event of selecting a sweet fruit. Which option below correctly represents the event P or S?

Choose one true answer:

- ☐ P or $S = \{$cherry, peach, plum, mango, orange$\}$
- ☐ P or $S = \{$cherry, peach, plum$\}$
- ☐ P or $S = \{$mango, orange$\}$
- ☐ P or $S = \{$cherry, mango$\}$

A.5.2 FINDING THE UNION OF TWO EVENTS

MCQ 40: Consider the roll of a standard six-sided die. Let event E be the event of rolling an even number, and let event F be the event of rolling a number less than 4. Which option below correctly represents the event E or F?

Choose one true answer:

- ☐ E or $F = \{2\}$
- ☐ E or $F = \{1, 2, 3, 4, 6\}$
- ☐ E or $F = \{1, 2, 3, 4, 5, 6\}$
- ☐ E or $F = \{1, 2, 3\}$

MCQ 41: Consider the roll of a standard six-sided die. Let event G be the event of rolling a number greater than 3, and let event H be the event of rolling a prime number. Which option below correctly represents the event G or H?

Choose one true answer:

- ☐ G or $H = \{2, 3, 4, 5, 6\}$
- ☐ G or $H = \{4, 5, 6\}$
- ☐ G or $H = \{2, 3, 5\}$
- ☐ G or $H = \{1, 2, 3, 4, 5, 6\}$

MCQ 42: Consider the roll of a standard six-sided die. Let event I be the event of rolling a number divisible by 3, and let event J be the event of rolling a number less than 5. Which option below correctly represents the event I or J?

Choose one true answer:

- ☐ I or $J = \{3, 6\}$
- ☐ I or $J = \{1, 2, 3, 4\}$
- ☐ I or $J = \{1, 2, 3, 4, 5, 6\}$
- ☐ I or $J = \{1, 2, 3, 4, 6\}$

MCQ 43: A family has three children. Let event A be the event of having at least two boys, and let event B be the event of having at least one girl. Which option below correctly represents the event A or B? (Use B for boy and G for girl.)

Choose one true answer:

- ☐ A or $B = \{$BBB$\}$
- ☐ A or $B = \{$BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG$\}$
- ☐ A or $B = \{$BBG, BGB, GBB, BGG, GBG, GGB$\}$
- ☐ A or $B = \{$GGG$\}$

A.5.3 FINDING THE UNION OF TWO EVENTS FROM A TABLE

MCQ 44: In a classroom, students are listed in a table with their names, ages, and genders. Consider a random selection of one student from this table:

Tableau : Élèves

Nom	Âge	Genre
A	15	Féminin
B	17	Masculin
C	16	Féminin
D	15	Masculin
E	14	Féminin
F	17	Féminin

Let event A be the event of selecting a girl, and event B be the event of selecting a student older or equal than 17 years. Which option below correctly represents the event A or B?

Choose one true answer:

- ☐ A or $B = \{\text{A, B, C, D, E, F}\}$
- ☐ A or $B = \{\text{C, F}\}$
- ☐ A or $B = \{\text{A, B, C, D, F}\}$
- ☐ A or $B = \{\text{A, B, C, E, F}\}$

MCQ 45: In a music class, students are listed in a table with their names, ages, and preferred instruments. Consider a random selection of one student from this table:

Table: Students

Name	Age	Instrument
G	14	Violin
H	15	Piano
I	17	Guitar
J	16	Drums
K	15	Flute
L	18	Violin

Let event X be the event of selecting a student who plays the violin, and event Y be the event of selecting a student aged 16 or older. Which option below correctly represents the event X or Y?

Choose one true answer:

- ☐ X or $Y = \{\text{G, I, J, L}\}$
- ☐ X or $Y = \{\text{G, H, I, J, L}\}$
- ☐ X or $Y = \{\text{G, L}\}$
- ☐ X or $Y = \{\text{I, J, L}\}$

MCQ 46: In a sports team, players are listed in a table with their names, heights, and positions. Consider a random selection of one player from this team:

Table: Players

Name	Height (cm)	Position
M	180	Forward
N	170	Goalkeeper
O	185	Defender
P	175	Midfielder
Q	165	Forward
R	190	Defender

Let event U be the event of selecting a defender, and event V be the event of selecting a player taller than 180 cm. Which option below correctly represents the event U or V?

Choose one true answer:

- ☐ U or $V = \{\text{M, O, R}\}$
- ☐ U or $V = \{\text{O, R}\}$
- ☐ U or $V = \{\text{M, N, O, R}\}$
- ☐ U or $V = \{\text{M, O, R}\}$

A.6 E AND F

A.6.1 FINDING THE INTERSECTION OF TWO EVENTS

MCQ 47: Let $E = \{A, C, D\}$ and $F = \{E, F, D\}$. Which option below correctly represents the event E and F?

Choose one true answer:

- ☐ E and $F = \{D\}$
- ☐ E and $F = \{A, C, D, E, F\}$
- ☐ E and $F = \{A, C, D\}$
- ☐ E and $F = \{E, F, D\}$

MCQ 48: A standard deck of 52 playing cards is used. Let event A be the event of drawing a red card, and let event B be the event of drawing a face card (Jack, Queen, or King). Which option below correctly represents the event A and B?

Choose one true answer:

- ☐ A and $B = \{\text{all red cards}\}$
- ☐ A and $B = \{\text{all red face cards}\}$
- ☐ A and $B = \{\text{all face cards}\}$
- ☐ A and $B = \{\text{all cards}\}$

MCQ 49: A weather forecast for a week lists possible conditions each day. Let event $W = \{\text{Monday, Tuesday, Wednesday}\}$ be the event of it being windy, and let event $R = \{\text{Wednesday, Thursday, Friday}\}$ be the event of it raining. Which option below correctly represents the event W and R?

Choose one true answer:

- ☐ W and $R = \{\text{Monday}\}$
- ☐ W and $R = \{\text{Monday, Tuesday, Wednesday, Thursday, Friday}\}$
- ☐ W and $R = \{\text{Wednesday}\}$
- ☐ W and $R = \{\text{Tuesday, Wednesday}\}$

MCQ 50: A fruit basket contains various fruits. Let event $P = \{\text{cherry, peach, plum}\}$ be the event of selecting a pitted fruit, and let event $S = \{\text{mango, orange, peach}\}$ be the event of selecting a sweet fruit. Which option below correctly represents the event P and S?

Choose one true answer:

- ☐ P and $S = \{\text{cherry, peach, plum, mango, orange}\}$
- ☐ P and $S = \{\text{peach}\}$
- ☐ P and $S = \{\text{mango, orange}\}$
- ☐ P and $S = \{\text{cherry, mango}\}$

A.6.2 FINDING THE INTERSECTION OF TWO EVENTS

MCQ 51: Consider the roll of a standard six-sided die. Let event E be the event of rolling an even number, and let event F be the event of rolling a number less than 4. Which option below correctly represents the event E and F?

Choose one true answer:

- ☐ E and $F = \{2\}$
- ☐ E and $F = \{1, 2, 3, 4, 6\}$
- ☐ E and $F = \{1, 2, 3\}$
- ☐ E and $F = \{2, 4, 6\}$

MCQ 52: Consider the roll of a standard six-sided die. Let event G be the event of rolling a number greater than 3, and let event H be the event of rolling a prime number. Which option below correctly represents the event G and H?

Choose one true answer:

- ☐ G and $H = \{2, 3, 4, 5, 6\}$
- ☐ G and $H = \{4, 5, 6\}$
- ☐ G and $H = \{5\}$
- ☐ G and $H = \{2, 3, 5\}$

MCQ 53: Consider the roll of a standard six-sided die. Let event I be the event of rolling a number divisible by 3, and let event J be the event of rolling a number less than 5. Which option below correctly represents the event I and J?

