# RATIOS

# A DEFINITION

# A.1 EXPRESSING RATIOS IN DIFFERENT FORMS

**Ex 1:** The ratio  $\frac{3}{5}$  to  $\frac{2}{5}$  is  $\frac{3}{2}$ .

Answer: The ratio 3 to 2 can be expressed as 3:2 or  $\frac{3}{2}$ .

**Ex 2:** The ratio 5 to 4 is 5: 4.

Answer: The ratio 5 to 4 can be expressed as 5:4 or  $\frac{5}{4}$ .

- **Ex 3:** The ratio 7 to 3 is 7:3.
- Answer: The ratio 7 to 3 can be expressed as 7:3 or  $\frac{7}{3}$ .
- **Ex 4:** The ratio 8 to 5 is 8:5.
- Answer: The ratio 8 to 5 can be expressed as 8:5 or  $\frac{8}{5}$ .
- **Ex 5:** The ratio 10 to 6 is 10: 6.

Answer: The ratio 10 to 6 can be expressed as 10:6 or  $\frac{10}{6}$ .

# **B PART-PART AND PART-WHOLE RATIOS**

# **B.1 FINDING RATIOS IN PART-PART**

**Ex 6:** What is the ratio of girls to boys?



Answer:

- There are 2 girls.
- There are 3 boys.
- The ratio of girls to boys is 2:3 or  $\frac{2}{3}$ .

**Ex 7:** What is the ratio of circles to rectangles?



Answer:

- There are 4 circles.
- There are 2 rectangles.

- The ratio of circles to rectangles is 4:2 or  $\frac{4}{2}$ .
- **Ex 8:** What is the ratio of squares to triangles?



Answer:

- There are 3 squares.
- There are 3 triangles.
- The ratio of squares to triangles is 3:3 or  $\frac{3}{3}$ .
- $\mathbf{Ex}~\mathbf{9:}$  What is the ratio of oranges to lemons?



Answer:

- There are 4 oranges.
- There are 3 lemons.
- The ratio of oranges to lemons is  $\frac{4}{3}$ , or 4:3.

**Ex 10:** What is the ratio of girls to boys?



Answer:

- There are 2 girls.
- There are 4 boys.
- The ratio of girls to boys is  $\frac{2}{4}$ , or 2 : 4.

#### **B.2 FINDING RATIOS IN PART-WHOLE**

**Ex 11:** What is the ratio of girls to kids?



Answer:

- There are 2 girls.
- There are 5 kids.
- The ratio of girls to kids is 2:5 or  $\frac{2}{\xi}$ .

**Ex 12:** What is the ratio of boys to kids?



Answer:

- There is 1 boy.
- There are 4 kids.
- The ratio of boys to kids is 1:4 or  $\frac{1}{4}$ .

**Ex 13:** Louis loves to play sports. In all, he has earned 5 swimming medals, 3 running medals, 6 cycling medals, and 2 triathlon medals.

What is the ratio of Louis's swimming medals to all of his medals?

5:16

Answer:

• Louis has earned 5 swimming medals.

•



Louis has earned 16 total medals.

• The ratio of swimming medals to all of his medals is 5:16 or  $\frac{5}{16}$ .

**Ex 14:** Anna loves to read books. In all, she has read 12 mystery novels, 8 science fiction novels, 5 fantasy novels, and 3 historical novels.

What is the ratio of Anna's mystery novels to all of her books?

12:28

Answer:

• Anna has read 12 mystery novels.

Anna has read 28 books in total.

• The ratio of mystery novels to all of her books is 12:28 or  $\frac{12}{28}$ .

12 + 8 + 5 + 3 = 28

**Ex 15:** The table shows the number of different types of birds that are swimming at a lake.

Bird	Number
Seagulls	1
Ducks	9
Geese	7
Swans	2

What is the ratio of swans to total birds?

2:19

Answer:

- There are 2 swans.
- There are 1 + 9 + 7 + 2 = 19 birds.
- The ratio of swans to total birds is 2:19 or  $\frac{2}{10}$ .

**Ex 16:** The table shows the number of different types of fruits in a basket.

Fruit	Number
Apples	3
Oranges	5
Bananas	4
Grapes	6

What is the ratio of apples to total fruits?

