PROPERTIES OF INTEGERS

A NUMBERS 1 AND 0

A.1 APPLYING NUMBER PROPERTIES

Ex 1: Calculate the following expression without using a calculator:

$$(4 \times 22 + 3 + 22 \div 2) \times 0 = \boxed{0}$$

Answer:

- Any number multiplied by 0 is 0. In the expression $(4 \times 22 + 3 + 22 \div 2) \times 0$, even if we calculated the value inside the parentheses, the entire result is multiplied by 0.
- Therefore, the final answer is 0.

Ex 2: Consider the following sequence of algebraic manipulations:

$$\begin{aligned} 0 \times 2 &= 0 && \text{Line 1} \\ 2 &= \frac{0}{0} && \text{Line 2} \quad \text{(dividing by 0)} \\ 2 &= \frac{1 \times \not{\emptyset}}{1 \times \not{\emptyset}} && \text{Line 3} \quad \text{(cancelling common factor)} \\ 2 &= 1 && \text{Line 4} \end{aligned}$$

This sequence appears to show that $2 = 1$, which is a false result. Identify the line where an invalid mathematical operation is performed.

$$\text{The error occurs in Line } \boxed{2}$$

Answer: The error is in Line 2, where the operation is division by 0. Division by zero is undefined in mathematics, making this step invalid and leading to the false conclusion.

Ex 3: Calculate the following expression without using a calculator:

$$2 + (120 - 45) \times (200 - 200) = \boxed{2}$$

Answer:

- Any number multiplied by 0 is 0. In the expression $2 + (120 - 45) \times (200 - 200)$, we notice that $(200 - 200) = 0$. Therefore, the expression becomes $2 + (120 - 45) \times 0$.
- Since $(120 - 45) \times 0 = 0$, the expression simplifies to $2 + 0$.
- Therefore, the final answer is 2.

Ex 4: Calculate the following expression without using a calculator:

$$(15 + 3 \times 5 - 30) \times (100 \times 11) = \boxed{0}$$

Answer:

- Any number multiplied by 0 is 0. In the expression $(15 + 3 \times 5 - 30) \times (100 \times 11)$, let's first simplify the part inside the first parentheses:

$$15 + 3 \times 5 - 30 = 15 + 15 - 30 = 30 - 30 = 0$$

- So, the expression becomes $0 \times (100 \times 11)$.
- Since $0 \times (100 \times 11) = 0$, the final answer is 0.

B DIVISION WITH REMAINDERS

B.1 CALCULATING THE DIVISION WITH REMAINDERS

Ex 5: Write the division with remainder of 21 by 5:

$$21 = 5 \times \boxed{4} + \boxed{1}$$

$$\begin{array}{r} 4 \\ 5\overline{)21} \\ \underline{20} \\ 1 \end{array}$$

Answer: As , $21 = 5 \times 4 + 1$

Ex 6: Write the division with remainder of 37 by 3:

$$37 = 3 \times \boxed{12} + \boxed{1}$$

$$\begin{array}{r} 12 \\ 3\overline{)37} \\ \underline{3} \\ 07 \\ \underline{6} \\ 1 \end{array}$$

Answer: As , $37 = 3 \times 12 + 1$

Ex 7: Write the division with remainder of 63 by 4:

$$63 = 4 \times \boxed{15} + \boxed{3}$$

$$\begin{array}{r} 15 \\ 4\overline{)63} \\ \underline{4} \\ 23 \\ \underline{20} \\ 3 \end{array}$$

Answer: As , $63 = 4 \times 15 + 3$

Ex 8: Write the division with remainder of 154 by 6:

$$154 = 6 \times \boxed{25} + \boxed{4}$$

$$\begin{array}{r} 25 \\ 6\overline{)154} \\ \underline{12} \\ 34 \\ \underline{30} \\ 4 \end{array}$$

Answer: As , $154 = 6 \times 25 + 4$

Ex 9: Write the division with remainder of 632 by 5:

$$632 = 5 \times \boxed{126} + \boxed{2}$$

$$\begin{array}{r} 126 \\ 5\overline{)632} \\ \underline{5} \\ 13 \\ \underline{10} \\ 32 \\ \underline{30} \\ 2 \end{array}$$

Answer: As , $632 = 5 \times 126 + 2$

B.2 SOLVING REAL-WORLD PROBLEMS

Ex 10: A farmer shares 243 eggs into boxes such that each box contains 6 eggs.
How many boxes are needed?

$\boxed{40}$ boxes

How many eggs remain without being placed in a box?

$\boxed{3}$ eggs

Answer:

$$\begin{array}{r} 40 \\ 6\overline{)243} \\ \underline{24} \\ 03 \end{array}$$

- As $243 = 6 \times 40 + 3$.
- The farmer needs 40 boxes.
- There are 3 eggs remaining.

Ex 11: A farmer's inheritance of 123 sheep is to be divided equally among 4 children.
How many sheep does each child receive?

$\boxed{30}$ sheep

How many sheep remain undistributed?

$\boxed{3}$ sheep

Answer:

$$\begin{array}{r} 30 \\ 4\overline{)123} \\ \underline{12} \\ 03 \end{array}$$

- As $123 = 4 \times 30 + 3$.
- Each child receives 30 sheep.
- There are 3 sheep remaining.

Ex 12: A gardener arranges 200 roses into bouquets such that each bouquet contains 12 roses.
How many bouquets are needed?

$\boxed{16}$ bouquets

How many roses remain without being placed in a bouquet?

$\boxed{8}$ roses

Answer:

$$\begin{array}{r} 16 \\ 12\overline{)200} \\ \underline{12} \\ 80 \\ \underline{72} \\ 8 \end{array}$$

- As $200 = 12 \times 16 + 8$.
- The gardener needs 16 bouquets.
- There are 8 roses remaining.

Ex 13: A child entering middle school decides to give his 300 marbles to his 7 cousins.
How many marbles does each cousin receive?