Choose one true answer:

- ☐ I and $J = \{3\}$
- ☐ I and $J = \{1, 2, 3, 4\}$
- ☐ I and $J = \{3, 6\}$
- ☐ I and $J = \{1, 2, 3, 4, 6\}$

MCQ 54: A family has three children. Let event A be the event of having at least two boys, and let event B be the event of having at least one girl. Which option below correctly represents the event A and B? (Use B for boy and G for girl.)

Choose one true answer:

- ☐ A and $B = \{\text{BBB}\}$
- ☐ A and $B = \{\text{BBG}, \text{BGB}, \text{GBB}\}$
- ☐ A and $B = \{\text{BBG}, \text{BGB}, \text{GBB}, \text{BGG}, \text{GBG}, \text{GGB}, \text{GGG}\}$
- ☐ A and $B = \{\text{GGG}\}$

A.6.3 FINDING THE INTERSECTION OF TWO EVENTS FROM A TABLE

MCQ 55: In a classroom, students are listed in a table with their names, ages, and genders. Consider a random selection of one student from this table:

Tableau : Élèves

Nom	Âge	Genre
A	15	Féminin
B	17	Masculin
C	16	Féminin
D	15	Masculin
E	14	Féminin
F	17	Féminin

Let event A be the event of selecting a girl, and event B be the event of selecting a student older than or equal to 17 years. Which option below correctly represents the event A and B?

Choose one true answer:

- ☐ A and $B = \{\text{A, C, E, F}\}$
- ☐ A and $B = \{\text{B, F}\}$
- ☐ A and $B = \{\text{F}\}$
- ☐ A and $B = \{\text{A, C, F}\}$

MCQ 56: In a music class, students are listed in a table with their names, ages, and preferred instruments. Consider a random selection of one student from this table:

Table: Students

Name	Age	Instrument
G	14	Violin
H	15	Piano
I	17	Guitar
J	16	Drums
K	15	Flute
L	18	Violin

Let event X be the event of selecting a student who plays the violin, and event Y be the event of selecting a student aged 16 or older. Which option below correctly represents the event X and Y?

Choose one true answer:

- ☐ X and $Y = \{\text{G, I, J, L}\}$
- ☐ X and $Y = \{\text{L}\}$
- ☐ X and $Y = \{\text{G, L}\}$
- ☐ X and $Y = \{\text{I, J, L}\}$

MCQ 57: In a sports team, players are listed in a table with their names, heights, and positions. Consider a random selection of one player from this team:

Table: Players

Name	Height (cm)	Position
M	180	Forward
N	170	Goalkeeper
O	185	Defender
P	175	Midfielder
Q	165	Forward
R	190	Defender

Let event U be the event of selecting a defender, and event V be the event of selecting a player taller than 180 cm. Which option below correctly represents the event U and V?

Choose one true answer:

- ☐ U and $V = \{\text{O, R}\}$
- ☐ U and $V = \{\text{M, O, R}\}$
- ☐ U and $V = \{\text{R}\}$
- ☐ U and $V = \{\text{M, N, O, R}\}$

A.7 MUTUALLY EXCLUSIVE

A.7.1 DETERMINING MUTUAL EXCLUSIVITY

MCQ 58: Consider rolling a standard six-sided die (numbered 1 to 6). Two events are defined as follows:

- Event E: Rolling an even number.
- Event F: Rolling an odd number.

Are the events E and F mutually exclusive?

Choose one true answer:

☐ Yes, the events are mutually exclusive.

☐ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

MCQ 59: Consider rolling a standard six-sided die (numbered 1 to 6). Two events are defined as follows:

- Event E: Rolling a prime number.
- Event F: Rolling an even number.

Are the events E and F mutually exclusive?

Choose one true answer:

☐ Yes, the events are mutually exclusive.

☐ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

MCQ 60: Consider a standard deck of 52 playing cards (no jokers). Two events are defined as follows:

- Event E: Drawing a Queen.
- Event F: Drawing a Heart.

Are the events E and F mutually exclusive?

Choose one true answer:

☐ Yes, the events are mutually exclusive.

☐ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

MCQ 61: Consider a family with exactly two children, where each child is a boy (B) or a girl (G). Two events are defined as follows:

- Event E: The family has only boys.
- Event F: The family has only girls.

Are the events E and F mutually exclusive?

Choose one true answer:

☐ Yes, the events are mutually exclusive.

☐ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

A.7.2 DETERMINING MUTUAL EXCLUSIVITY FROM TABLES

MCQ 62: Consider a sports club where members are listed with their preferred sport and age group. A member is selected at random from the following table:

Table: Sports Club Members

Name	Preferred Sport	Age Group
S	Football	Under 16
T	Basketball	16-18
U	Tennis	Over 18
V	Swimming	Under 16
W	Football	16-18
X	Basketball	Over 18

Let event C be selecting a member whose preferred sport is Football, and event D be selecting a member from the Over 18 age group. Are events C and D mutually exclusive?

Choose one true answer:

☐ Yes, the events are mutually exclusive.

☐ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

MCQ 63: Consider a bakery where items are listed with their type and topping. An item is selected at random from the following table:

Table: Bakery Items

Item	Type	Topping
Muffin	Pastry	Chocolate
Cookie	Cookie	Sprinkles
Cake	Cake	Chocolate
Donut	Pastry	Glaze
Brownie	Cake	None
Croissant	Pastry	None

Let event T be selecting a Pastry, and event U be selecting an item with Chocolate topping. Are events T and U mutually exclusive?

Choose one true answer:

☐ Yes, the events are mutually exclusive.

☐ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

MCQ 64: Consider a library where books are listed with their genre and checkout status. A book is selected at random from the following table:

Table: Library Books

Title	Genre	Status
A	Mystery	Checked Out
B	Fantasy	Available
C	Mystery	Available
D	Romance	Checked Out
E	Fantasy	Checked Out
F	Romance	Available

Let event P be selecting a Mystery book, and event Q be selecting a Checked Out book. Are events P and Q mutually exclusive?

Choose one true answer:

☐ Yes, the events are mutually exclusive.

☐ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

MCQ 65: Consider a zoo where animals are listed with their type and habitat. An animal is selected at random from the following table:

Table: Zoo Animals

Name	Type	Habitat
Lion	Mammal	Savanna
Penguin	Bird	Arctic
Crocodile	Reptile	Swamp
Elephant	Mammal	Savanna
Parrot	Bird	Jungle
Snake	Reptile	Jungle

Let event R be selecting a Mammal, and event S be selecting an animal from the Arctic habitat. Are events R and S mutually exclusive?

Choose one true answer:

☐ Yes, the events are mutually exclusive.

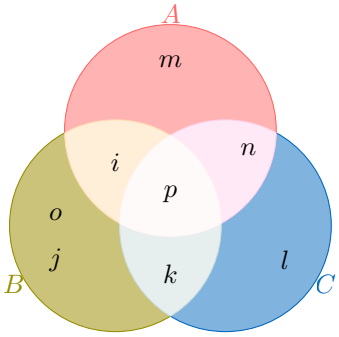
☐ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

A.8 VENN DIAGRAM

A.8.1 FINDING THE UNION OF TWO EVENTS IN A VENN DIAGRAM

MCQ 66: You are given this populated Venn diagram representing events A, B, and C:



Which option below correctly shows the union of A and B?

Choose one true answer:

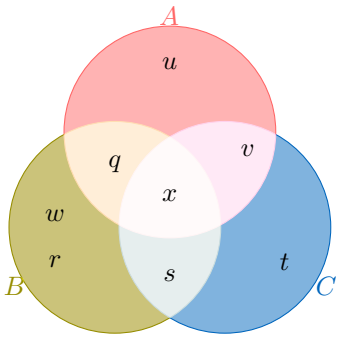
☐ A or $B = \{i, j, k, l, m, n\}$

☐ A or $B = \{i, j, k, m, o, p\}$

☐ A or $B = \{i, j, k, l, m\}$

☐ A or $B = \{i, j, k, l, m, o, p\}$

MCQ 67: You are given this populated Venn diagram representing events A, B, and C:



Which option below correctly shows the union of A and B?