3:18

Answer:

- There are 3 apples.
- There are 3 + 5 + 4 + 6 = 18 fruits.
- The ratio of apples to total fruits is 3:18 or  $\frac{3}{18}$ .

**Ex 17:** The table shows the number of different types of vehicles in a parking lot.

Vehicle	Number
Cars	10
Bicycles	6
Motorcycles	4
Trucks	2

What is the ratio of trucks to total vehicles?

2 | 22

• There are 2 trucks.

Answer:

- There are 10 + 6 + 4 + 2 = 22 vehicles.
- The ratio of trucks to total vehicles is 2:22 or  $\frac{2}{22}$ .



# C EQUAL RATIOS

## C.1 MULTIPLYING THE RATIOS

**Ex 18:** Multiply the ratio by 2:

$$3:5=6:10$$

Answer:

• 
$$\frac{3}{5} = \frac{6}{10}$$
$$\frac{3}{5} = \frac{3 \times 2}{5 \times 2}$$
$$= \frac{6}{10}$$

• 
$$3:5=6:10.$$

### **Ex 19:** Multiply the ratio by 3:

$$4:7 = 12:21$$

Answer:

• 
$$\frac{4}{7} = \frac{12}{21}$$

$$\frac{4}{7} = \frac{4 \times 3}{7 \times 3}$$

$$= \frac{12}{21}$$

• 
$$4:7=12:21$$
.

**Ex 20:** Multiply the ratio by 4:

$$5:3 = 20:12$$

Answer:

• 
$$\frac{5}{3} = \frac{20}{12}$$
$$\frac{5}{3} = \frac{5 \times 4}{3 \times 4}$$
$$= \frac{20}{12}$$

• 5:3=20:12.

### **Ex 21:** Multiply the ratio by 5:

$$: 5 = 10 : 25$$

2

Answer:

•  $\frac{2}{5} = \frac{10}{25}$  $\frac{2}{5} = \frac{2 \times 5}{5 \times 5}$  $= \frac{10}{25}$ 

• 2:5=10:25.

# C.2 FINDING THE MISSING VALUE

Ex 22:

Answer:



1:2=2:4

Ex 23:

Answer:

• 1:2=2:4.

2:3 = 4:6



Ex 24:

• 2:3=4:6.

3:5=9:15

Answer:



www.commeunjeu.com

• 
$$3:5=9:15.$$

Ex 25:

$$4:7 = 8:14$$

Answer:



$$\frac{4}{7} = \frac{4 \times 2}{7 \times 2}$$
$$= \frac{8}{14}$$

• 
$$4:7=8:14$$

Ex 26:

$$2:3=8:12$$

Answer:



$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4}$$
$$= \frac{8}{12}$$

• 
$$2:3=8:12$$

Ex 27:

3:2 = 30:20

Answer:



 $\frac{3}{2} = \frac{3 \times 10}{2 \times 10}$  $= \frac{30}{20}$ 

• 3:2=30:20.

# D PROPORTION

#### D.1 IDENTIFYING THE PROPORTION

MCQ 28: Two vinaigrettes are being prepared:

- Vinaigrette A is made with 2 mL of oil and 1 mL of vinegar.
- Vinaigrette B is made with 4 mL of oil and 2 mL of vinegar.

Will these two vinaigrettes taste the same?

 $\boxtimes$  Yes

 $\square$  No

Answer:



• Since both ratios are equal, there is a proportion. So, the vinaigrettes will taste the same.

MCQ 29: On the cement package, it is indicated: 2 kilos of cement for 3 liters of water.

A worker prepares a mixture with 4 kilos of cement and 6 liters of water.

Did he follow the recommended proportions?

 $\boxtimes$  Yes

 $\square$  No

Answer:



• Since the two ratios are equal, the worker followed the recommended proportions.

MCQ 30: Two smoothie recipes are being prepared:

- Smoothie A is made with 3 cups of fruit and 2 cups of yogurt.
- Smoothie B is made with 6 cups of fruit and 4 cups of yogurt.

Will these two smoothies taste the same?

