

SQUARE ROOTS

A SQUARE ROOTS

A.1 CALCULATING SQUARE ROOTS OF PERFECT SQUARES

Ex 1: Calculate:

$$\sqrt{4} = \boxed{2}$$

Answer: Since $2 \times 2 = 4$, we have $\sqrt{4} = 2$.

Ex 2: Without using a calculator, calculate:

$$\sqrt{36} = \boxed{6}$$

Answer: Since $6 \times 6 = 36$, we have $\sqrt{36} = 6$.

Ex 3: Calculate:

$$\sqrt{64} = \boxed{8}$$

Answer: Since $8 \times 8 = 64$, we have $\sqrt{64} = 8$.

Ex 4: Calculate:

$$\sqrt{49} = \boxed{7}$$

Answer: Since $7 \times 7 = 49$, we have $\sqrt{49} = 7$.

Ex 5: Calculate:

$$\sqrt{100} = \boxed{10}$$

Answer: Since $10 \times 10 = 100$, we have $\sqrt{100} = 10$.

Ex 6: Calculate:

$$\sqrt{81} = \boxed{9}$$

Answer: Since $9 \times 9 = 81$, we have $\sqrt{81} = 9$.

Ex 7: Calculate:

$$\sqrt{0} = \boxed{0}$$

Answer: Since $0 \times 0 = 0$, we have $\sqrt{0} = 0$.

A.2 CALCULATING SQUARE ROOTS OF FRACTIONS

Ex 8: Write in fraction form:

$$\sqrt{\frac{1}{4}} = \boxed{\frac{1}{2}}$$

Answer: Since $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, we have $\sqrt{\frac{1}{4}} = \frac{1}{2}$.

Ex 9: Write in fraction form:

$$\sqrt{\frac{1}{25}} = \boxed{\frac{1}{5}}$$

Answer: Since $\frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$, we have $\sqrt{\frac{1}{25}} = \frac{1}{5}$.

Ex 10: Write in fraction form:

$$\sqrt{\frac{1}{9}} = \boxed{\frac{1}{3}}$$

Answer: Since $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$, we have $\sqrt{\frac{1}{9}} = \frac{1}{3}$.

Ex 11: Write in fraction form:

$$\sqrt{\frac{1}{16}} = \boxed{\frac{1}{4}}$$

Answer: Since $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$, we have $\sqrt{\frac{1}{16}} = \frac{1}{4}$.

Ex 12: Write in fraction form:

$$\sqrt{\frac{9}{16}} = \boxed{\frac{3}{4}}$$

Answer: Since $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$, we have $\sqrt{\frac{9}{16}} = \frac{3}{4}$.


Ex 13: Write in fraction form:

$$\sqrt{\frac{4}{9}} = \boxed{\frac{2}{3}}$$

Answer: Since $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$, we have $\sqrt{\frac{4}{9}} = \frac{2}{3}$.


B CALCULATING SQUARE ROOTS

B.1 USING A CALCULATOR

Ex 14:  Using a calculator, evaluate $\sqrt{2}$ (round to 2 decimal places).


$$\sqrt{2} \approx \boxed{1.41}$$

Answer: By entering $\sqrt{2}$ and pressing the equal button, the calculator displays: 1.41421356237...
So $\sqrt{2} \approx 1.41$ (rounded to two decimal places).

Ex 15:  Using a calculator, evaluate $\sqrt{10}$ (round to 2 decimal places).


$$\sqrt{10} \approx \boxed{3.16}$$

Answer: By entering $\sqrt{10}$ and pressing the equal button, the calculator displays: 3.16227766017...
So $\sqrt{10} \approx 3.16$ (rounded to two decimal places).

Ex 16:  Using a calculator, evaluate $\sqrt{50}$ (round to 2 decimal places).

$$\sqrt{50} \approx \boxed{7.07}$$

Answer: By entering $\sqrt{50}$ and pressing the equal button, the calculator displays: 7.07106781187...
So $\sqrt{50} \approx 7.07$ (rounded to two decimal places).

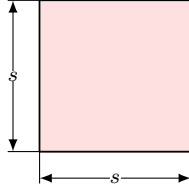
Ex 17:  Using a calculator, evaluate $\sqrt{0.5}$ (round to 2 decimal places).

$$\sqrt{0.5} \approx \boxed{0.71}$$

Answer: By entering $\sqrt{0.5}$ and pressing the equal button, the calculator displays: 0.70710678118...
So $\sqrt{0.5} \approx 0.71$ (rounded to two decimal places).