Choose one true answer:

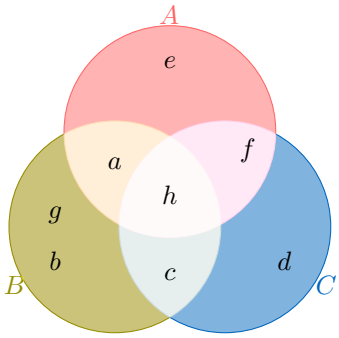
☐ A or $B = \{q, r, s, t, u, v\}$

☐ A or $B = \{q, r, s, u, w, x\}$

☐ A or $B = \{q, r, s, t, u\}$

☐ A or $B = \{q, r, s, t, u, w, x\}$

MCQ 68: You are given this populated Venn diagram representing events A, B, and C:



Which option below correctly shows the union of A and C?

Choose one true answer:

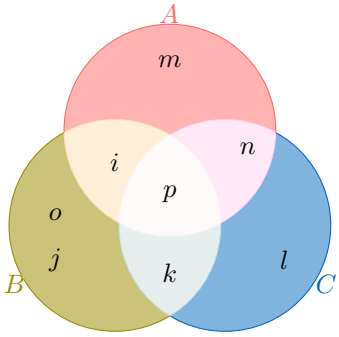
☐ A or $C = \{a, c, d, e, f, h\}$

☐ A or $C = \{a, b, e, g, h\}$

☐ A or $C = \{a, b, c, d, e\}$

☐ A or $C = \{a, b, c, d, e, g, h\}$

MCQ 69: You are given this populated Venn diagram representing events A, B, and C:



Which option below correctly shows the union of B and C?

Choose one true answer:

☐ B or $C = \{i, j, k, l, m, n\}$

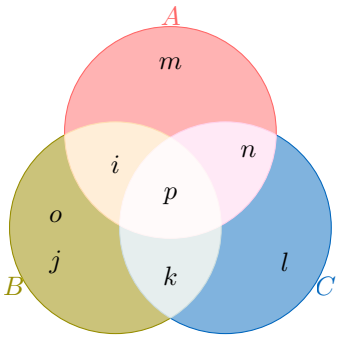
☐ B or $C = \{i, j, k, l, o, p\}$

☐ B or $C = \{i, j, k, l, o, p\}$

☐ B or $C = \{i, j, k, l, o, p\}$

A.8.2 FINDING THE INTERSECTION OF TWO EVENTS IN A VENN DIAGRAM

MCQ 70: You are given this populated Venn diagram representing events A, B, and C:

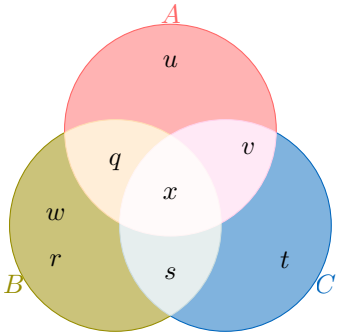


Which option below correctly shows the intersection of A and B?

Choose one true answer:

- ☐ A and $B = \{i, p\}$
- ☐ A and $B = \{i, j, k, m, o, p\}$
- ☐ A and $B = \{i, j, k, l, m\}$
- ☐ A and $B = \{j, k, o\}$

MCQ 71: You are given this populated Venn diagram representing events A, B, and C:

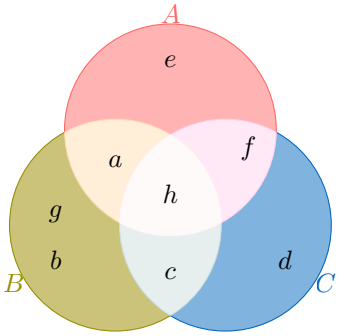


Which option below correctly shows the intersection of A and B?

Choose one true answer:

- ☐ A and $B = \{q, x\}$
- ☐ A and $B = \{q, r, s, u, w, x\}$
- ☐ A and $B = \{r, s, w\}$
- ☐ A and $B = \{q, r, s, t, u\}$

MCQ 72: You are given this populated Venn diagram representing events A, B, and C:

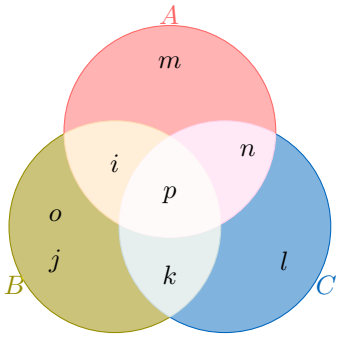


Which option below correctly shows the intersection of A and C?

Choose one true answer:

- ☐ A and $C = \{a, c, d, e, f, h\}$
- ☐ A and $C = \{a, e\}$
- ☐ A and $C = \{f, h\}$
- ☐ A and $C = \{c, d, f\}$

MCQ 73: You are given this populated Venn diagram representing events A, B, and C:



Which option below correctly shows the intersection of B and C?

Choose one true answer:

- ☐ B and $C = \{i, j, k, l, o, p\}$
- ☐ B and $C = \{k, p\}$
- ☐ B and $C = \{j, k, o\}$
- ☐ B and $C = \{l, n, p\}$

B PROBABILITY

B.1 DEFINITION

B.1.1 DETERMINING THE PROBABILITY

MCQ 74: Keziah eats rice often. Let E be the event that Keziah eats rice this week. Find $P(E)$, the probability that Keziah eats rice this week.

- ☐ $P(E) = 1\%$
- ☐ $P(E) = 50\%$
- ☐ $P(E) = 99\%$

MCQ 75: Emily drinks water every day. Let E be the event that Emily drinks water tomorrow. Find $P(E)$, the probability that Emily drinks water tomorrow.

- ☐ $P(E) = 50\%$
- ☐ $P(E) = 90\%$
- ☐ $P(E) = 100\%$

MCQ 76: It almost never snows in July in the Sahara Desert. Let E be the event that it snows this July in the Sahara Desert. Find $P(E)$, the probability that it snows this July.

- ☐ $P(E) = 0.01\%$

☐ $P(E) = 5\%$

☐ $P(E) = 99.9\%$

MCQ 77: Samuel loves playing basketball. Let E be the event that Samuel plays basketball this weekend. Find $P(E)$, the probability that Samuel plays this weekend.

☐ $P(E) = 5\%$

☐ $P(E) = 20\%$

☐ $P(E) = 90\%$

MCQ 78: Benjamin rolls a die. Let E be the event that Benjamin rolls a number bigger than 7. Find $P(E)$, the probability that Benjamin rolls a number bigger than 7.

☐ $P(E) = 0\%$

☐ $P(E) = 50\%$

☐ $P(E) = 100\%$

B.1.2 FINDING PROBABILITY FOR MUTUALLY EXCLUSIVE EVENTS

Ex 79: Let $P(G) = 0.6$ and $P(H) = 0.2$. Assume that events G and H are mutually exclusive.
Calculate $P(G \text{ or } H)$.

$$P(G \text{ or } H) = \square$$

Ex 80: Let $P(A) = 0.5$ and $P(B) = 0.3$. Assume that events A and B are mutually exclusive.
Calculate $P(A \text{ or } B)$.

$$P(A \text{ or } B) = \square$$

Ex 81: Let $P(C) = 0.4$ and $P(D) = 0.5$. Assume that events C and D are mutually exclusive.
Calculate $P(C \text{ or } D)$.

$$P(C \text{ or } D) = \square$$

Ex 82: Let $P(E) = 0.7$ and $P(F) = 0.1$. Assume that events E and F are mutually exclusive.
Calculate $P(E \text{ or } F)$.

$$P(E \text{ or } F) = \square$$

B.2 PROBABILITY RULES

B.2.1 APPLYING THE COMPLEMENT RULE

Ex 83: I toss a fair coin. The probability of getting heads is $\frac{1}{2}$.
Find the probability of getting tails.

$$P(\text{"Getting tails"}) = \frac{\square}{\square}$$

Ex 84: A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.
Find the probability that a student does not laugh at the joke.

$$P(\text{"Not laughing"}) = \square\%$$

Ex 85: I randomly select a student in the class. The probability that a girl is selected is $\frac{9}{10}$.
Find the probability that a boy is selected.

$$P(\text{"Selecting a boy"}) = \frac{\square}{\square}$$

Ex 86: The weather forecast predicts that there is a 70% chance of rain tomorrow.
Find the probability that it will not rain tomorrow.

$$P(\text{"No rain"}) = \square\%$$

Ex 87: A survey shows that 70% of the students in a school love Math.
Find the probability that a randomly chosen student does not love Math.

$$P(\text{"Not loving Math"}) = \square\%$$

MCQ 88: A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.
Find the probability that a student does not laugh at the joke.