 $\boxtimes$  Yes

 $\square$  No

Answer:



• Since both ratios are equal, the proportions are the same. Therefore, the smoothies will taste the same.



MCQ 31: A gardener uses a fertilizer mix:

- The recommended mix is 5 grams of fertilizer per 2 liters of water.
- The gardener prepares a mixture with 10 grams of fertilizer and 4 liters of water.

Did the gardener follow the recommended proportions?

 $\Box$  Yes

□ No

Answer:



• Since both ratios are equals, the gardener followed the recommended proportions.

## E UNITARY METHOD

#### **E.1 BRINGING TO THE UNIT**

**Ex 32:** A satellite makes 4 orbits around the Earth in 24 hours. How many hours does it take to complete one orbit?

6 hours





Thus, to make 1 orbit, it takes 6 hours.

**Ex 33:** A car travels 500 kilometers in 5 hours. How many kilometers does it travel in 1 hour?

## 100 kilometers

Answer: For 5 hours, the car travels 500 kilometers. Therefore, to maintain this proportion for 1 hour, we divide both the number of kilometers and the number of hours by 5:



Thus, to travel 1 hour, the car covers 100 kilometers.

**Ex 34:** A factory produces 720 widgets in 8 hours. How many widgets does it produce in 1 hour?

#### 90 widgets

Answer: For 8 hours, the factory produces 720 widgets. Therefore, to maintain this proportion for 1 hour, we divide both the number of widgets and the number of hours by 8:



Thus, in 1 hour, the factory produces 90 widgets.

**Ex 35:** A baker uses 2 kilograms of flour to make 4 loaves of bread. How many kilograms of flour does it take to make 1 loaf of bread?

0.5 kilograms

Answer: For 4 loaves, the baker uses 2 kilograms of flour. Therefore, to maintain this proportion for 1 loaf, we divide both the number of kilograms and the number of loaves by 4:



Thus, to make 1 loaf of bread, it takes 0.5 kilograms of flour.

### **E.2 CALCULATING FROM THE UNIT**

**Ex 36:** To make 1 chocolate cake, 4 eggs are needed. How many eggs are needed to make 2 cakes?

8 eggs

Answer: For 1 cake, it takes 4 eggs. Therefore, to maintain this proportion for 2 cakes, you multiply both the number of cakes and the number of eggs by 2:



Thus, to make 2 chocolate cakes, you need 8 eggs.

**Ex 37:** The price of 1 kilogram of apples is \$2.5. What is the price for 3 kilograms of apples?



Answer: The price for 1 kilogram of apples is 2.5. Therefore, to maintain this proportion for 3 kilograms, you multiply the price by 3:



Thus, the price for 3 kilograms of apples is 7.5.

**Ex 38:** To build 1 bookshelf, 10 wooden planks are needed. How many wooden planks are needed to build 3 bookshelves?

#### 30 wooden planks

Answer: For 1 bookshelf, it takes 10 wooden planks. Therefore, to maintain this proportion for 3 bookshelves, you multiply both the number of bookshelves and the number of wooden planks by 3:





Thus, to build 3 bookshelves, you need 30 wooden planks.

**Ex 39:** To paint  $1 \text{ m}^2$ , 0.2 liters of paint are needed. How many liters of paint are needed to paint  $3 \text{ m}^2$ ?

Answer: For  $1 \text{ m}^2$ , it takes 0.2 liters of paint. Therefore, to maintain this proportion for  $3 \text{ m}^2$ , you multiply both the area and the number of liters of paint by 3:



Thus, to paint  $3 \text{ m}^2$ , you need 0.6 liters of paint.

#### E.3 CONVERTING TO AND FROM THE UNIT

**Ex 40:** To make a special juice mix, you need 5 apples for every 15 oranges. How many oranges do you need if you have 3 apples?

## 9 oranges

Answer:

• To the unit: For 5 apples, you need 15 oranges. Therefore, to maintain this proportion for 1 apple, we divide both the number of oranges and the number of apples by 5:



Thus, for 1 apple, you need 3 oranges.