$\boxed{42}$ marbles

How many marbles remain undistributed?

$\boxed{6}$ marbles

Answer:

$$\begin{array}{r} 42 \\ 7\overline{)300} \\ \underline{28} \\ 20 \\ \underline{14} \\ 6 \end{array}$$

- As $300 = 7 \times 42 + 6$.
- Each cousin receives 42 marbles.
- There are 6 marbles remaining.

Ex 14: A coach organizes 37 soccer players into teams such that each team contains 5 players. The remaining players are substitutes.
How many full teams can be formed?

$\boxed{7}$ teams

How many players are substitutes?

$\boxed{2}$ players

Answer:

$$\begin{array}{r} 7 \\ 5\overline{)37} \\ \underline{35} \\ 2 \end{array}$$

- As $37 = 5 \times 7 + 2$.
- The coach can form 7 full teams.
- There are 2 substitute players.

C DIVISIBILITY

C.1 DETERMINING MULTIPLES

MCQ 15: Is 73 a multiple of 9?

☐ Yes

☒ No

Answer: As 73 does not appear on the multiplication table of 9,

$$\begin{aligned} 9 \times 1 &= 9 \\ 9 \times 2 &= 18 \\ 9 \times 3 &= 27 \\ 9 \times 4 &= 36 \\ 9 \times 5 &= 45 \\ 9 \times 6 &= 54 \\ 9 \times 7 &= 63 \\ 9 \times 8 &= 72 \\ 9 \times 9 &= 81 \\ 9 \times 10 &= 90 \end{aligned}$$

73 is not a multiple of 9.

MCQ 16: Is 77 a multiple of 11?

☒ Yes

☐ No

Answer: As 77 appears on the multiplication table of 11,

$$11 \times 1 = 11$$
$$11 \times 2 = 22$$
$$11 \times 3 = 33$$
$$11 \times 4 = 44$$
$$11 \times 5 = 55$$
$$11 \times 6 = 66$$
$$11 \times 7 = 77$$
$$11 \times 8 = 88$$
$$11 \times 9 = 99$$
$$11 \times 10 = 110$$

77 is a multiple of 11.

MCQ 17: Is 50 a multiple of 4?

☐ Yes

☒ No

Answer: As 50 does not appear on the multiplication table of 4,

$$4 \times 1 = 4$$
$$4 \times 2 = 8$$
$$4 \times 3 = 12$$
$$4 \times 4 = 16$$
$$4 \times 5 = 20$$
$$4 \times 6 = 24$$
$$4 \times 7 = 28$$
$$4 \times 8 = 32$$
$$4 \times 9 = 36$$
$$4 \times 10 = 40$$
$$4 \times 11 = 44$$
$$4 \times 12 = 48$$
$$4 \times 13 = 52$$
$$4 \times 14 = 56$$

50 is not a multiple of 4.

MCQ 18: Is 100 a multiple of 12?

☐ Yes

☒ No

Answer: As 100 does not appear on the multiplication table of 12,

$$12 \times 1 = 12$$
$$12 \times 2 = 24$$
$$12 \times 3 = 36$$
$$12 \times 4 = 48$$
$$12 \times 5 = 60$$
$$12 \times 6 = 72$$
$$12 \times 7 = 84$$
$$12 \times 8 = 96$$
$$12 \times 9 = 108$$
$$12 \times 10 = 120$$

100 is not a multiple of 12.

C.2 DETERMINING DIVISIBILITY

MCQ 19: Is 10 divisible by 5?

☒ Yes

☐ No

Answer: 10 is divisible by 5 because the remainder of the division

$$\begin{array}{r} 2 \\ 5\overline{)10} \\ \underline{10} \\ 0 \end{array}$$

is 0: .

MCQ 20: Is 82 divisible by 4?

☐ Yes

☒ No

Answer: 82 is not divisible by 4 because the remainder of the

$$\begin{array}{r} 20 \\ 4\overline{)82} \\ \underline{8} \\ 02 \end{array}$$

division is 2: .

MCQ 21: Is 72 divisible by 5?

☐ Yes

☒ No

Answer: 72 is not divisible by 5 because the remainder of the

$$\begin{array}{r} 14 \\ 5\overline{)72} \\ \underline{5} \\ 22 \\ \underline{20} \\ 2 \end{array}$$

division is 2: .

MCQ 22: Is 234 divisible by 3?

☒ Yes

☐ No

Answer: 234 is divisible by 3 because the remainder of the division

$$\begin{array}{r} 78 \\ 3\overline{)234} \\ \underline{21} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

is 0: .

C.3 DETERMINING FACTORS

MCQ 23: Is 10 a factor of 60?

☒ Yes

☐ No

Answer: 10 is a factor of 60 if 10 times another whole number equals 60.
As $10 \times 6 = 60$, 10 is a factor of 60.

MCQ 24: Which of the following numbers are factors of 64?
Choose all answers that apply:

☒ 2

☒ 4

☒ 8

☒ 32

Answer:

- 2 is a factor of 64 since $2 \times 32 = 64$.
- 4 is a factor of 64 since $4 \times 16 = 64$.
- 8 is a factor of 64 since $8 \times 8 = 64$.
- 32 is a factor of 64 since $32 \times 2 = 64$.

All these numbers are factors of 64.

MCQ 25: Which equation shows that 5 is a factor of 45?
Choose 1 answer:

☐ $45 = 5 + 40$

☐ $45 = 50 - 5$

☐ $45 = 225 \div 5$

☒ $45 = 5 \times 9$

Answer: 5 is a factor of 45 if 5 times another whole number equals 45.
Since $5 \times 9 = 45$, the correct equation is:

$$45 = 5 \times 9$$

MCQ 26: List all the factors of 6.
Choose 1 answer:

☐ $1, 2, 3, 4, 6$

☒ $1, 2, 3, 6$

☐ $1, 2, 3, 6, 12$

Answer: The factors of 6, determined by:

- $1 \times 6 = 6$
- $2 \times 3 = 6$
- $3 \times 2 = 6$
- $6 \times 1 = 6$

are 1, 2, 3, and 6.