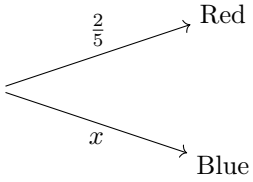
☐ $P(\text{"Not laughing"}) = 30\%$

☐ $P(\text{"Not laughing"}) = 70\%$

☐ $P(\text{"Not laughing"}) = 50\%$

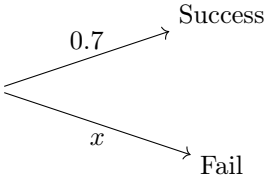
B.2.2 COMPLETING A PROBABILITY TREE DIAGRAM

Ex 89: From a bag containing red balls and blue balls, the probability of choosing a red ball is $\frac{2}{5}$. Find the probability x of choosing a blue ball.



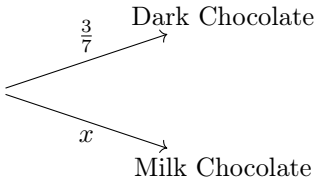
$$x = \frac{\square}{\square}$$

Ex 90: Jasper is playing basketball. The probability that he makes his first shot is 0.7. Find the probability x that he misses his first shot.



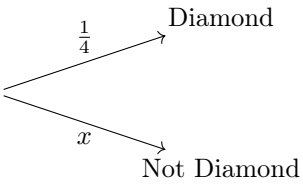
$$x = \square$$

Ex 91: In a box of assorted chocolates, the probability of picking a dark chocolate is $\frac{3}{7}$. Find the probability x of picking a milk chocolate.



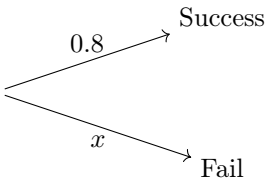
$x =$ $\square$

Ex 92: In a deck of cards, the probability of drawing a card from the suit of diamonds is $\frac{1}{4}$. Find the probability x of drawing a card that is not a diamond.



$x =$ $\square$

Ex 93: Emma is playing a video game. The probability that she completes a level is 0.8. Find the probability x that she fails to complete the level.



$x =$ $\square$

B.2.3 CALCULATING PROBABILITIES FOR UNION OF EVENTS

Ex 94: Let $P(A) = 0.5$, $P(B) = 0.3$ and $P(A \text{ and } B) = 0.1$. Calculate $P(A \text{ or } B)$.

$$P(A \text{ or } B) = \square$$

Ex 95: Let $P(E) = 0.8$, $P(F) = 0.3$ and $P(E \text{ and } F) = 0.2$. Calculate $P(E \text{ or } F)$.

$$P(E \text{ or } F) = \square$$

Ex 96: Let $P(G) = 0.6$, $P(H) = 0.2$ and $P(G \text{ and } H) = 0.1$ Calculate $P(G \text{ or } H)$.

$$P(G \text{ or } H) = \square$$

Ex 97: Let $P(X) = 0.7$, $P(Y) = 0.4$ and $P(X \text{ and } Y) = 0.2$. Calculate $P(X \text{ or } Y)$.

$$P(X \text{ or } Y) = \square$$

B.2.4 CALCULATING PROBABILITIES FOR UNION OF EVENTS IN REAL-WORLD PROBLEMS

Ex 98: In a school survey, the probability of a student liking math is 0.6, and the probability of liking science is 0.4. The probability of a student liking both math and science is 0.25.
What is the probability that a randomly selected student likes either math or science?

$$P(\text{Math or Science}) = \square$$

Ex 99: In a city survey, the probability of a resident using public transportation is 0.7, and the probability of using a bicycle is 0.3. The probability of a resident using both public transportation and a bicycle is 0.15.
What is the probability that a randomly selected resident uses either public transportation or a bicycle?

$$P(\text{Public Transport or Bicycle}) = \square$$

Ex 100: In a company survey, the probability of an employee enjoying team meetings is 0.5, and the probability of enjoying training sessions is 0.4. The probability of an employee enjoying both team meetings and training sessions is 0.2.
What is the probability that a randomly selected employee enjoys either team meetings or training sessions?

$$P(\text{Team Meetings or Training}) = \square$$

Ex 101: In a neighborhood survey, the probability of a household owning a dog is 0.5, and the probability of owning a cat is 0.35. The probability of a household owning both a dog and a cat is 0.2.
What is the probability that a randomly selected household owns either a dog or a cat?

$$P(\text{Dog or Cat}) = \square$$

B.3 EQUALLY LIKELY

B.3.1 DETERMINING THE PROBABILITY

Ex 102: A ball is chosen randomly from a bag containing 2 red balls, 3 blue balls.
Find the probability that we choose a red ball.

$$P(\text{"choosing a red ball"}) = \frac{\square}{\square}$$

Ex 103: A card is drawn at random from a standard deck of 52 playing cards. Determine the probability of drawing an Ace and express your answer as a simplified fraction.

$$P(\text{"drawing an Ace"}) = \frac{\square}{\square}$$

Ex 104: A six-sided die is rolled once. Determine the probability of obtaining an even number.

$$P(\text{"rolling an even number"}) = \frac{\square}{\square}$$

MCQ 105: A fruit is selected randomly from a basket containing 3 apples, 2 oranges, and 5 bananas.
Find the probability that the selected fruit is an orange (simplify the fraction).

$$P(\text{"selecting an orange"}) = \frac{\square}{\square}$$

B.3.2 DETERMINING THE PROBABILITY IN MULTI-STEP RANDOM EXPERIMENTS

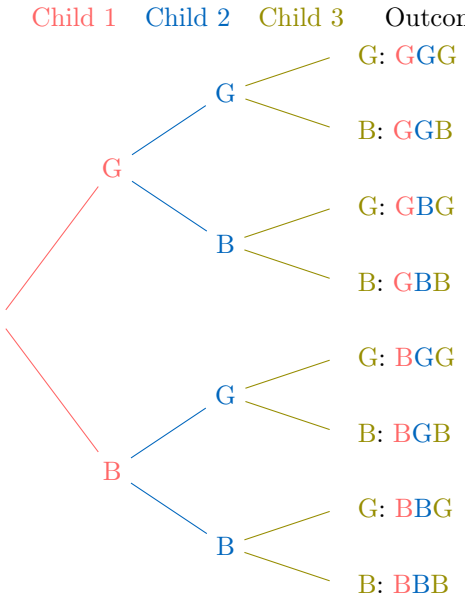
Ex 106: A coach selects two players at random from a group of four players, labeled A, B, C, and D, without replacement (once a player is chosen, they are not available for the next selection). The table below shows all possible outcomes for selecting Player 1 and Player 2, where an "X" indicates an impossible outcome due to the same player being selected twice.

