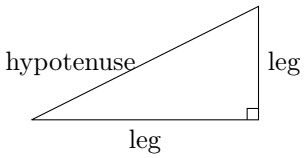
PYTHAGOREAN THEOREM

One of the oldest and most famous mathematical concepts is the Pythagorean theorem. Named after the ancient Greek mathematician Pythagoras, this theorem establishes a fundamental relationship between the sides of a right-angled triangle.

A RIGHT-ANGLED TRIANGLE

Definition Right-Angled Triangle

A **right-angled triangle** is a triangle with one right angle. The two sides forming this right angle are called the **legs**, and the longest side, opposite the right angle, is called the **hypotenuse**.

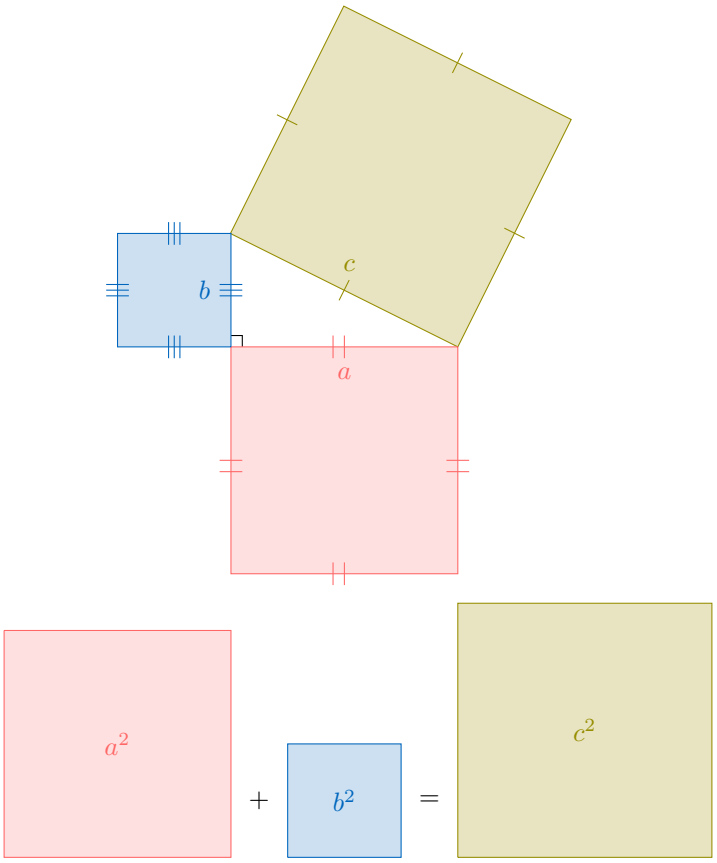


B PYTHAGOREAN THEOREM

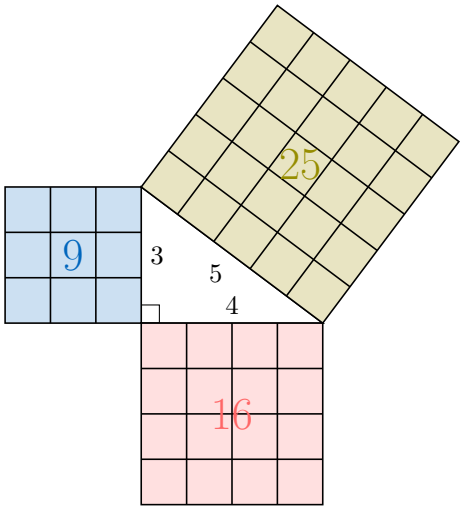
Theorem Pythagorean Theorem

For any right-angled triangle with legs a and b and hypotenuse c, the following holds:

$$a^2 + b^2 = c^2$$

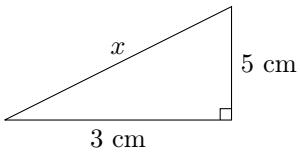


Ex:



$$4^2 + 3^2 = 5^2$$
$$16 + 9 = 25$$

Ex: Find the length of the hypotenuse:



Answer:

$$x^2 = 3^2 + 5^2 \quad \text{(Pythagorean theorem)}$$
$$x^2 = 9 + 25$$
$$x^2 = 34$$
$$x = \sqrt{34} \quad \text{(since the length of a triangle side is positive)}$$

Thus, the hypotenuse has length $\sqrt{34}$ cm.

C VERIFYING RIGHT-ANGLED TRIANGLES

Theorem Converse of the Pythagorean Theorem

For any triangle with sides of lengths a, b, and c, if $a^2 + b^2 = c^2$, then the triangle is right-angled.

Ex: Is a triangle with sides of lengths 3, 4, and 5 right-angled?

Answer: The two shorter sides are 3 and 4:

$$3^2 + 4^2 = 9 + 16 = 25$$
$$5^2 = 25$$

Since $3^2 + 4^2 = 5^2$, the triangle is right-angled by the converse of the Pythagorean theorem.

Theorem Contrapositive of the Pythagorean Theorem

For any triangle with sides of lengths a, b, and c, where $c \geq a$ and $c \geq b$, if $a^2 + b^2 \neq c^2$, then the triangle is not right-angled.

Ex: Is a triangle with sides of lengths 5, 8, and 9 right-angled?

Answer: The two shorter sides are 5 and 8, and the longest side is 9:

$$5^2 + 8^2 = 25 + 64 = 89$$
$$9^2 = 81$$

Since $5^2 + 8^2 \neq 9^2$, the triangle is not right-angled by the contrapositive of the Pythagorean theorem.