MCQ 27: List all the factors of 24.
Choose 1 answer:

☒ $1, 2, 3, 4, 6, 8, 12, 24$

☐ $1, 2, 3, 4, 6, 8$

☐ $1, 2, 3, 4, 5, 6, 8, 12, 24$

Answer: The factors of 24 are determined by:

- $1 \times 24 = 24$
- $2 \times 12 = 24$
- $3 \times 8 = 24$
- $4 \times 6 = 24$
- $6 \times 4 = 24$
- $8 \times 3 = 24$
- $12 \times 2 = 24$
- $24 \times 1 = 24$

Thus, the factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

MCQ 28: List all the factors of 40.
Choose 1 answer:

☐ $1, 2, 4, 5, 8, 10, 40$

☐ $1, 2, 4, 5, 8, 10, 12, 20, 40$

☒ $1, 2, 4, 5, 8, 10, 20, 40$

Answer: The factors of 40 are determined by:

- $1 \times 40 = 40$
- $2 \times 20 = 40$
- $4 \times 10 = 40$
- $5 \times 8 = 40$
- $8 \times 5 = 40$
- $10 \times 4 = 40$
- $20 \times 2 = 40$
- $40 \times 1 = 40$

Thus, the factors of 40 are 1, 2, 4, 5, 8, 10, 20, and 40.

D DIVISIBILITY CRITERIA

D.1 DETERMINING DIVISIBILITY FOR 2 AND 5

MCQ 29: Is 98 divisible by 2?

☒ Yes

☐ No

Answer: To apply the rule for 2, we need to look at the last digit of the number and check whether it is even (0, 2, 4, 6, 8).
For 98, the last digit is 8, which is even, so 98 is divisible by 2.

MCQ 30: Is 315 divisible by 2?

☐ Yes

☒ No

Answer: For 315, the last digit is 5, which is odd, so 315 is not divisible by 2.

MCQ 31: Is 462 divisible by 2?

☒ Yes

☐ No

Answer: For 462, the last digit is 2, which is even, so 462 is divisible by 2.

MCQ 32: Is 799 divisible by 2?

☐ Yes

☒ No

Answer: For 799, the last digit is 9, which is odd, so 799 is not divisible by 2.

MCQ 33: Is 45 divisible by 5?

☒ Yes

☐ No

Answer: To apply the rule for 5, we need to look at the last digit and check whether it is either 0 or 5.
For 45, the last digit is 5, so 45 is divisible by 5.

MCQ 34: Is 80 divisible by 5?

☒ Yes

☐ No

Answer: For 80, the last digit is 0, so 80 is divisible by 5.

MCQ 35: Is 126 divisible by 5?

☐ Yes

☒ No

Answer: For 126, the last digit is 6, which is neither 0 nor 5, so 126 is not divisible by 5.

MCQ 36: Is 301 divisible by 5?

☐ Yes

☒ No

Answer: For 301, the last digit is 1, so 301 is not divisible by 5.

D.2 DETERMINING DIVISIBILITY FOR 3 AND 9

MCQ 37: Is 162 divisible by 3?

☒ Yes

☐ No

Answer: To apply the rule for 3, add the digits of the number and check if the sum is divisible by 3.
For 162, we have $1 + 6 + 2 = 9$, which is divisible by 3, so 162 is divisible by 3.

MCQ 38: Is 305 divisible by 3?

☐ Yes

☒ No

Answer: For 305, the sum of the digits is $3 + 0 + 5 = 8$, which is not divisible by 3, so 305 is not divisible by 3.

MCQ 39: Is 888 divisible by 3?

☒ Yes

☐ No

Answer: For 888, the sum of the digits is $8 + 8 + 8 = 24$, which is divisible by 3 (since $3 \times 8 = 24$), so 888 is divisible by 3.

MCQ 40: Is 504 divisible by 3?

☒ Yes

☐ No

Answer: For 504, the digits sum to $5 + 0 + 4 = 9$, which is divisible by 3, so 504 is divisible by 3.

MCQ 41: Is 126 divisible by 9?

☒ Yes

☐ No

Answer: To apply the rule for 9, add the digits of the number. For 126, we have $1 + 2 + 6 = 9$, which is divisible by 9, so 126 is divisible by 9.

MCQ 42: Is 235 divisible by 9?

☐ Yes

☒ No

Answer: For 235, the sum of the digits is $2 + 3 + 5 = 10$, which is not divisible by 9, so 235 is not divisible by 9.

MCQ 43: Is 369 divisible by 9?

☒ Yes

☐ No

Answer: For 369, the sum of the digits is $3 + 6 + 9 = 18$, which is divisible by 9, so 369 is divisible by 9.

MCQ 44: Is 441 divisible by 9?

☒ Yes

☐ No

Answer: For 441, the sum of the digits is $4 + 4 + 1 = 9$, which is divisible by 9, so 441 is divisible by 9.

D.3 DETERMINING DIVISIBILITY FOR 4

MCQ 45: Is 188 divisible by 4?

☒ Yes

☐ No

Answer: To apply the rule for 4, look at the last two digits of the number. For 188, the last two digits form 88, and since $88 = 4 \times 22$, 188 is divisible by 4.

MCQ 46: Is 373 divisible by 4?

☐ Yes

☒ No

Answer: For 373, the last two digits form 73, and since $73 = 4 \times 18 + 1$, 73 is not divisible by 4, so 373 is not divisible by 4.

MCQ 47: Is 412 divisible by 4?

☒ Yes

☐ No

Answer: For 412, the last two digits form 12, and since $12 = 4 \times 3$, 412 is divisible by 4.

MCQ 48: Is 256 divisible by 4?