Player 1 \ Player 2	A	B	C	D
A	X	AB	AC	AD
B	BA	X	BC	BD
C	CA	CB	X	CD
D	DA	DB	DC	X

Calculate the probability that player C is selected as either Player 1 or Player 2, simplifying the fraction.

$$P(\text{"selecting player C"}) = \frac{\square}{\square}$$

Ex 107: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



Calculate the probability that the family has at least two girls, simplifying the fraction.

$$P(\text{"at least two girls"}) = \frac{\square}{\square}$$

Ex 108: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

red die \ blue die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is exactly 7, simplifying the fraction.

$$P(\text{"sum is 7"}) = \frac{\square}{\square}$$

Ex 109: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

red die \ blue die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is greater than or equal to 11, simplifying the fraction.

$$P(\text{"sum } \geq 11\text{"}) = \frac{\square}{\square}$$

Ex 110: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

red die \ blue die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is exactly 6 or 8, simplifying the fraction.

$$P(\text{"sum is 6 or 8"}) = \frac{\square}{\square}$$

B.3.3 CRACKING PROBABILITIES

Ex 111: In a race with 20 horses, you bet on 3 horses to finish first, second, and third in exact order (a "triple forecast"). What's the probability of winning your bet?

$$P(\text{"}Winning\text{"}) = \square$$

Ex 112: In a race with 20 horses, you bet on 3 horses to finish in the top 3 positions (first, second, and third) in any order (a "trio" bet). What's the probability of winning your bet?

$$P(\text{Winning}) = \square$$

Ex 113: A combination lock uses a 3-digit code, where each digit is chosen from 0 to 9 (10 possible digits), and digits cannot repeat. You guess a specific 3-digit code to unlock it. What's the probability of guessing the correct code on your first try?

$$P(\text{Correct Guess}) = \square$$

Ex 114: The slogan associated with an American lottery is *You can't win if you don't play.* It's true, but what's the probability? In Lotto, you pick 6 numbers from a grid of 49. What is the probability of winning the jackpot (i.e., matching all 6 numbers)?

$$P(\text{"Winning"}) = \square$$

Ex 115: The slogan associated with an American lottery is *You can't win if you don't play.* It's true, but what's the probability? In Lotto, you pick 6 numbers from a grid of 49. What is the probability of winning a small prize (i.e., matching exactly 5 out of the 6 numbers drawn)?

$$P(\text{"Small Prize"}) = \square$$

Ex 116: In a class of 30 students, what is the probability that at least two students have the same birthday? Assume a year has 365 days and birthdays are equally likely. (round to 2 decimal places)

$$P(\text{"At least two share a birthday"}) = \square$$

C CONDITIONAL PROBABILITY

C.1 DEFINITION

C.1.1 EXPLORING PROBABILITIES WITH TWO-WAY TABLES

Ex 117: Consider a two-way table showing students preferences for loving math, categorized by gender:

	Loves Math	Not Love Math	Total
Girls	35	16	51
Boys	30	19	49
Total	65	35	100

A student is randomly selected from the class. Find the probability that the selected student is a girl.

$$P(\text{"Girl"}) = \square$$

Ex 118: Consider a two-way table showing students' preferences for participating in a school drama club, categorized by gender:

	Likes Drama Club	Dislikes Drama Club	Total
Girls	28	12	40
Boys	32	18	50
Total	60	30	90

A student is randomly selected from the group. Find the probability that the selected student likes the drama club.

$$P(\text{"Likes Drama Club"}) = \square$$

Ex 119: Consider a two-way table showing students' preferences for a short school trip, categorized by grade level:

	Likes Trip	Dislikes Trip	Total
Grade 9	15	5	20
Grade 10	25	15	40
Total	40	20	60

A student is randomly selected from the group. Find the probability that the selected student is in Grade 9 and likes the trip.

$$P(\text{"Grade 9 and Likes Trip"}) = \square$$

Ex 120: Consider a two-way table showing students' preferences for science, categorized by gender:

	Likes Science	Dislikes Science	Total
Girls	18	12	30
Boys	24	6	30
Total	42	18	60

A student is randomly selected from the group. Find the probability that the selected student is a boy and likes science.

$$P(\text{"Boy and Likes Science"}) = \square$$

Ex 121: Consider a two-way table showing students' preferences for science, categorized by gender:

	Likes Science	Dislikes Science	Total
Girls	18	12	30
Boys	24	6	30
Total	42	18	60

A student is randomly selected from the group. Find the probability that the selected student likes science, given that the student is a boy.

$$P(\text{"Likes Science"} \mid \text{"Boy"}) = \square$$

Ex 122: Consider a two-way table showing students' preferences for music, categorized by grade level:

	Likes Music	Dislikes Music	Total
Grade 9	20	10	30
Grade 10	15	15	30
Total	35	25	60

A student is randomly selected from the group. Find the probability that the selected student likes music, given that the student is in Grade 9.

$$P(\text{"Likes Music"} \mid \text{"Grade 9"}) = \square$$

Ex 123: Consider a two-way table showing students' preferences for art, categorized by grade level:

	Likes Art	Dislikes Art	Total
Grade 9	12	8	20
Grade 10	18	12	30
Total	30	20	50

A student is randomly selected from the group. Find the probability that the selected student is in Grade 9, given that the student likes art.

$$P(\text{"Grade 9"} \mid \text{"Likes Art"}) = \square$$

Ex 124: Consider a two-way table showing students' preferences for art, categorized by grade level:

	Likes Art	Dislikes Art	Total
Grade 9	12	8	20
Grade 10	18	12	30
Total	30	20	50

A student is randomly selected from the group. Find the probability that the selected student likes art, given that the student is in Grade 9.

$$P(\text{"Likes Art"} \mid \text{"Grade 9"}) = \square$$

Ex 125: Consider a two-way table showing students' preferences for music, categorized by grade level:

	Likes Music	Dislikes Music	Total
Grade 9	20	10	30
Grade 10	15	15	30
Total	35	25	60

A student is randomly selected from the group. Find the probability that the selected student is in Grade 9, given that the student likes music.

$$P(\text{"Grade 9"} \mid \text{"Likes Music"}) = \square$$

C.1.2 CALCULATING CONDITIONAL PROBABILITIES

Ex 126: Given that $P(E \text{ and } F) = 0.1$ and $P(F) = 0.4$, find :

$$P(E \mid F) = \square$$

Ex 127: Given that $P(A \text{ and } B) = 0.15$ and $P(B) = 0.5$, find :

$$P(A \mid B) = \square$$

Ex 128: Given that $P(X \text{ and } Y) = 0.12$ and $P(Y) = 0.3$, find :

$$P(X \mid Y) = \square$$

Ex 129: Given that $P(M \text{ and } N) = 0.25$ and $P(N) = 0.8$, find :

$$P(M \mid N) = \square$$

C.1.3 CALCULATING CONDITIONAL PROBABILITIES IN REAL-WORLD PROBLEMS

Ex 130: In a certain town, it is found that 30% of the families own a pet dog, and 15% of the families own both a pet dog and a cat. If a randomly selected family owns a dog, what is the probability that they also own a cat?

$$P(\text{"Own a cat"} \mid \text{"Own a dog"}) = \square$$

Ex 131: In a school, it is found that 40% of the students enjoy reading books, and 20% of the students enjoy both reading books and watching movies. If a randomly selected student enjoys reading books, what is the probability that they also enjoy watching movies?

$$P(\text{"Enjoy movies"} \mid \text{"Enjoy books"}) = \square$$

Ex 132: In a neighborhood, it is found that 60% of the households own a car, and 24% of the households own both a car and a bicycle. If a randomly selected household owns a car, what is the probability that they also own a bicycle?

$$P(\text{"Own a bicycle"} \mid \text{"Own a car"}) = \square$$

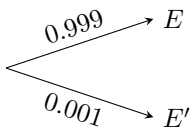
Ex 133: In a club, it is found that 25% of the members play soccer, and 10% of the members play both soccer and basketball. If a randomly selected member plays soccer, what is the probability that they also play basketball?

