# PROPERTIES OF INTEGERS

## A NUMBERS 1 AND 0

Zero (0) and one (1) are very special numbers which have important properties.

**Proposition Additive Identity Property**

Adding 0 to any number results in the number itself.
For any number $a$,

$$0 + a = a$$

**Ex:** $7 + 0 = 7$

**Proposition Multiplicative Identity Property**

Multiplying 1 by any number results in the number itself.
For any number $a$,

$$1 \times a = a$$

**Ex:** $1 \times 7 = 7$

**Discover:**

- **Multiplying by** $0$:
  - **Any number multiplied by 0 always results in 0.**
  - For example, $5 \times 0 = 0 + 0 + 0 + 0 + 0 = 0$.
- **Dividing by** $0$:
  - Division by 0 is **undefined** in mathematics because it leads to a contradiction.
  - For example, if $a = 5 \div 0$, then $a \times 0 = 5$. But this is impossible because $a \times 0$ is always 0, not 5.

**Proposition Multiplying by** $0$

For any number $a$,

$$a \times 0 = 0$$

**Proposition Dividing by** $0$

Division by 0 is **undefined**.

## B DIVISION WITH REMAINDERS

**Theorem Division with Remainder**

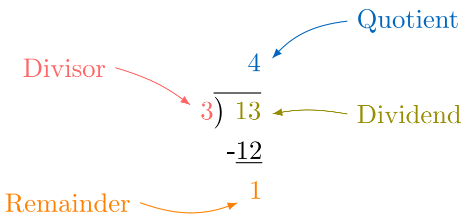
For any integers $a$ and nonzero $b$, there exist unique integers $q$ and $r$ such that:

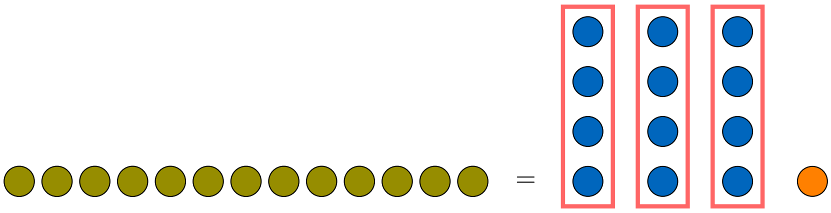
$$a = b \times q + r \quad \text{with} \quad 0 \leq r < b$$

- $a$ is the dividend (the number to be shared),
- $b$ is the divisor (the number of groups),
- $q$ is the quotient (the size of each group),
- $r$ is the remainder.

**Ex:**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 13 | = | 3 | × | 4 | + | 1 | with | 1 < 3 |
| Dividend | = | Divisor | × | Quotient | + | Remainder | with | Remainder < Divisor |





## C DIVISIBILITY

**Definition Divisible**

A whole number is said to be **divisible** by another non-zero whole number if the remainder is zero when you divide the first number by the second number. In this case, we say that the second number is a **divisor** of the first number.

**Ex:** Is 10 divisible by 5?

*Answer:* Yes, 10 is divisible by 5 because the remainder of the division is 0: $10 = 5 \times 2 + 0$.

**Ex:** Is 13 divisible by 5?

*Answer:* No, 13 is not divisible by 5 because the remainder of the division is 3: $13 = 5 \times 2 + 3$.

**Definition Multiple**

A whole number is said to be a **multiple** of another non-zero whole number if the first number can be obtained by multiplying the second number by some whole number. In other words, the first number appears in the multiplication table of the second number. In this case, we say that the second number is a **factor** of the first number.

**Ex:** Is 10 a multiple of 5?

*Answer:* Yes, 10 is a multiple of 5 because $5 \times 2 = 10$. So 5 is a factor of 10.

**Theorem Divisible ⇔ Multiple**

For two whole numbers, the first number is **divisible** by the second number if and only if the first number is a **multiple** of the second number.

**Ex:** Is 1000 divisible by 10?

*Answer:* Yes, as $1000 = 100 \times 10$, 1000 is a multiple of 10. Therefore, 1000 is divisible by 10.

## D DIVISIBILITY CRITERIA

Divisibility criteria are methods that allow us to quickly determine if a whole number is divisible by another whole number without performing long division. These rules are useful for simplifying calculations and understanding number properties. Here are some common divisibility criteria:

**Proposition Divisibility Criteria for 2 and 5**

- A number is divisible by 2 if its last digit is even (0, 2, 4, 6, or 8).
- A number is divisible by 5 if its last digit is 0 or 5.