☒ Yes

☐ No

Answer: For 256, the last two digits form 56, and since $56 = 4 \times 14$, 256 is divisible by 4.

MCQ 49: Is 179 divisible by 4?

☐ Yes

☒ No

Answer: For 179, the last two digits form 79, and since $79 = 4 \times 19 + 3$, 179 is not divisible by 4.

MCQ 50: Is 520 divisible by 4?

☒ Yes

☐ No

Answer: For 520, the last two digits form 20, and since $20 = 4 \times 5$, 520 is divisible by 4.

MCQ 51: Is 567 divisible by 4?

☐ Yes

☒ No

Answer: For 567, the last two digits form 67, and since $67 = 4 \times 16 + 3$, 567 is not divisible by 4.

E PRIME NUMBER

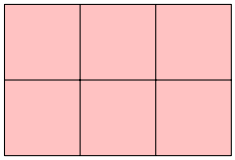
E.1 CHECKING IF PRIME

MCQ 52: State whether 6 is a prime number.

☐ Yes

☒ No

Answer:

- As $2 \times 3 = 6$, 6 is not a prime number because it is divisible by 2 and 3 (as well as 1 and 6). So 6 is a composite number.
- 
$3 \times 2 = 6$

MCQ 53: State whether 5 is a prime number.

☒ Yes

☐ No

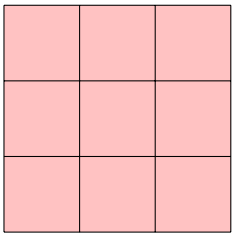
Answer: 5 is a prime number because it is divisible only by 1 and 5. We cannot divide it by 2, 3 or 4 (there would be a remainder).

MCQ 54: State whether 9 is a prime number.

☐ Yes

☒ No

Answer:

- 9 is not a prime number because $3 \times 3 = 9$. It is divisible by 3 (as well as 1 and 9).
- 
$3 \times 3 = 9$

MCQ 55: State whether 7 is a prime number.

☒ Yes

☐ No

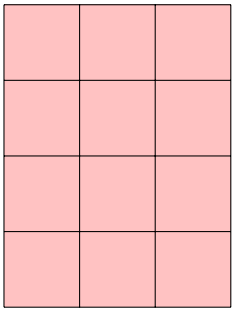
Answer: 7 is a prime number because it is divisible only by 1 and 7. We cannot divide it by 2, 3, 4, 5, or 6 (there would be a remainder).

MCQ 56: State whether 12 is a prime number.

☐ Yes

☒ No

Answer:

- 12 is not a prime number because $3 \times 4 = 12$. It is divisible by 3 and 4 (and by 2, 6 as well as 1 and 12).
- 
$3 \times 4 = 12$

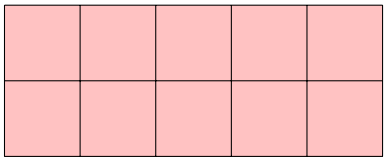
MCQ 57: State whether 10 is a prime number.

☐ Yes

☒ No

Answer:

- 10 is not a prime number because $2 \times 5 = 10$. It is divisible by 2 and 5 (and by 1 and 10).



$2 \times 5 = 10$

-

MCQ 58: State whether 13 is a prime number.

☒ Yes

☐ No

Answer: 13 is a prime number because it is divisible only by 1 and 13. We cannot divide it by $2, 3, 4, 5, 6, \ldots, 12$ (there would be a remainder).

MCQ 59: State whether 11 is a prime number.

☒ Yes

☐ No

Answer: 11 is a prime number because it is divisible only by 1 and 11. We cannot divide it by $2, 3, 4, 5, 6, 7, 8, 9$, or 10 (there would be a remainder).

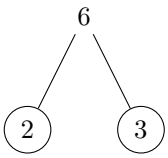
F PRIME FACTORIZATION

F.1 WRITING IN PRIME FACTORS: LEVEL 1

Ex 60: Write the number as a product of prime factors :

$$6 = \boxed{2 \times 3}$$

Answer:

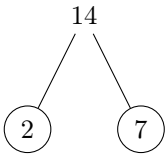


The prime factorization is $6 = 2 \times 3$ or $6 = 3 \times 2$.

Ex 61: Write the number as a product of prime factors :

$$14 = \boxed{2 \times 7}$$

Answer:

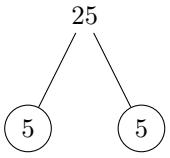


The prime factorization is $14 = 2 \times 7$ or $14 = 7 \times 2$.

Ex 62: Write the number as a product of prime factors :

$$25 = \boxed{5 \times 5}$$

Answer:

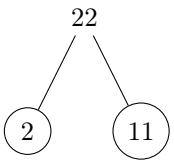


The prime factorization is $25 = 5 \times 5$.

Ex 63: Write the number as a product of prime factors :

$$22 = \boxed{2 \times 11}$$

Answer:



The prime factorization is $22 = 2 \times 11$ or $22 = 11 \times 2$.

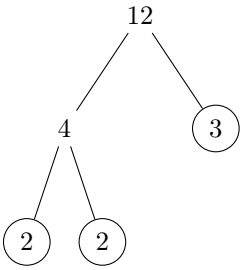
F.2 WRITING IN PRIME FACTORS: LEVEL 2

Ex 64: Write the number as a product of prime factors :

$$12 = \boxed{2 \times 2 \times 3}$$

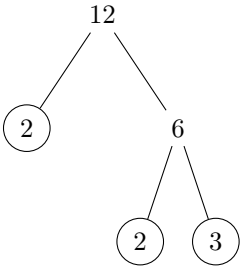
Answer:

- Solution 1:



A prime factorization is $12 = 2 \times 2 \times 3$.

- Solution 2:



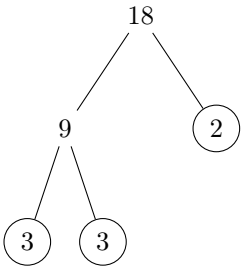
A prime factorization is $12 = 2 \times 2 \times 3$.

