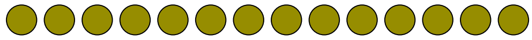
PROPERTIES OF INTEGERS

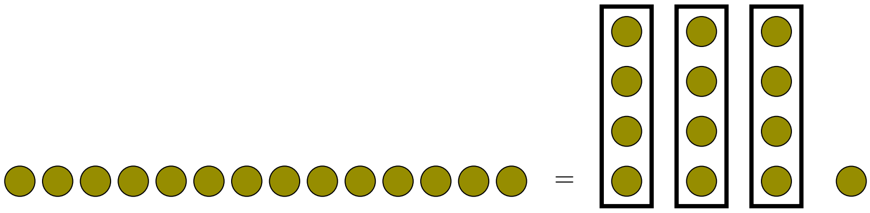
A DIVISION WITH REMAINDERS

Discover: Hugo has 13 marbles to divide evenly into 3 groups.



How many marbles will be in each group, and how many will be left over?

Answer:



There are 4 marbles in each group, with 1 marble left over. We call the leftover marble the remainder. We can write this division as:

$$13 \div 3 = 4\text{R}1$$

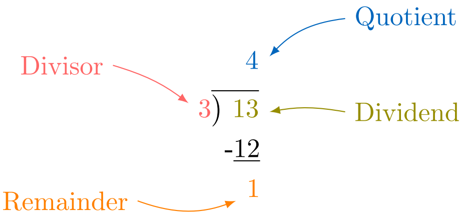
To check our answer, we can use multiplication and addition:

$$13 = 3 \times 4 + 1.$$

Theorem Division with Remainder

For any two whole numbers, a dividend and a non-zero divisor, there exists a unique quotient and a unique remainder that are also whole numbers, such that:

13	=	3	×	4	+	1	with	$1 < 3$
Dividend	=	Divisor	×	Quotient	+	Remainder	with	Remainder < Divisor



B DIVISIBILITY

Definition Divisible

A whole number is said to be **divisible** by another non-zero whole number if the remainder is zero when you divide the first number by the second number. In this case, we say that the second number is a **divisor** of the first number.

Ex: Is 10 divisible by 5?

Answer: Yes, 10 is divisible by 5 because the remainder of the division is 0: $10 = 5 \times 2 + 0$.

Ex: Is 13 divisible by 5?

Answer: No, 13 is not divisible by 5 because the remainder of the division is 3: $13 = 5 \times 2 + 3$.

Definition Multiple

A whole number is said to be a **multiple** of another non-zero whole number if the first number can be obtained by multiplying the second number by some whole number. In other words, the first number appears in the multiplication table of the second number. In this case, we say that the second number is a **factor** of the first number.

Ex: Is 10 a multiple of 5?

Answer: Yes, 10 is a multiple of 5 because $5 \times 2 = 10$. So 5 is a factor of 10.

Theorem Divisible ⇔ Multiple

For two whole numbers, the first number is **divisible** by the second number if and only if the first number is a **multiple** of the second number.

Proof

This is a direct consequence of the uniqueness of the quotient and remainder in the division with remainder theorem. If the remainder is zero, then the dividend is exactly the product of the divisor and the quotient, which is the definition of a multiple. Conversely, if a number is a multiple of another, then by definition it can be written as a product, implying a zero remainder upon division.

Ex: Is 1000 divisible by 10?

Answer: Yes, as $1000 = 100 \times 10$, 1000 is a multiple of 10. Therefore, 1000 is divisible by 10.

C DIVISIBILITY CRITERIA

Divisibility criteria are methods that allow us to quickly determine if a whole number is divisible by another whole number without performing long division. These rules are useful for simplifying calculations and understanding number properties. Here are some common divisibility criteria:

Proposition Divisibility Criteria for 2 and 5

- A number is divisible by 2 if its last digit is even (0, 2, 4, 6, or 8).
- A number is divisible by 5 if its last digit is 0 or 5.

Ex: Determine whether 946 is divisible by 2.

Answer: 946 is divisible by 2 because its last digit is 6, which is even.

Ex: Determine whether 947 is divisible by 5.

Answer: 947 is not divisible by 5 because its last digit is 7, which is not 0 or 5.

Proposition Divisibility Criteria for 3 and 9

- A number is divisible by 3 if the sum of its digits is divisible by 3.
- A number is divisible by 9 if the sum of its digits is divisible by 9.

Ex: Determine whether 948 is divisible by 3.

Answer: 948 is divisible by 3 because the sum of its digits, $9 + 4 + 8 = 21$, is divisible by 3 ($21 = 3 \times 7$).

Ex: Determine whether 948 is divisible by 9.

Answer: 948 is not divisible by 9 because the sum of its digits, $9 + 4 + 8 = 21$, is not divisible by 9 ($21 = 9 \times 2 + 3$).

Proposition Divisibility Criteria for 4

A number is divisible by 4 if the number formed by its last two digits is divisible by 4.

Ex: Determine whether 917 is divisible by 4.

Answer: 917 is not divisible by 4 because the number formed by its last two digits, 17, is not divisible by 4 ($17 = 4 \times 4 + 1$).

D NUMBERS 1 AND 0

Zero (0) and one (1) are very special numbers which have important properties.