$$P(\text{"Play basketball"} \mid \text{"Play soccer"}) = \square$$

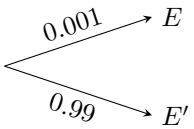
C.2 CONDITIONAL PROBABILITY TREE DIAGRAMS

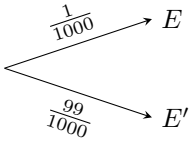
C.2.1 CONDITIONAL PROBABILITY TREE DIAGRAMS: LEVEL 1

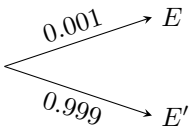
MCQ 134: Consider a rare disease that affects 1 in every 1 000 people.
Let E be the event of having the disease.
A person is randomly selected from the population.
Choose the correct probability tree diagram:



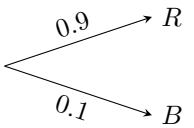
□

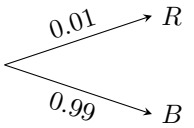
□ 

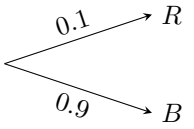
□ 

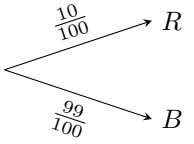
□ 

MCQ 135: Imagine a bag containing 100 marbles, of which 10 are red (R) and the rest are blue (B). A marble is randomly drawn from the bag.
Let R be the event of drawing a red marble.
Choose the correct probability tree diagram:

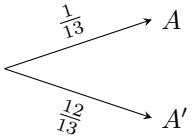
□ 

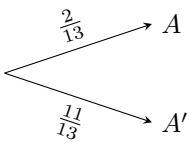
□ 

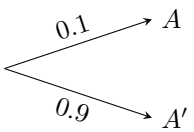
□ 

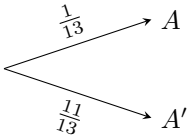
□ 

MCQ 136: A deck of cards contains 52 cards, of which 4 are aces. A card is randomly drawn from the deck.
Let A be the event of drawing an ace.
Choose the correct probability tree diagram:

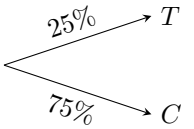
□ 

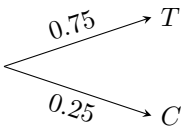
□ 

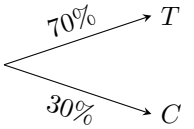
□ 

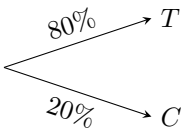
□ 

MCQ 137: A survey shows that 75% of a town's population prefers tea (T) over coffee (C). A person is randomly selected from the town's population.
Let T be the event of a person preferring tea.
Choose the correct probability tree diagram:

□ 

□ 

□ 

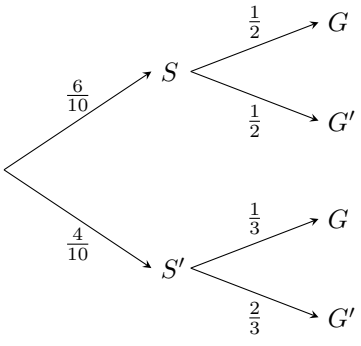
□ 

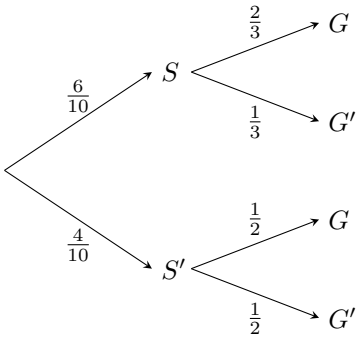
C.2.2 CONDITIONAL PROBABILITY TREE DIAGRAMS: LEVEL 2

MCQ 138: The probability that Sam is coaching a game is $\frac{6}{10}$, and the probability that Alex is coaching is $\frac{4}{10}$.

- With Coach Sam, the probability of being Goalkeeper is $\frac{1}{2}$.
- With Coach Alex, the probability of being Goalkeeper is $\frac{2}{3}$.

Let S be the event that Sam is the coach.
Let G be the event of being the goalkeeper.
Choose the correct probability tree diagram:

□ 



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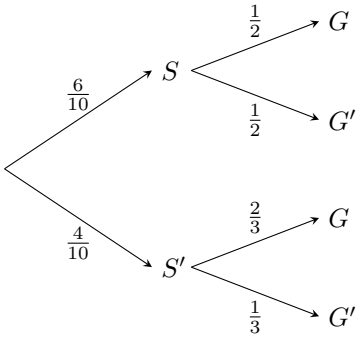

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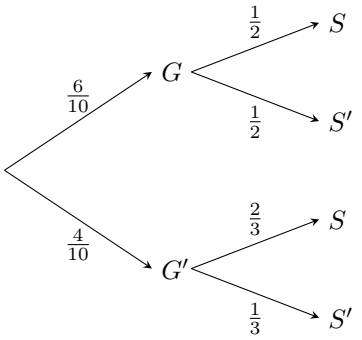

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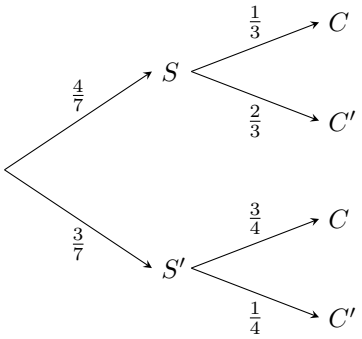
MCQ 139: In a city, the probability of a randomly chosen day being sunny is $\frac{4}{7}$, and the probability of it being rainy is $\frac{3}{7}$.

- On a sunny day, the probability that the park is crowded is $\frac{2}{3}$.
- On a rainy day, the probability that the park is crowded is $\frac{1}{4}$.

Let S be the event that it's a sunny day.
Let C be the event of the park being crowded.

Choose the correct probability tree diagram:



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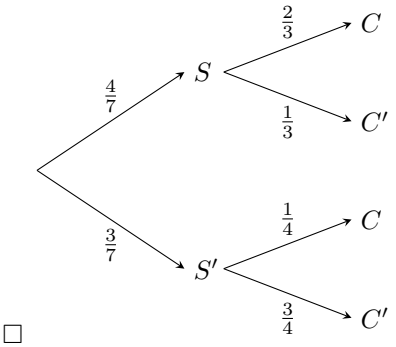

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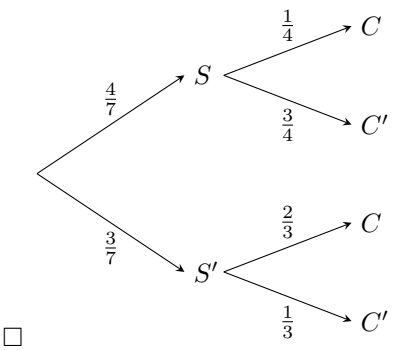

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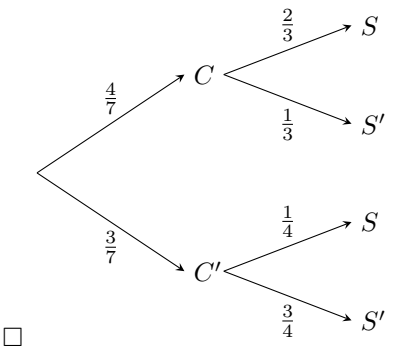
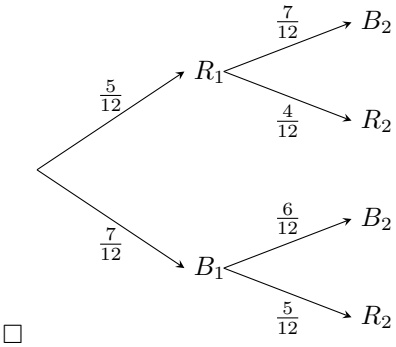


□

MCQ 140: A bag contains 5 red (R) and 7 blue (B) marbles. One marble is drawn and not replaced, and then a second marble is drawn.
Let R_1 be the event of drawing a red marble first, and let B_1 be the event of drawing a blue marble first.
Let R_2 be the event of drawing a red marble second, and let B_2 be the event of drawing a blue marble second.