Ex 65: Write the number as a product of prime factors :

$$18 = \boxed{2 \times 3 \times 3}$$

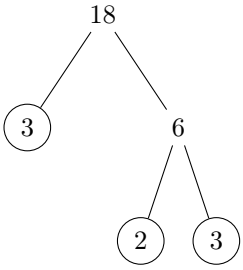
Answer:

- Solution 1:



A prime factorization is $18 = 3 \times 3 \times 2$.

- Solution 2:



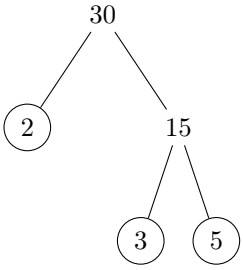
A prime factorization is $18 = 3 \times 2 \times 3$.

Ex 66: Write the number as a product of prime factors :

$$30 = \boxed{2 \times 3 \times 5}$$

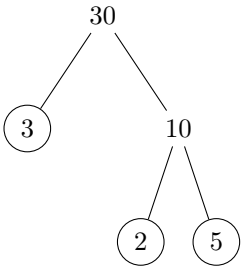
Answer:

- Solution 1:



A prime factorization is $30 = 2 \times 3 \times 5$.

- Solution 2:



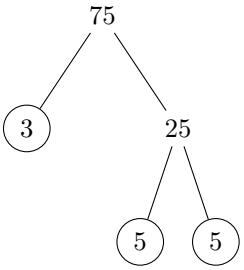
A prime factorization is $30 = 3 \times 2 \times 5$.

Ex 67: Write the number as a product of prime factors :

$$75 = \boxed{3 \times 5 \times 5}$$

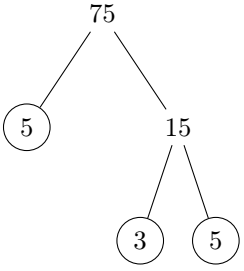
Answer:

- Solution 1:



A prime factorization is $75 = 3 \times 5 \times 5$.

- Solution 2:



A prime factorization is $75 = 5 \times 3 \times 5$.

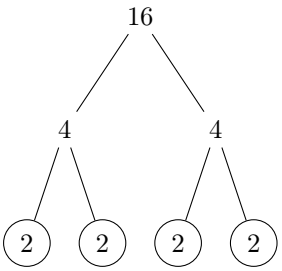
F.3 WRITING IN PRIME FACTORS: LEVEL 3

Ex 68: Write the number as a product of prime factors :

$$16 = \boxed{2 \times 2 \times 2 \times 2}$$

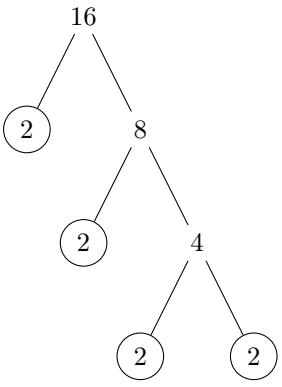
Answer:

- Solution 1:



A prime factorization is $16 = 2 \times 2 \times 2 \times 2$.

- Solution 2:



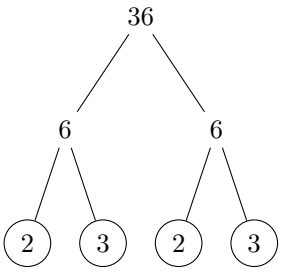
A prime factorization is $16 = 2 \times 2 \times 2 \times 2$.

Ex 69: Write the number as a product of prime factors :

$$36 = \boxed{2 \times 2 \times 3 \times 3}$$

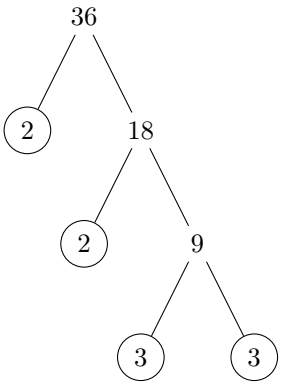
Answer:

- Solution 1:



A prime factorization is $36 = 2 \times 2 \times 3 \times 3$.

- Solution 2:



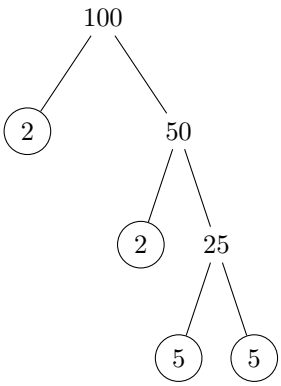
A prime factorization is $36 = 2 \times 2 \times 3 \times 3$.

Ex 70: Write the number as a product of prime factors :

$$100 = \boxed{2 \times 2 \times 5 \times 5}$$

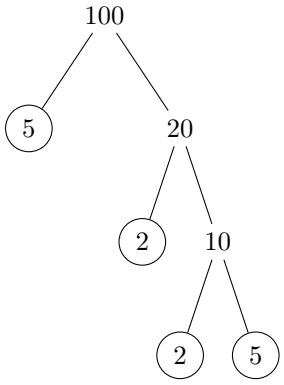
Answer:

- Solution 1:



A prime factorization is $100 = 2 \times 2 \times 5 \times 5$.

- Solution 2:



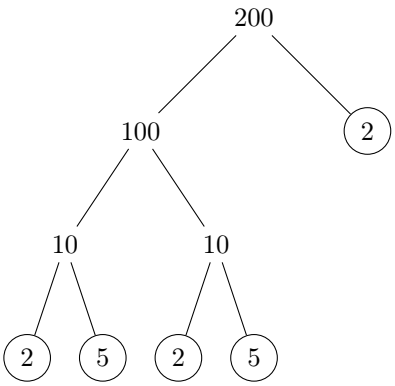
A prime factorization is $100 = 5 \times 2 \times 2 \times 5$.