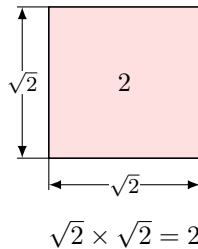
B.2 FINDING THE SIDE LENGTH OF A SQUARE

Ex 18: The area of a square is 2 m^2 . What is the length of the side of the square, s ?

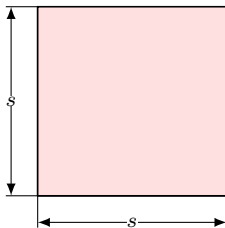


$$s \approx \boxed{1.41} \text{ m (round your answer to 2 decimal places)}$$

Answer: The area of a square is s^2 , so $s^2 = 2$.
Therefore, $s = \sqrt{2} \approx 1.41 \text{ m}$.

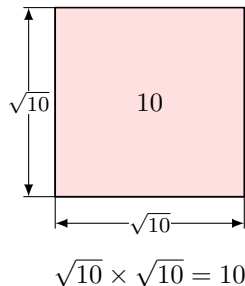


Ex 19: The area of a square is 10 m^2 . What is the length of the side of the square, s ?



$$s \approx \boxed{3.16} \text{ m (round your answer to 2 decimal places)}$$

Answer: The area of a square is s^2 , so $s^2 = 10$.
Therefore, $s = \sqrt{10} \approx 3.16 \text{ m}$ (rounded to two decimal places).



C CUBE ROOT

C.1 CALCULATING CUBE ROOTS OF PERFECT CUBES

Ex 20: Calculate:

$$\sqrt[3]{8} = \boxed{2}$$

Answer: Since $2 \times 2 \times 2 = 8$, we have $\sqrt[3]{8} = 2$.

Ex 21: Without using a calculator, calculate:

$$\sqrt[3]{27} = \boxed{3}$$

Answer: Since $3 \times 3 \times 3 = 27$, we have $\sqrt[3]{27} = 3$.

Ex 22: Calculate:

$$\sqrt[3]{64} = \boxed{4}$$

Answer: Since $4 \times 4 \times 4 = 64$, we have $\sqrt[3]{64} = 4$.

Ex 23: Calculate:

$$\sqrt[3]{125} = \boxed{5}$$

Answer: Since $5 \times 5 \times 5 = 125$, we have $\sqrt[3]{125} = 5$.

Ex 24: Calculate:

$$\sqrt[3]{1\,000} = \boxed{10}$$

Answer: Since $10 \times 10 \times 10 = 1\,000$, we have $\sqrt[3]{1\,000} = 10$.

Ex 25: Calculate:

$$\sqrt[3]{0} = \boxed{0}$$

Answer: Since $0 \times 0 \times 0 = 0$, we have $\sqrt[3]{0} = 0$.

D LAWS OF SQUARE ROOTS

D.1 WRITING AS A SINGLE ROOT: LEVEL 1

Ex 26: Write as a single square root:

$$\sqrt{3}\sqrt{4} = \boxed{\sqrt{12}}$$

Answer:

$$\begin{aligned}\sqrt{3}\sqrt{4} &= \sqrt{3 \times 4} \\ &= \sqrt{12}\end{aligned}$$

Ex 27: Write as a single square root:

$$\sqrt{5}\sqrt{20} = \boxed{\sqrt{100}}$$

Answer:

$$\begin{aligned}\sqrt{5}\sqrt{20} &= \sqrt{5 \times 20} \\ &= \sqrt{100}\end{aligned}$$

Ex 28: Write as a single square root:

$$\sqrt{6}\sqrt{6} = \boxed{\sqrt{36}}$$

Answer:

$$\begin{aligned}\sqrt{6}\sqrt{6} &= \sqrt{6 \times 6} \\ &= \sqrt{36}\end{aligned}$$

Ex 29: Write as a single square root:

$$\sqrt{9}\sqrt{4} = \boxed{\sqrt{36}}$$

Answer:

$$\begin{aligned}\sqrt{9}\sqrt{4} &= \sqrt{9 \times 4} \\ &= \sqrt{36}\end{aligned}$$