Choose the correct probability tree diagram:



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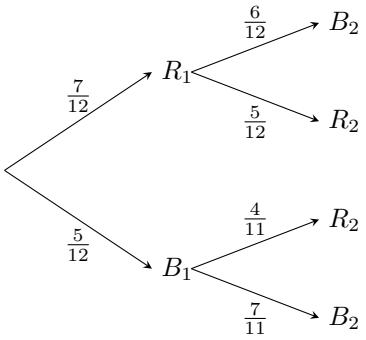

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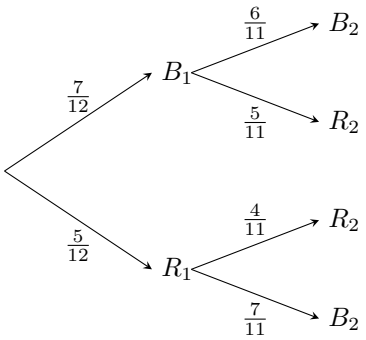

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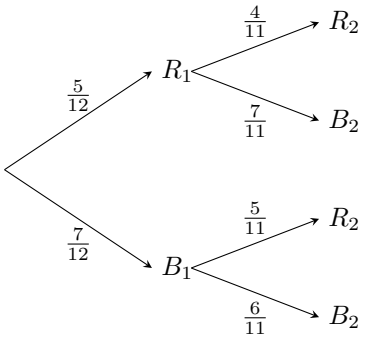


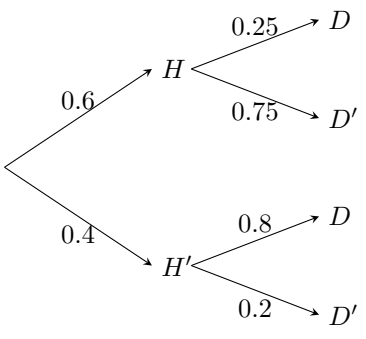
□

MCQ 141: In a town, the probability that a randomly chosen morning has heavy traffic is 0.6, and the probability that it has light traffic is 0.4.

- On a morning with heavy traffic, the probability of a bus being delayed is 0.75.
- On a morning with light traffic, the probability of a bus being delayed is 0.2.

Let H be the event that the morning has heavy traffic.
Let D be the event that the bus is delayed.

Choose the correct probability tree diagram:



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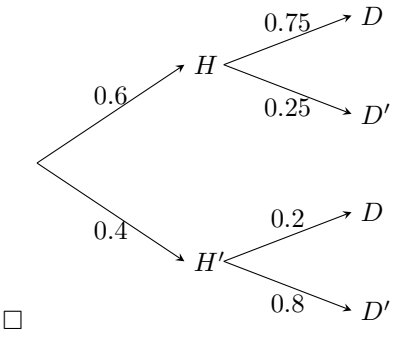

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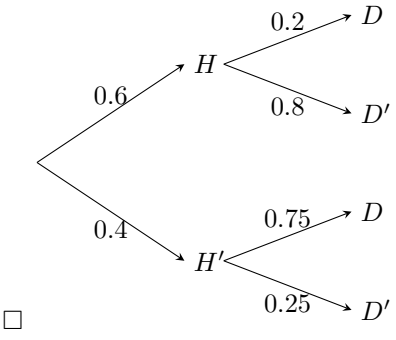

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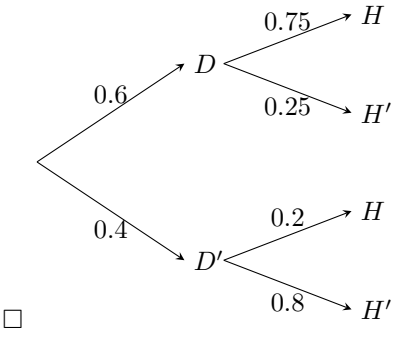

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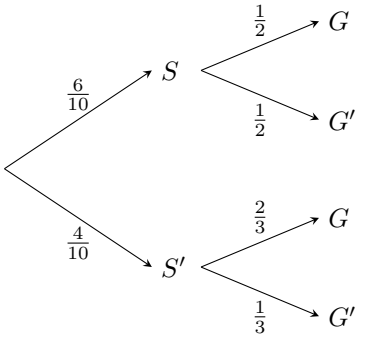
C.3 JOINT PROBABILITY: $P(E$ and $F)$

C.3.1 CALCULATING JOINT PROBABILITIES WITH TREES

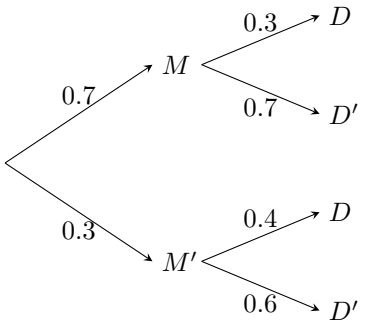
Ex 142: For this probability tree diagram:



find the probability :

$$P(S \text{ and } G) = \square$$

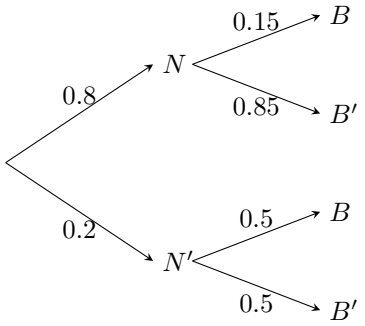
Ex 143: For this probability tree diagram:



find the probability :

$$P(M' \text{ and } D') = \square$$

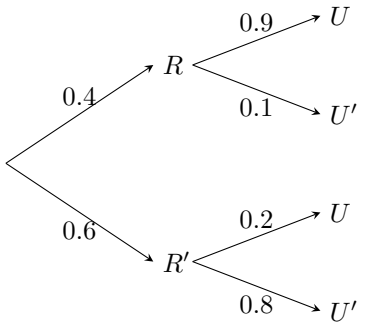
Ex 144: For this probability tree diagram:



find the probability :

$$P(N' \text{ and } B) = \square$$

Ex 145: For this probability tree diagram:



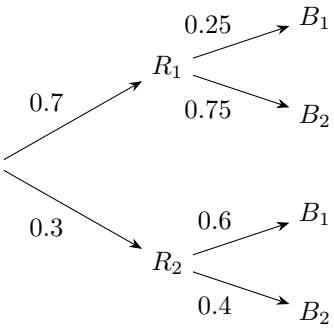
find the probability :

$$P(R \text{ and } U) = \square$$

C.4 LAW OF TOTAL PROBABILITY

C.4.1 CALCULATING PROBABILITIES WITH TREES

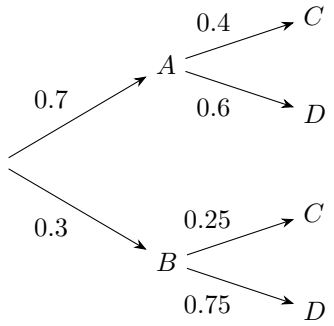
Ex 146: For this probability tree:



calculate the probability :

$$P(B_1) = \square$$

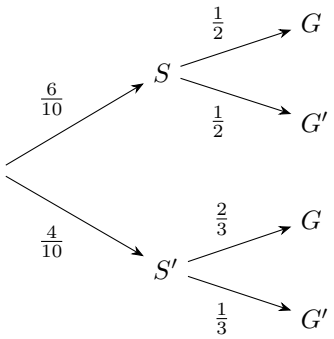
Ex 147: For this probability tree:



calculate the probability :

$$P(D) = \square$$

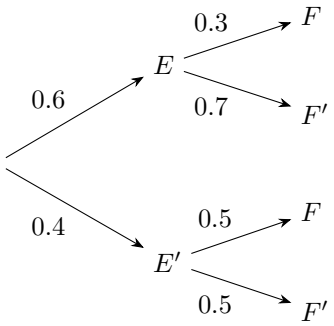
Ex 148: For this probability tree,



calculate the probability :

$$P(G) = \square$$

Ex 149: For this probability tree:



calculate the probability :

