QUADRATIC FUNCTIONS

A DEFINITION

Definition Quadratic Function

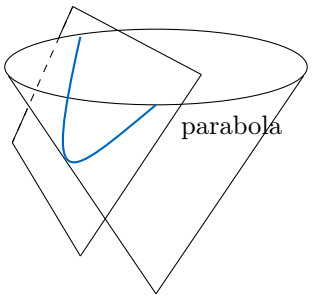
A **quadratic function** is $x \mapsto ax^2 + bx + c$ where $a \neq 0$.

Ex: For $f(x) = x^2 - 3x + 1$, evaluate $f(2)$.

Answer: $f(2) = (2)^2 - 3(2) + 1$
$= 4 - 6 + 1$
$= -1$

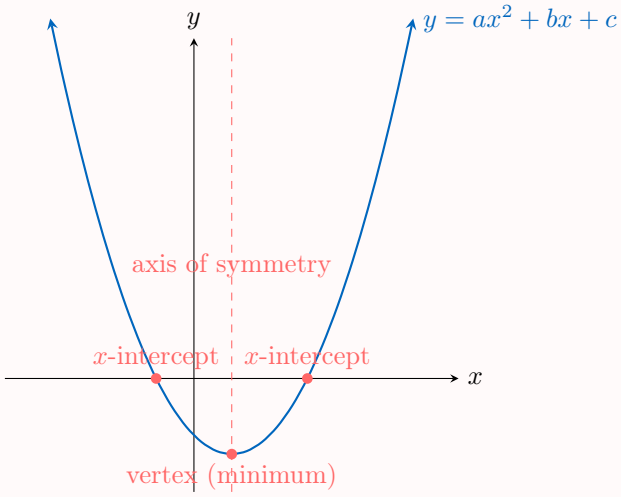
B GRAPH

The parabola is one of the conic sections, which are the group of curves obtained by intersecting a cone with a plane. A parabola is produced by intersecting the cone with a plane parallel to its generating line. By intersecting the cone at other angles, we can produce circles, hyperbolas, and ellipses.



Definition Parabola

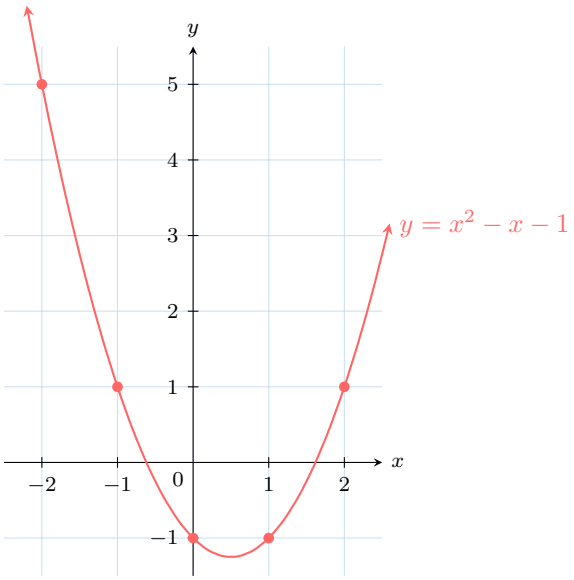
Given a quadratic function $x \mapsto ax^2 + bx + c$ where $a \neq 0$, its graph is called a **parabola**.



Ex: Sketch the graph of $x \mapsto x^2 - x - 1$.

Answer: A table of values is

x	-2	-1	0	1	2
y	5	1	-1	-1	1



To understand the concavity based on the sign of a, use the GeoGebra animation at https://www.geogebra.org/m/gn3c2sqe.

Proposition Concavity

For any quadratic function $x \mapsto ax^2 + bx + c$, $a \neq 0$:

- If $a > 0$, the graph is concave up: ∪.
- If $a < 0$, the graph is concave down: ∩.

C SOLVING $f(x) = y$

Method Solving $f(x) = y$

When solving for a value of $f(x) = y$, we obtain a quadratic equation in x. Since it is quadratic, there may be 0, 1, or 2 real solutions for x.

Ex: For $f(x) = 2x^2 - 5x + 2$, find the x-intercepts of the function.

Answer: Set $f(x) = 0$: $2x^2 - 5x + 2 = 0$, with $a = 2$, $b = -5$, $c = 2$.

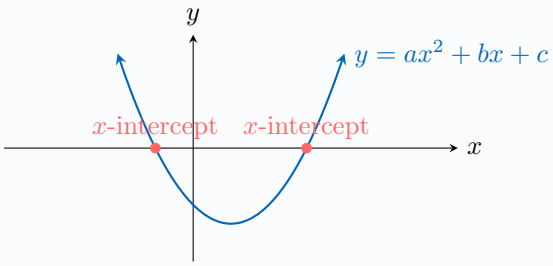
- $\Delta = b^2 - 4ac$
  $= (-5)^2 - 4(2)(2)$
  $= 25 - 16$
  $= 9$
- As $\Delta > 0$, there are 2 distinct roots.
- $x = \dfrac{-b - \sqrt{\Delta}}{2a}$ or $x = \dfrac{-b + \sqrt{\Delta}}{2a}$
  $x = \dfrac{-(-5) - \sqrt{9}}{2 \cdot 2}$ or $x = \dfrac{-(-5) + \sqrt{9}}{2 \cdot 2}$
  $x = \dfrac{5 - 3}{4}$ or $x = \dfrac{5 + 3}{4}$
  $x = \dfrac{2}{4}$ or $x = \dfrac{8}{4}$
  $x = \dfrac{1}{2}$ or $x = 2$

The x-intercepts are at $x = \frac{1}{2}$ and $x = 2$.

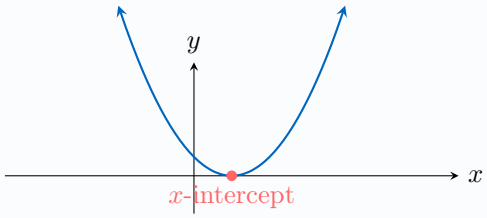
Proposition **Relative position between the graph and the x-axis**

For any quadratic function $x \mapsto ax^2 + bx + c$ and the discriminant $\Delta = b^2 - 4ac$:

- If $\Delta > 0$, then the graph intersects the x-axis twice.



- If $\Delta = 0$, then the graph touches the x-axis at one point.



- If $\Delta < 0$, then the graph does not intersect the x-axis.