PYTHAGOREAN THEOREM

One of the oldest and most famous mathematical concepts is the Pythagorean theorem. Named after the ancient Greek mathematician Pythagoras, this theorem establishes a fundamental relationship between the sides of a right-angled triangle.

A RIGHT-ANGLED TRIANGLE

Definition Right-Angled Triangle

A **right-angled triangle** is a triangle with one right angle. The two sides forming this right angle are called the **legs**, and the longest side, opposite the right angle, is called the **hypotenuse**.



B PYTHAGOREAN THEOREM

Discover:

1. Draw two right-angled triangles, similar to the one shown below:



2. Measure the sides of these right-angled triangles to the nearest millimeter and complete the table:

	a	b	с	a^2	b^2	c^2
Triangle 1						
Triangle 2						

- 3. Can you identify any relationship between the values in the columns for a^2 , b^2 , and c^2 ?
- 4. Can you express this relationship as an equation?

Answer:

1. You have drawn two right-angled triangles. For example:



2. Here are example measurements of the sides and their squares:

	a	b	c	a^2	b^2	c^2
Triangle 1	4	3	5	16	9	25
Triangle 2	6	2.5	6.5	36	6.25	42.25

3. From the table, the sum of the squares of the legs $(a^2 + b^2)$ equals the square of the hypotenuse (c^2) for both triangles:

$$16 + 9 = 25$$

 $36 + 6.25 = 42.25$

4. This relationship is expressed as the equation $a^2 + b^2 = c^2$, known as the Pythagorean theorem, which states that the sum of the squares of the legs equals the square of the hypotenuse in a right-angled triangle.



Ex:



Proof

- The geometric proof involves rearranging four identical right-angled triangles within a square of side length a + b.
 - 1. In the first arrangement, the four right-angled triangles form a smaller square in the center of the larger square:



2. In the second arrangement, the four right-angled triangles form two smaller rectangles within the larger square:



3. Since the area not covered by the triangles is equal in both arrangements, we have:

$$a^2 + b^2 = c^2$$

• Algebraic proof: Using four copies of the same triangle arranged symmetrically around a square with side c:



The larger square has side a + b, with area $(a + b)^2$. The four triangles and the central square of side c have the same area as the larger square:

$$(a+b)^2 = \underbrace{central square area}_{c^2} + 4 \times \underbrace{\frac{ab}{2}}_{c^2}$$
$$a^2 + 2ab + b^2 = c^2 + 2ab$$
$$a^2 + b^2 = c^2$$

Ex: Find the length of the hypotenuse:



Answer:

 $x^2 = 3^2 + 5^2$ (Pythagorean theorem) $x^2 = 9 + 25$ $x^2 = 34$ $x = \sqrt{34}$ (since the length of a triangle side is positive)

Thus, the hypotenuse has length $\sqrt{34}$ cm.

C VERIFYING RIGHT-ANGLED TRIANGLES

. Theorem Converse of the Pythagorean Theorem -

For any triangle with sides of lengths a, b, and c, if $a^2 + b^2 = c^2$, then the triangle is right-angled.

Ex: Is a triangle with sides of lengths 3, 4, and 5 right-angled?

Answer: The two shorter sides are 3 and 4:

$$3^2 + 4^2 = 9 + 16 = 25$$

 $5^2 = 25$

Since $3^2 + 4^2 = 5^2$, the triangle is right-angled by the converse of the Pythagorean theorem.

. Theorem Contrapositive of the Pythagorean Theorem .

For any triangle with sides of lengths a, b, and c, where $c \ge a$ and $c \ge b$, if $a^2 + b^2 \ne c^2$, then the triangle is not right-angled.

Proof

This is the contrapositive of the Pythagorean theorem.

Ex: Is a triangle with sides of lengths 5, 8, and 9 right-angled?

Answer: The two shorter sides are 5 and 8, and the longest side is 9:

$$5^2 + 8^2 = 25 + 64 = 89$$
$$9^2 = 81$$

Since $5^2 + 8^2 \neq 9^2$, the triangle is not right-angled by the contrapositive of the Pythagorean theorem.