**Ex:** Determine whether 946 is divisible by 2.

*Answer:* 946 is divisible by 2 because its last digit is 6, which is even.

**Ex:** Determine whether 947 is divisible by 5.

*Answer:* 947 is not divisible by 5 because its last digit is 7, which is not 0 or 5.

**Proposition Divisibility Criteria for 3 and 9**

- A number is divisible by 3 if the sum of its digits is divisible by 3.
- A number is divisible by 9 if the sum of its digits is divisible by 9.

**Ex:** Determine whether 948 is divisible by 3.

*Answer:* 948 is divisible by 3 because the sum of its digits, $9 + 4 + 8 = 21$, is divisible by 3 ($21 = 3 \times 7$).

**Ex:** Determine whether 948 is divisible by 9.

*Answer:* 948 is not divisible by 9 because the sum of its digits, $9 + 4 + 8 = 21$, is not divisible by 9 ($21 = 9 \times 2 + 3$).

**Proposition Divisibility Criteria for 4**

A number is divisible by 4 if the number formed by its last two digits is divisible by 4.

**Ex:** Determine whether 917 is divisible by 4.

*Answer:* 917 is not divisible by 4 because the number formed by its last two digits, 17, is not divisible by 4 ($17 = 4 \times 4 + 1$).

## E PRIME NUMBER

**Definition Prime Number**

A **prime number** is a whole number greater than 1 that has **only two different divisors**: 1 and itself.
A whole number greater than 1 that is not prime is called a **composite number**.

**Ex:** State whether 6 is a prime number.

*Answer:* As $2 \times 3 = 6$, 6 is not a prime number because it is divisible by 2 and 3, in addition to 1 and 6. So 6 is a composite number.

**Ex:** State whether 5 is a prime number.

*Answer:* 5 is a prime number because it is divisible **entirely** only by 1 and 5. We cannot divide it **entirely** by $2, 3$ or 4 (there would be a remainder).

**Proposition First 25 Prime Numbers**

Here is the list of the first 25 prime numbers:

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97$$

## F PRIME FACTORIZATION

**Definition Prime Factorization**

**Prime factorization** is writing a number as a product of only prime numbers. In other words, it's finding which prime numbers you need to multiply together to get the original number.

**Ex:** Find the prime factorization of 12.

*Answer:* The prime factorization is $12 = 2 \times 2 \times 3$.
The order is not important. You can write $12 = 3 \times 2 \times 2$.
The prime factorization is not $12 = 2 \times 6$ as 6 is a composite number.

**Method Factor Tree**

The **Factor tree method** involves breaking down a composite number into smaller factors, then breaking down those factors further until you have only prime factors.

1. Place the number at the top of the factor tree.
2. **Check if the number is prime.**
   (a) **If the number is prime:** Circle it. You are done with this branch.
   (b) **If the number is composite:** Factorize it into two smaller factors. Write these two factors as branches below the number. Repeat step 2 for each of these new factors.
3. The prime factorization is the product of all circled prime numbers on the tree.

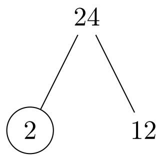
**Ex:** Find a prime factorization of 24.

*Answer:*

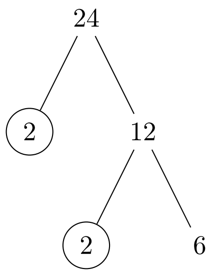
- Step 1:

$$24$$

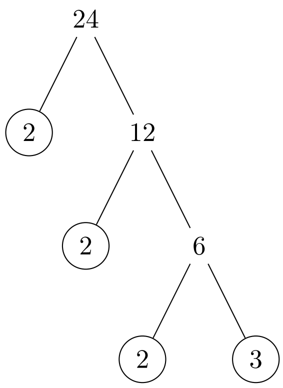
- Step 2: 24 is a composite number. $24 = 2 \times 12$.




- Step 3: 12 is a composite number. $12 = 2 \times 6$.




- Step 4: 6 is a composite number. $6 = 2 \times 3$.




A prime factorization is $24 = 2 \times 2 \times 2 \times 3$.