Ex 71: Write the number as a product of prime factors :

$$200 = \boxed{2 \times 2 \times 2 \times 5 \times 5}$$

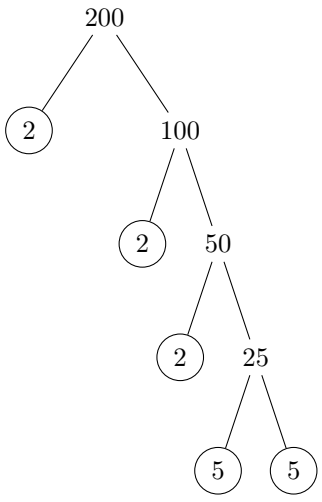
Answer:

- Solution 1:



A prime factorization is $200 = 2 \times 5 \times 2 \times 5 \times 2$.

- Solution 2:



A prime factorization is $200 = 2 \times 2 \times 2 \times 5 \times 5$.

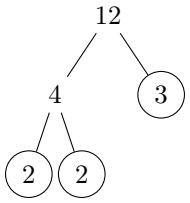
G GREATEST COMMON DIVISOR (GCD)

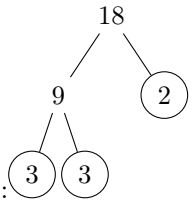
G.1 FINDING GCD

Ex 72: Find the GCD of 12 and 18:

$$\gcd(12, 18) = \boxed{6}$$

Answer:

- $12 = \boxed{2} \times 2 \times \boxed{3}$: 

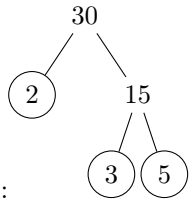
- $18 = \boxed{2} \times \boxed{3} \times 3$: 

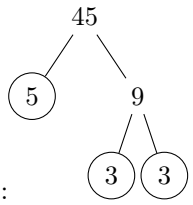
- Common prime factors: $\boxed{2}$ and $\boxed{3}$
- So, $\gcd(12, 18) = 2 \times 3 = 6$

Ex 73: Find the GCD of 30 and 45:

$$\gcd(30, 45) = \boxed{15}$$

Answer:

- $30 = 2 \times \boxed{3} \times \boxed{5}$: 

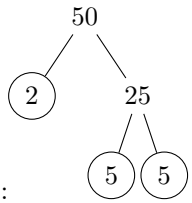
- $45 = 3 \times \boxed{3} \times \boxed{5}$: 

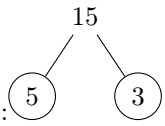
- Common prime factors: $\boxed{3}$ and $\boxed{5}$
- So, $\gcd(30, 45) = 3 \times 5 = 15$

Ex 74: Find the GCD of 50 and 15.

$$\gcd(50, 15) = \boxed{5}$$

Answer:

- $50 = 2 \times \boxed{5} \times 5$: 

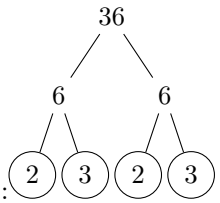
- $15 = 3 \times \boxed{5}$: 

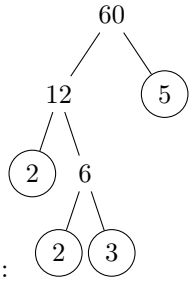
- Common prime factor: $\boxed{5}$
- So, $\gcd(50, 15) = 5$

Ex 75: Find the GCD of 36 and 60:

$$\gcd(36, 60) = \boxed{12}$$

Answer:

- $36 = \boxed{2} \times \boxed{2} \times \boxed{3} \times 3$: 

- $60 = \boxed{2} \times \boxed{2} \times \boxed{3} \times 5$: 

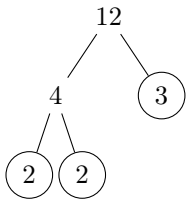
- Common prime factors: $\boxed{2}$, $\boxed{2}$ and $\boxed{3}$
- So, $\gcd(36, 60) = 2 \times 2 \times 3 = 12$

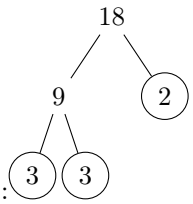
G.2 EXPRESSING FRACTIONS IN SIMPLEST FORM

Ex 76: Simplify:

$$\frac{12}{18} = \frac{\boxed{2}}{\boxed{3}}$$

Answer:

- $12 = \boxed{2} \times 2 \times \boxed{3}$: 

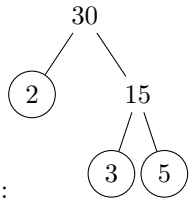
- $18 = \boxed{2} \times \boxed{3} \times 3$: 

- $$\frac{12}{18} = \frac{\boxed{2} \times 2 \times \boxed{3}}{\boxed{2} \times \boxed{3} \times 3} \quad \text{(simplifying by GCD)}$$
$$= \frac{2}{3}$$

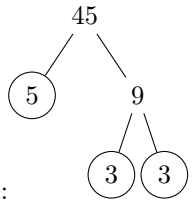
Ex 77: Simplify:

$$\frac{30}{45} = \frac{\boxed{2}}{\boxed{3}}$$

Answer:



- $30 = 2 \times \boxed{3} \times \boxed{5}$:

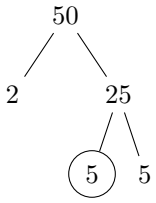


- $45 = 3 \times \boxed{3} \times \boxed{5}$:
- $$\frac{30}{45} = \frac{2 \times \boxed{3} \times \boxed{5}}{3 \times \boxed{3} \times \boxed{5}} \quad \text{(simplifying by gcd)}$$
$$= \frac{2}{3}$$

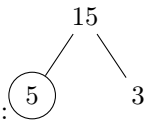
Ex 78: Simplify:

$$\frac{50}{15} = \frac{\boxed{10}}{\boxed{3}}$$

Answer:



- $50 = 2 \times \boxed{5} \times 5$:

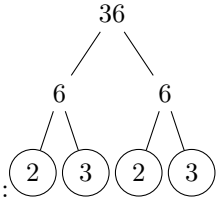


- $15 = 3 \times \boxed{5}$:
- $$\frac{50}{15} = \frac{2 \times \boxed{5} \times 5}{3 \times \boxed{5}} \quad \text{(simplifying by gcd)}$$
$$= \frac{2 \times 5}{3} = \frac{10}{3}$$

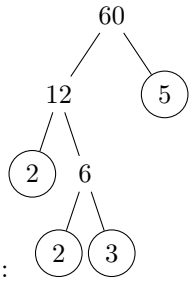
Ex 79: Simplify:

$$\frac{36}{60} = \frac{\boxed{3}}{\boxed{5}}$$

Answer:



- $36 = \boxed{2} \times \boxed{2} \times \boxed{3} \times 3$:



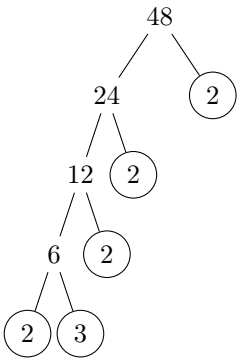
- $60 = \boxed{2} \times \boxed{2} \times \boxed{3} \times 5$:
- $$\frac{36}{60} = \frac{\boxed{2} \times \boxed{2} \times \boxed{3} \times 3}{\boxed{2} \times \boxed{2} \times \boxed{3} \times 5} \quad \text{(simplifying by gcd)}$$
$$= \frac{3}{5}$$

G.3 REAL-LIFE PROBLEMS WITH THE GREATEST COMMON DIVISOR (GCD)

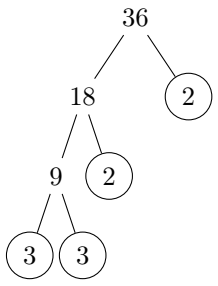
Ex 80: A baker has 48 chocolate muffins and 36 blueberry muffins. She wants to arrange them in boxes so that each box has the same number of each type of muffin, and all muffins are used. What is the largest number of boxes she can prepare?

$\boxed{12}$ boxes

Answer: The problem asks for the greatest number of identical boxes, which means we need the greatest common divisor (gcd) of 48 and 36.



- $48 = 2 \times 2 \times 2 \times 2 \times 3$

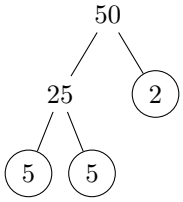


- $36 = 2 \times 2 \times 3 \times 3$
- Common prime factors: $2 \times 2 \times 3 = 12$
- So, $\gcd(48, 36) = 12$
- Therefore, the baker can make $\boxed{12}$ boxes.
- Each box will have $\frac{48}{12} = 4$ chocolate muffins and $\frac{36}{12} = 3$ blueberry muffins.

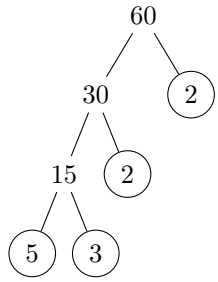
Ex 81: Maria and Jamal are organizing a community garden and need to divide their rectangular plot into square sections for planting. The garden measures 50 meters by 60 meters. They want the squares to be as large as possible while ensuring the entire area is used without any leftover space. What should be the side length of each square section?

$\boxed{10}$ meters

Answer: To find the largest possible square side, we need the greatest common divisor (gcd) of 50 and 60.



- $50 = 2 \times 5 \times 5$

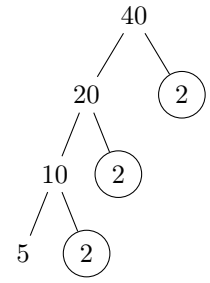


- $60 = 2 \times 2 \times 3 \times 5$
- Common prime factors: $2 \times 5 = 10$
- So, $\gcd(50, 60) = 10$
- The side of each square should be $\boxed{10}$ meters.
- There will be $50 \div 10 = 5$ squares along one side and $60 \div 10 = 6$ squares along the other, for a total of $5 \times 6 = 30$ squares.

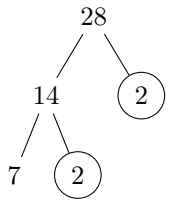
Ex 82: A gym teacher has 40 basketballs and 28 soccer balls. She wants to divide them into sports kits so that each kit has the same number of each type of ball, and all balls are used. What is the largest number of kits she can prepare?

$\boxed{4}$ kits

Answer: The problem asks for the greatest number of identical kits, which means we need the greatest common divisor (gcd) of 40 and 28.



- $40 = \boxed{2} \times \boxed{2} \times 2 \times 5$



- $28 = \boxed{2} \times \boxed{2} \times 7$
- Common prime factors: $2 \times 2 = 4$
- So, $\gcd(40, 28) = 4$
- Therefore, the teacher can make $\boxed{4}$ kits.
- Each kit will have $\frac{40}{4} = 10$ basketballs and $\frac{28}{4} = 7$ soccer balls.

H LEAST COMMON MULTIPLE (LCM)

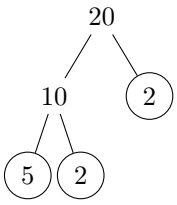
H.1 FINDING LCM

Ex 83: Find the LCM of 20 and 12.

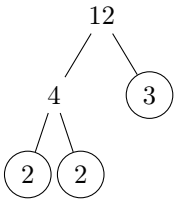
$$LCM(20, 12) = \boxed{60}$$

Answer:

- **Multiple Table Method:**
  - The multiples of 20 are: 20, 40, 60, 80, 100, ...
  - The multiples of 12 are: 12, 24, 36, 48, 60, 72, ...
  - The least common multiple is 60.
- **Prime Factorization Method:**
  - $20 = 2^2 \times 5$:



  - $12 = 2^2 \times 3$:



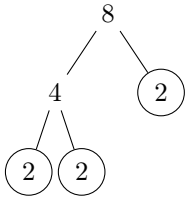
  - Take the highest powers: 2^2, 3^1, 5^1
  - $LCM(20, 12) = 2^2 \times 3^1 \times 5^1 = 4 \times 3 \times 5 = 60$

Ex 84: Find the LCM of 8 and 12.