Discover: Imagine you have 3 apples in your basket.

If someone gives you 0 more apples, how many apples do you have in total?

Answer: Well, if you get no extra apples, you still have exactly the same number you started with. You still have 3 apples.This is what the Additive Identity Property tells us: adding zero doesn't change the amount. In mathematics, we write this as:

$$3 + 0 = 3$$

And this is true for any number of apples, or any number at all! Adding zero to any number leaves the number unchanged.

Proposition Additive Identity Property

Adding 0 to any number results in the number itself. For any number a,

$$0 + a = a$$

Ex: $7 + 0 = 7$

Discover: Properties of Zero in Multiplication and Division

- **Multiplying by** 0:
  - **Any number multiplied by 0 always results in 0.**
  - For example, $5 \times 0 = 0 + 0 + 0 + 0 + 0 = 0$.
- **Dividing by** 0:
  - Division by 0 is **undefined** in mathematics because it leads to a contradiction.
  - For example, if $x = 10 \div 0$, then $x \times 0 = 10$. But this is impossible because $0 \times x$ is always 0, not 10.

Proposition Multiplying by 0

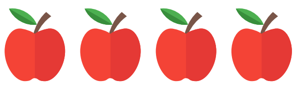
For any number a,

$$a \times 0 = 0$$

Proposition Dividing by 0

Division by 0 is **undefined**.

Discover: Imagine you have a basket with 4 delicious apples.



If you have 1 such basket, how many apples do you have in total?Well, if you have just 1 basket, you have exactly the number of apples that are in that one basket. You still have 4 apples.This is what the Multiplicative Identity Property tells us: multiplying by one doesn't change the amount. In mathematics, we write this as:

$$1 \times 4 = 4$$

And this is true for any number of apples in a basket, or any number at all! Multiplying any number by one leaves the number unchanged.

Answer: Well, if you have just 1 basket, you have exactly the same number of apples as in that basket. You still have 4 apples.This is what the Multiplicative Identity Property tells us: multiplying by one doesn't change the amount. In mathematics, we write this as:

$$1 \times 4 = 4$$

And this is true for any number of apples, or any number at all! Multiplying any number by one leaves the number unchanged.

Proposition Multiplicative Identity Property

Multiplying 1 by any number results in the number itself.
For any number a,

$$1 \times a = a$$

Ex: $1 \times 7 = 7$

E PRIME NUMBER

Definition Prime Number

Un **nombre premier** est un nombre entier plus grand que 1 qui a **seulement deux diviseurs différents** : 1 et lui-même.
Un nombre entier plus grand que 1 qui n'est pas premier est appelé un **nombre composé**.

Ex: State whether 6 is a prime number.

Answer: As $2 \times 3 = 6$, 6 is not a prime number because it is divisible by 2 and 3, in addition to 1 and 6. So 6 is a composite number.

Ex: State whether 5 is a prime number.

Answer: 5 is a prime number because it is divisible **entirely** only by 1 and 5. We cannot divide it **entirely** by $2, 3$ or 4 (there would be a remainder).

Proposition First 25 Prime Numbers

Here is the list of the first 25 prime numbers:

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97$$

F PRIME FACTORIZATION

Definition Prime Factorization

Prime factorization is writing a number as a product of only prime numbers. In other words, it's finding which prime numbers you need to multiply together to get the original number.

Ex: Find the prime factorization of 12.

Answer: The prime factorization is $12 = 2 \times 2 \times 3$.
The order is not important. You can write $12 = 3 \times 2 \times 2$.
The prime factorization is not $12 = 2 \times 6$ as 6 is a composite number.

Method Factor Tree

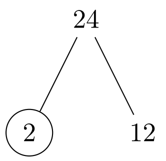
The **Factor tree method** involves breaking down a composite number into smaller factors, then breaking down those factors further until you have only prime factors.

1. Place the number at the top of the factor tree.
2. **Check if the number is prime.**
   (a) **If the number is prime:** Circle it. You are done with this branch.
   (b) **If the number is composite:** Factorize it into two smaller factors. Write these two factors as branches below the number. Repeat step 2 for each of these new factors.
3. The prime factorization is the product of all circled prime numbers on the tree.

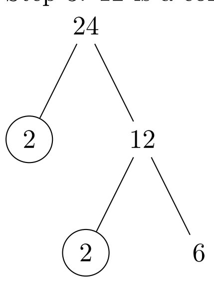
Ex: Find the prime factorization of 24.

Answer:

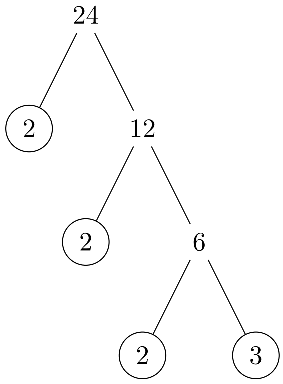
- Step 1: 24
- Step 2: 24 is a composite number. $24 = 2 \times 12$.



- Step 3: 12 is a composite number. $12 = 2 \times 6$.



- Step 4: 6 is a composite number. $6 = 2 \times 3$.



The prime factorization is $24 = 2 \times 2 \times 2 \times 3$.