Ex 30: Write as a single square root:

$$\sqrt{2}\sqrt{8} = \boxed{\sqrt{16}}$$

Answer:

$$\begin{aligned}\sqrt{2}\sqrt{8} &= \sqrt{2 \times 8} \\ &= \sqrt{16}\end{aligned}$$

D.2 WRITING AS A SINGLE ROOT: LEVEL 2

Ex 31: Write as a single square root:

$$\sqrt{2}\sqrt{3}\sqrt{5} = \boxed{\sqrt{30}}$$

Answer:

$$\begin{aligned}\sqrt{2}\sqrt{3}\sqrt{5} &= \sqrt{2 \times 3 \times 5} \\ &= \sqrt{30}\end{aligned}$$

Ex 32: Write as a single square root:

$$\sqrt{5}\sqrt{2}\sqrt{10} = \boxed{\sqrt{100}}$$

Answer:

$$\begin{aligned}\sqrt{5}\sqrt{2}\sqrt{10} &= \sqrt{5 \times 2 \times 10} \\ &= \sqrt{100}\end{aligned}$$

Ex 33: Write as a single square root:

$$(\sqrt{3})^3 = \boxed{\sqrt{27}}$$

Answer:

$$\begin{aligned}(\sqrt{3})^3 &= \sqrt{3}\sqrt{3}\sqrt{3} \\ &= \sqrt{3 \times 3 \times 3} \\ &= \sqrt{27}\end{aligned}$$

Ex 34: Write as a single square root:

$$(\sqrt{2})^3 \sqrt{3} = \boxed{\sqrt{24}}$$

Answer:

$$\begin{aligned}(\sqrt{2})^3 \sqrt{3} &= \sqrt{2}\sqrt{2}\sqrt{2}\sqrt{3} \\ &= \sqrt{2 \times 2 \times 2 \times 3} \\ &= \sqrt{24}\end{aligned}$$

D.3 UNDERSTANDING SQUARE ROOT OPERATIONS

MCQ 35: Is $\sqrt{2} + \sqrt{3} = \sqrt{2+3}$?

☐ True

☒ False

Answer: **The statement is incorrect.** The product of square roots can be combined into a single root by multiplying inside:

$$\sqrt{2}\sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$$

But the sum of square roots **cannot** be combined in this way:

$$\sqrt{2} + \sqrt{3} \neq \sqrt{2+3}$$

because $\sqrt{2} + \sqrt{3} \approx 1.41 + 1.73 = 3.14$ and $\sqrt{5} \approx 2.24$.

MCQ 36: Is $\sqrt{2}\sqrt{3} = \sqrt{6}$?

☒ True

☐ False

Answer: **The statement is correct.** The product of square roots can be written as a single root:

$$\sqrt{2}\sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$$

This is a property of square roots: $\sqrt{a}\sqrt{b} = \sqrt{ab}$.

MCQ 37: Is $\sqrt{3} + \sqrt{3} = \sqrt{3+3}$?

☐ True

☒ False

Answer: **The statement is incorrect.** The sum of square roots **cannot** be simplified by adding inside:

$$\sqrt{3} + \sqrt{3} = 2\sqrt{3}$$

but

$$\sqrt{3+3} = \sqrt{6}$$

and $2\sqrt{3} \approx 3.46$ while $\sqrt{6} \approx 2.45$, so they are not equal. The product of roots, however, can be written as a single root:

$$\sqrt{3}\sqrt{3} = \sqrt{3 \times 3} = \sqrt{9} = 3$$

MCQ 38: Is $\sqrt{3} + \sqrt{3} = 3$?

☐ True

☒ False

Answer: **The statement is incorrect.** $\sqrt{3} + \sqrt{3}$ means you add two times the value of $\sqrt{3}$:

$$\sqrt{3} + \sqrt{3} = 2\sqrt{3} \approx 2 \times 1.73 = 3.46$$

So $\sqrt{3} + \sqrt{3} \neq 3$.