• From the unit: For 1 apple, you need 3 oranges. Therefore, to maintain this proportion for 3 apples, we multiply both the number of oranges and the number of apples by 3:



Thus, for 3 apples, you need 9 oranges.



**Ex 41:** A baker uses 2 kilograms of flour to make 4 loaves of bread. How many kilograms of flour does it take to make 3 loaf of bread?

1.5 kilograms

Answer:

• To the unit: For 4 loaves, the baker uses 2 kilograms of flour. Therefore, to maintain this proportion for 1 loaf, we divide both the number of kilograms and the number of loaves by 4:



Thus, to make 1 loaf of bread, it takes 0.5 kilograms of flour.

• From the unit: For 1 loaves, the baker uses 0.5 kilograms of flour. Therefore, to maintain this proportion for 3 loaf, we multiply both the number of kilograms and the number of loaves by 3:



Thus, to make 3 loaf of bread, it takes 1.5 kilograms of flour.



**Ex 42:** An artist mixes 3 liters of red paint with 6 liters of blue paint to create a purple shade. How many liters of red paint are needed to mix with 9 liters of blue paint to maintain the same shade of purple?



Answer:

• To the unit: For 6 liters of blue paint, the artist uses 3 liters of red paint. Therefore, to maintain this proportion for 1 liter of blue paint, we divide both the number of liters of red paint and blue paint by 6:



Thus, to mix with 1 liter of blue paint, it takes 0.5 liters of red paint.

• From the unit: For 1 liter of blue paint, the artist uses 0.5 liters of red paint. Therefore, to maintain this proportion for 9 liters of blue paint, we multiply both the number of liters of red paint and blue paint by 9:



(°<u>+</u>°)

Thus, to mix with 9 liters of blue paint, it takes 4.5 liters of red paint.



**Ex 43:** To make a magic potion, you need 10 drops of dragon's blood for every 5 drops of phoenix tears. How many drops of phoenix tears do you need if you have 4 drops of dragon's blood?

2 drops of phoenix tears

Answer:

• To the unit: For 10 drops of dragon's blood, you need 5 drops of phoenix tears. Therefore, to maintain this proportion for 1 drop of dragon's blood, we divide both the number of drops of phoenix tears and the number of drops of dragon's blood by 10:



Thus, for 1 drop of dragon's blood, you need 0.5 drops of phoenix tears.

• From the unit: For 1 drop of dragon's blood, you need 0.5 drops of phoenix tears. Therefore, to maintain this proportion for 4 drops of dragon's blood, we multiply both the number of drops of phoenix tears and the number of drops of dragon's blood by 4:



Thus, for 4 drops of dragon's blood, you need 2 drops of phoenix tears.



E.4 SOLVING NUMERATOR Ex 44:  $\frac{6}{2} = \frac{9}{3}$ Answer:  $\div 2 \times 3$ 



Ex 45:

Answer:

Ex 46:

Answer:

Ex 47:

Answer:









7.5

5





## E.5 SOLVING DENOMINATOR

Ex 48:

Answer:



 $\frac{5}{10} = \frac{2}{4}$ 





(\*<u>+</u>)

Answer:

www.commeunjeu.com



+-×= Ex 50:

> 3  $\overline{5}$ 3.75

Answer:



Ex 51:



Answer:



## F CROSS-MULTIPLICATION METHOD

## F.1 FINDING A QUANTITY

4 tickets cost 28 dollars. Find the cost of 6 tickets. Ex 52: 42 dollars

Answer:

• Method 1: Cross Multiplication in a Table



So, 6 tickets cost 42 dollars.

• Method 2: Unitary Rate with Equivalent Ratios



So, 6 tickets cost 42 dollars.

# • Method 3: Cross Multiplication Equation

$$\frac{28}{4} = \frac{x}{6}$$

$$4 \times x = 28 \times 6 \quad (\text{cross multiplication})$$

$$x = \frac{28 \times 6}{4}$$

$$x = 42$$

So, 6 tickets cost 42 dollars.

### • Method 4: Unit Rate in Words

- 4 tickets cost 28 dollars, so 1 ticket costs  $28 \div 4 = 7$ dollars.
- -6 tickets cost  $7 \times 6 = 42$  dollars.