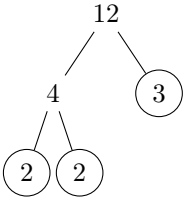
$$LCM(8, 12) = \boxed{24}$$

Answer:

- **Multiple Table Method:**
  - The multiples of 8 are: $8, 16, 24, 32, 40, \ldots$
  - The multiples of 12 are: $12, 24, 36, 48, \ldots$
  - The least common multiple is 24.
- **Prime Factorization Method:**
  - $8 = 2^3$:



  - $12 = 2^2 \times 3^1$:



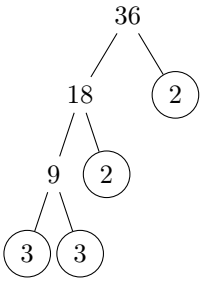
- Take the highest powers: 2^3 and 3^1
- $LCM(8, 12) = 2^3 \times 3^1 = 8 \times 3 = 24$

Ex 85: Find the LCM of 36 and 30.

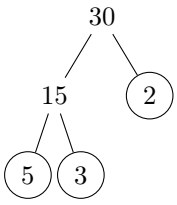
$$LCM(36, 30) = \boxed{180}$$

Answer:

- **Multiple Table Method:**
  - The multiples of 36 are: 36, 72, 108, 144, 180, 216, ...
  - The multiples of 30 are: 30, 60, 90, 120, 150, 180, 210, ...
  - The least common multiple is 180.
- **Prime Factorization Method:**
  - $36 = 2^2 \times 3^2$:



  - $30 = 2^1 \times 3^1 \times 5^1$:



  - Take the highest powers: 2^2, 3^2, 5^1
  - $LCM(36, 30) = 2^2 \times 3^2 \times 5^1 = 4 \times 9 \times 5 = 180$

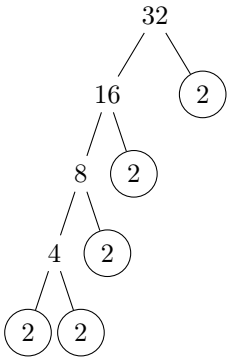
H.2 REAL-LIFE PROBLEMS WITH THE LEAST COMMON MULTIPLE (LCM)

Ex 86: A lighthouse emits a white light every 32 seconds and a green light every 24 seconds. At midnight, both lights flash together. After how many seconds will both lights flash together again?

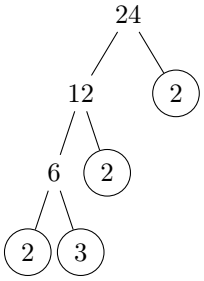
$\boxed{96}$ seconds

Answer: We need to find the **least common multiple (LCM)** of 32 and 24 to know when both lights will flash together again.

- **Multiple Table Method:**
  - Multiples of 32 : 32, 64, 96, 128, ...
  - Multiples of 24 : 24, 48, 72, 96, 120, ...
  - The least common multiple is 96.
- **Prime Factorization Method:**
  - $32 = 2^5$:



  - $24 = 2^3 \times 3$:



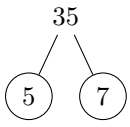
  - Take the highest powers: 2^5 and 3^1
  - $LCM(32, 24) = 2^5 \times 3^1 = 32 \times 3 = 96$
- So, both lights will flash together every $\boxed{96}$ seconds.

Ex 87: A ferry departs every 35 minutes from Port A and every 10 minutes from Port B. If both ferries leave their respective ports together at 6:00 AM, after how many minutes will they next leave at the same time?

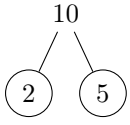
$\boxed{70}$ minutes

Answer: We need to find the **least common multiple (LCM)** of 35 and 10 to determine after how many minutes both ferries will next depart together.

- **Multiple Table Method:**
  - Multiples of 35 : 35, 70, 105, 140, ...
  - Multiples of 10 : 10, 20, 30, 40, 50, 60, 70, 80, ...
  - The least common multiple is 70.
- **Prime Factorization Method:**
  - $35 = 5 \times 7$:



  - $10 = 2 \times 5$:



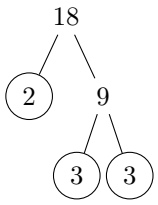
  - Take the highest powers: 2^1, 5^1, 7^1
  - $LCM(35, 10) = 2 \times 5 \times 7 = 70$
- So, both ferries will next depart together every $\boxed{70}$ minutes.

Ex 88: Two digital billboards in a city display their special animations at different intervals: one every 18 minutes, the other every 30 minutes. If both animations start at the same time at 9:00 AM, after how many minutes will both billboards display their animations together again?

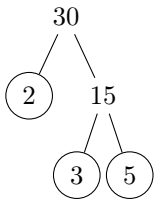
$\boxed{90}$ minutes

Answer: We need to find the **least common multiple (LCM)** of 18 and 30 to know after how many minutes both billboards will display their animation together again.

- **Multiple Table Method:**
  - Multiples of 18 : 18, 36, 54, 72, 90, 108, ...
  - Multiples of 30 : 30, 60, 90, 120, ...
  - The least common multiple is 90.
- **Prime Factorization Method:**
  - $18 = 2 \times 3 \times 3$:



  - $30 = 2 \times 3 \times 5$:



  - Take the highest powers: $2^1,\ 3^2,\ 5^1$
  - $LCM(18, 30) = 2^1 \times 3^2 \times 5^1 = 2 \times 9 \times 5 = 90$
- So, both billboards will display their special animation together every $\boxed{90}$ minutes.