$$P(F') = \square$$

C.4.2 CALCULATING PROBABILITIES IN REAL-WORLD PROBLEMS

Ex 150: A company produces two types of parts: A and B. 20% of parts are type A and 80% are type B. The probability that a part is defective given type A is 0.02, and the probability that a part is defective given type B is 0.01.
Find the probability that a part is defective :

$$P(\text{"Defective"}) = \square$$

Ex 151: A meteorologist observes cloud conditions to predict rain. On a given day, 40% of the time the sky is cloudy, and 60% of the time it is clear. The probability of rain given a cloudy sky is 0.75, and the probability of rain given a clear sky is 0.15.
Find the probability that it rains :

$$P(\text{Rain}) = \square$$

Ex 152: An urn contains 1 red ball and 4 blue balls. A first ball is drawn without replacement. Then a second ball is drawn from the remaining balls.
Find the probability that the second ball drawn is red :

$$P(R_2) = \square$$

Ex 153: A population is tested for a disease. 30% of the population has the disease. The probability that a test is positive given the person has the disease is 0.95, and the probability that a test is positive given the person does not have the disease is 0.10.
Find the probability that a test is positive :

$$P(\text{Positive}) = \square$$

C.5 BAYES' THEOREM

C.5.1 UNVEILING THE HIDDEN CAUSE: BAYES' THEOREM IN RARE EVENT DETECTION

Ex 154: Consider a rare disease that affects approximately 1 in every 1,000 people. A medical test developed for detecting this disease has the following characteristics:

- Sensitivity: If a person has the disease, the test correctly returns a positive result 99% of the time.
- Specificity: If a person does not have the disease, the test correctly returns a negative result 95% of the time.

Find the probability in percent that a person actually has the disease if their test result is positive (round to 1 decimal place):

$$P(\text{Disease} \mid \text{Test positive}) = \square\%$$

Ex 155: Consider a rare disease that affects approximately 1 in every 1,000 people. A medical test developed for detecting this disease has the following characteristics:

- Sensitivity: If a person has the disease, the test correctly returns a positive result 99% of the time.
- Specificity: If a person does not have the disease, the test correctly returns a negative result 95% of the time.

Find the probability in percent that a person actually has the disease if their test result is positive (round at 1 decimal place):

$$P(\text{"Disease"} \mid \text{"Test positive"}) = \square\%$$

Ex 156: Consider a rare alien signal that is present in approximately 1 out of every 10,000 radio scans conducted by a space observatory. A signal detector has the following characteristics:

- Sensitivity: If an alien signal is present, the detector correctly identifies it as positive 98% of the time.
- Specificity: If no alien signal is present, the detector correctly identifies it as negative 96% of the time.

Find the probability in percent that an alien signal is actually present if the detector returns a positive result (round to 1 decimal place):

$$P(\text{Signal} \mid \text{Positive}) = \square\%$$

Ex 157: In a city, 1 out of every 100 drivers drives with alcohol in their system. The probability of having an accident given that a driver has alcohol is $1/2$, and the probability of having an accident given that a driver has no alcohol is $1/1000$.
Find the probability in percent that a driver has alcohol in their system if they have had an accident (round to 1 decimal place):

$$P(\text{Alcohol} \mid \text{Accident}) = \square\%$$

Ex 158: In a futuristic society, 1 out of every 500 devices contains a rare quantum crystal as its power source. A crystal detector has been invented with the following properties:

- Sensitivity: If a device has a quantum crystal, the detector correctly registers it as active 90% of the time.
- Specificity: If a device does not have a quantum crystal, the detector correctly registers it as inactive 97% of the time.

Find the probability in percent that a device actually has a quantum crystal if the detector registers it as active (round to 1 decimal place):

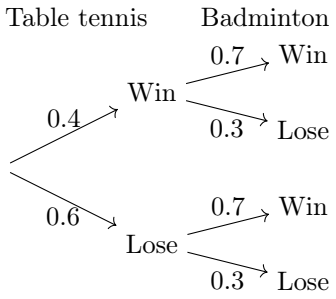
$$P(\text{Crystal} \mid \text{Active}) = \square\%$$

D PROBABILITY OF INDEPENDENT EVENTS

D.1 DEFINITION

D.1.1 READING PROBABILITY TREE

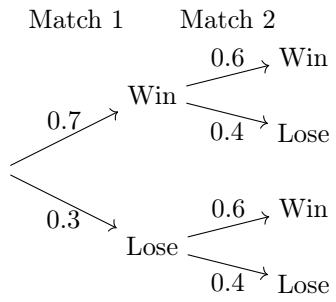
Ex 159: Niamh plays a game of table tennis on Saturday and a game of badminton on Sunday. The probability tree is represented:



Calculate the probability that Niamh wins both games.

$$P(\text{"Win both"}) = \square$$

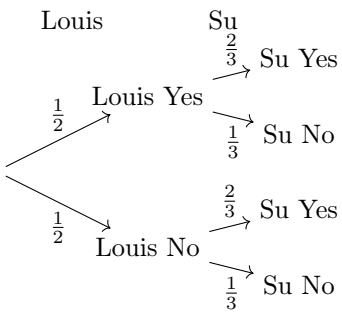
Ex 160: Sam is playing an online multiplayer game. The probability that Sam wins their first match is 0.7, and the probability that Sam wins their second match is 0.6.



Calculate the probability that Sam loses both the first and second matches.

$$P(\text{"Both lose"}) = \square$$

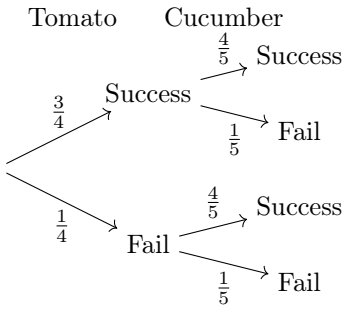
Ex 161: A party is happening this weekend! Louis might come with a probability of $\frac{1}{2}$, and Su might come with a probability of $\frac{2}{3}$.



Calculate the probability that both Louis and Su come to the party (simplify the fraction).

$$P(\text{"Both come"}) = \frac{\square}{\square}$$

Ex 162: Mia takes care of her garden. The probability that her tomato plants grow is $\frac{3}{4}$, and the probability that her cucumber plants grow is $\frac{4}{5}$.



Calculate the probability that both the tomato plants and the cucumber plants fail to grow (simplify the fraction).

$$P(\text{"Both fail"}) = \frac{\square}{\square}$$

D.1.2 FINDING THE PROBABILITY WITH INDEPENDENT EVENTS

Ex 163: Imagine you're at a carnival playing a game. You pick a ball from a bag containing 2 red balls and 3 blue balls, then roll a fair six-sided die. Find the probability of choosing a red ball **and** rolling a 6 (simplify the fraction).

$$P(\text{"Red" and "6"}) = \frac{\square}{\square}$$

Ex 164: Imagine you're at a carnival playing another game. You flip a fair coin and draw a card from a standard deck of 52 playing cards. Find the probability of getting heads **and** drawing an Ace (simplify the fraction).

$$P(\text{"Heads" and "Ace"}) = \frac{\square}{\square}$$

Ex 165: Imagine you're at a carnival playing a dice game. You roll a fair six-sided die two times in a row. Find the probability of getting a number greater than 4 (a 5 or 6) on both rolls (simplify the fraction).

$$P(\text{"Number} > 4\text{" and "Number} > 4\text{"}) = \frac{\square}{\square}$$

Ex 166: Sam is playing an online multiplayer game. The probability that Sam wins their first match is 0.7, and the probability that Sam wins their second match is 0.6.
Calculate the probability that Sam loses both the first and second matches.

$$P(\text{"Both lose"}) = \square$$

Ex 167: Mia takes care of her garden. The probability that her tomato plants grow is $\frac{3}{4}$, and the probability that her cucumber plants grow is $\frac{4}{5}$.
Calculate the probability that both the tomato plants and the cucumber plants fail to grow (simplify the fraction).

$$P(\text{"Both fail"}) = \frac{\square}{\square}$$