A recipe requires 200 grams of flour to make 8 Ex 53: cookies. How much flour is needed to make 12 cookies.

300 grams

Answer:

• Method 1: Cross Multiplication in a Table

Flour (grams)	200 ÷	$\frac{12 \times 200}{8} = 300$
Number of cookies	8	× 12

So, 12 cookies need 300 grams of flour.

• Method 2: Unitary Rate with Equivalent Ratios



So, 12 cookies need 300 grams of flour.

• Method 3: Cross Multiplication Equation

$$\frac{200}{8} = \frac{x}{12}$$

$$8 \times x = 200 \times 12 \quad (\text{cross multiplication})$$

$$x = \frac{200 \times 12}{8}$$

$$x = 300$$

So, 12 cookies need 300 grams of flour.

#### • Method 4: Unit Rate in Words

- 8 cookies need 200 grams of flour, so 1 cookie needs  $200 \div 8 = 25$  grams of flour.
- -12 cookies need  $25 \times 12 = 300$  grams of flour.

To make a certain shade of paint, you need 1.5Ex 54: liters of blue paint for every 3 liters of base paint. How much blue paint is needed if you have 4.5 liters of base paint.



2.25 | liters

Answer:

# • Method 1: Cross Multiplication in a Table

Blue paint (liters)	1.5	$\frac{4.5 \times 1.5}{3} = 2.25$
Base paint (liters)	3	× 4.5

So, you need 2.25 liters of blue paint.

## • Method 2: Unitary Rate with Equivalent Ratios



So, you need 2.25 liters of blue paint.

# • Method 3: Cross Multiplication Equation

$$\frac{1.5}{3} = \frac{x}{4.5}$$
  
 $3 \times x = 1.5 \times 4.5$  (cross multiplication)  
 $x = \frac{1.5 \times 4.5}{3}$   
 $x = 2.25$ 

So, you need 2.25 liters of blue paint.

## • Method 4: Unit Rate in Words

- -3 liters of base paint need 1.5 liters of blue paint, so 1 liter of base paint needs  $1.5 \div 3 = 0.5$  liters of blue paint.
- -4.5 liters of base paint need  $0.5 \times 4.5 = 2.25$  liters of blue paint.

A car travels 120 kilometers on 7.5 liters of fuel. Ex 55: Assuming the car's fuel consumption rate is constant, how much fuel is needed to travel 200 kilometers.

12.5 liters

Answer:

# • Method 1: Cross Multiplication in a Table



So, you need 12.5 liters of fuel.

• Method 2: Unitary Rate with Equivalent Ratios

So, you need 12.5 liters of fuel.

• Method 3: Cross Multiplication Equation

 $\frac{7.5}{120} = \frac{x}{200}$  $120 \times x = 7.5 \times 200$ (cross multiplication)  $x = \frac{7.5 \times 200}{120}$ x = 12.5

So, you need 12.5 liters of fuel.

- Method 4: Unit Rate in Words
  - 120 kilometers need 7.5 liters of fuel, so 1 kilometer needs  $7.5 \div 120 = 0.0625$  liters of fuel.
  - -200 kilometers need  $0.0625 \times 200 = 12.5$  liters of fuel.

On a map, 4 centimeters represents a real distance Ex 56: of 10 kilometers. If two cities are 6 centimeters apart on the map, what is the actual distance between them in kilometers.

15 kilometers

Answer:

• Method 1: Cross Multiplication in a Table



So, the actual distance is 15 kilometers.

• Method 2: Unitary Rate with Equivalent Ratios



So, the actual distance is 15 kilometers.

• Method 3: Cross Multiplication Equation

$$\frac{10}{4} = \frac{x}{6}$$

$$4 \times x = 10 \times 6 \quad (\text{cross multiplication})$$

$$x = \frac{10 \times 6}{4}$$

$$x = 15$$

So, the actual distance is 15 kilometers.

- Method 4: Unit Rate in Words
  - -4 centimeters on the map represent 10 kilometers in reality, so 1 centimeter represents  $10 \div 4 = 2.5$ kilometers.
  - -6 centimeters on the map represent  $2.5 \times 6 = 15$ kilometers in reality.