D.4 SIMPLIFYING THE SQUARE ROOT OF A PERFECT SQUARE: LEVEL 1

Ex 39: Simplify:

$$\sqrt{4} = \boxed{2}$$

Answer:

$$\begin{aligned}\sqrt{4} &= \sqrt{2 \times 2} \\ &= 2\end{aligned}$$

Ex 40: Simplify:

$$\sqrt{36} = \boxed{6}$$

Answer:

$$\begin{aligned}\sqrt{36} &= \sqrt{6 \times 6} \\ &= 6\end{aligned}$$

Ex 41: Simplify:

$$\sqrt{10^2} = \boxed{10}$$

Answer:

$$\begin{aligned}\sqrt{10^2} &= \sqrt{10 \times 10} \\ &= 10\end{aligned}$$

Ex 42: Simplify:

$$\sqrt{x^2} = \boxed{x}$$

Answer:

$$\begin{aligned}\sqrt{x^2} &= \sqrt{x \times x} \\ &= x\end{aligned}$$

Ex 43: Simplify:

$$\sqrt{(2x)^2} = \boxed{2x}$$

Answer:

$$\begin{aligned}\sqrt{(2x)^2} &= \sqrt{(2x) \times (2x)} \\ &= 2x\end{aligned}$$

D.5 SIMPLIFYING THE SQUARE ROOT OF A PERFECT SQUARE: LEVEL 2

Ex 44: Simplify:

$$\sqrt{9x^2} = \boxed{3x}$$

Answer:

$$\begin{aligned}\sqrt{9x^2} &= \sqrt{(3x) \times (3x)} \\ &= 3x\end{aligned}$$

Ex 45: Simplify:

$$\sqrt{x^4} = \boxed{x^2}$$

Answer:

$$\begin{aligned}\sqrt{x^4} &= \sqrt{x^2 \times x^2} \\ &= x^2\end{aligned}$$

Ex 46: Simplify:

$$\sqrt{4x^2} + x = \boxed{3x}$$

Answer:

$$\begin{aligned}\sqrt{4x^2} + x &= \sqrt{(2x) \times (2x)} + x \\ &= 2x + x \\ &= 3x\end{aligned}$$

Ex 47: Simplify:

$$\sqrt{12}\sqrt{3} = \boxed{6}$$

Answer:

$$\begin{aligned}\sqrt{12}\sqrt{3} &= \sqrt{12 \times 3} \\ &= \sqrt{2 \times 2 \times 3 \times 3} \\ &= 2 \times 3 \\ &= 6\end{aligned}$$

D.6 SIMPLIFYING SQUARE ROOTS

Ex 48: Simplify:

$$\sqrt{18} = \boxed{3\sqrt{2}}$$

Answer:

$$\begin{aligned}\sqrt{18} &= \sqrt{3 \times 3 \times 2} \\ &= 3\sqrt{2}\end{aligned}$$

Ex 49: Simplify:

$$\sqrt{50} = \boxed{5\sqrt{2}}$$

Answer:

$$\begin{aligned}\sqrt{50} &= \sqrt{5 \times 5 \times 2} \\ &= 5\sqrt{2}\end{aligned}$$

Ex 50: Simplify:

$$\sqrt{32} = \boxed{4\sqrt{2}}$$

Answer:

$$\begin{aligned}\sqrt{32} &= \sqrt{4 \times 4 \times 2} \\ &= 4\sqrt{2}\end{aligned}$$

Ex 51: Simplify:

$$\sqrt{20} = \boxed{2\sqrt{5}}$$

Answer:

$$\begin{aligned}\sqrt{20} &= \sqrt{2 \times 2 \times 5} \\ &= 2\sqrt{5}\end{aligned}$$

D.7 SIMPLIFYING QUOTIENTS OF SQUARE ROOTS

Ex 52: Simplify:

$$\frac{\sqrt{10}}{\sqrt{5}} = \boxed{\sqrt{2}}$$

Answer: $\frac{\sqrt{10}}{\sqrt{5}} = \sqrt{\frac{10}{5}}$
 $= \sqrt{2}$

Ex 53: Simplify:

$$\frac{\sqrt{75}}{\sqrt{25}} = \boxed{\sqrt{3}}$$

Answer: $\frac{\sqrt{75}}{\sqrt{25}} = \sqrt{\frac{75}{25}}$
 $= \sqrt{3}$

Ex 54: Simplify:

$$\frac{\sqrt{18}}{\sqrt{3}} = \boxed{\sqrt{6}}$$

Answer: $\frac{\sqrt{18}}{\sqrt{3}} = \sqrt{\frac{18}{3}}$
 $= \sqrt{6}$

Ex 55: Simplify:

$$\frac{\sqrt{20}}{\sqrt{2}} = \boxed{\sqrt{10}}$$

Answer: $\frac{\sqrt{20}}{\sqrt{2}} = \sqrt{\frac{20}{2}}$
 $= \sqrt{10}$

E ALGEBRAIC OPERATIONS

E.1 ADDING AND SUBTRACTING LIKE RADICALS: LEVEL 1

Ex 56: Simplify:

$$2\sqrt{3} + 5\sqrt{3} = \boxed{7\sqrt{3}}$$

Answer: $2\sqrt{3} + 5\sqrt{3} = (2 + 5)\sqrt{3}$ (factorisation)
 $= 7\sqrt{3}$

Ex 57: Simplify:

$$4\sqrt{5} + 7\sqrt{5} = \boxed{11\sqrt{5}}$$

Answer: $4\sqrt{5} + 7\sqrt{5} = (4 + 7)\sqrt{5}$ (factorisation)
 $= 11\sqrt{5}$

Ex 58: Simplify:

$$3\sqrt{6} - \sqrt{6} = \boxed{2\sqrt{6}}$$

Answer: $3\sqrt{6} - \sqrt{6} = (3 - 1)\sqrt{6}$ (factorisation)
 $= 2\sqrt{6}$

Ex 59: Simplify:

$$3\sqrt[3]{7} + 5\sqrt[3]{7} = \boxed{8\sqrt[3]{7}}$$

Answer: $3\sqrt[3]{7} + 5\sqrt[3]{7} = (3 + 5)\sqrt[3]{7}$ (factorisation)
 $= 8\sqrt[3]{7}$

Ex 60: Simplify:

$$2\sqrt{2} - 4\sqrt{2} = \boxed{-2\sqrt{2}}$$

Answer: $2\sqrt{2} - 4\sqrt{2} = (2 - 4)\sqrt{2}$ (factorisation)
 $= -2\sqrt{2}$

Ex 61: Simplify:

$$2\sqrt{7} - 5\sqrt{7} = \boxed{-3\sqrt{7}}$$

Answer: $2\sqrt{7} - 5\sqrt{7} = (2 - 5)\sqrt{7}$ (factorisation)
 $= -3\sqrt{7}$

E.2 ADDING AND SUBTRACTING LIKE RADICALS: LEVEL 2

Ex 62: Simplify:

$$\sqrt{8} - \sqrt{2} = \boxed{\sqrt{2}}$$

Answer: $\sqrt{8} - \sqrt{2} = \sqrt{2 \times 2 \times 2} - \sqrt{2}$
 $= 2\sqrt{2} - \sqrt{2}$
 $= (2 - 1)\sqrt{2}$
 $= \sqrt{2}$

Ex 63: Simplify:

$$\sqrt{12} + 3\sqrt{3} = \boxed{5\sqrt{3}}$$

Answer: $\sqrt{12} + 3\sqrt{3} = \sqrt{2 \times 2 \times 3} + 3\sqrt{3}$
 $= 2\sqrt{3} + 3\sqrt{3}$
 $= (2 + 3)\sqrt{3}$
 $= 5\sqrt{3}$

Ex 64: Simplify:

$$5\sqrt{3} - \sqrt{12} = \boxed{3\sqrt{3}}$$

Answer:

$$\begin{aligned}5\sqrt{3} - \sqrt{12} &= 5\sqrt{3} - \sqrt{2 \times 2 \times 3} \\&= 5\sqrt{3} - 2\sqrt{3} \\&= (5 - 2)\sqrt{3} \\&= 3\sqrt{3}\end{aligned}$$

Ex 65: Simplify:

$$2\sqrt{7} + 3\sqrt{28} = \boxed{8\sqrt{7}}$$

Answer:

$$\begin{aligned}2\sqrt{7} + 3\sqrt{28} &= 2\sqrt{7} + 3 \times \sqrt{2 \times 2 \times 7} \\&= 2\sqrt{7} + 3 \times 2\sqrt{7} \\&= 2\sqrt{7} + 6\sqrt{7} \\&= (2 + 6)\sqrt{7} \\&= 8\sqrt{7}\end{aligned}$$