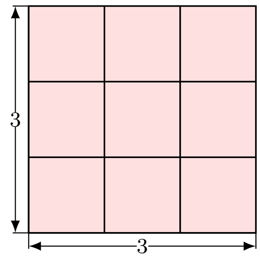
ROOTS

A SQUARE ROOTS

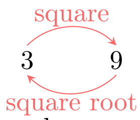
Discover:

- When we **square** a number, we multiply it by itself.
  For example, 3 squared is 3×3, which can be written as 3^2.

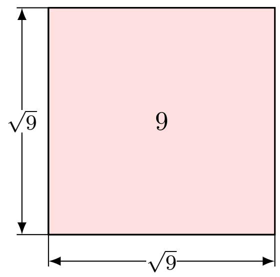


$3^2 = 9$ is the area of a square with side length 3.

- On the other hand, taking the **square root** of a number is the reverse process: it is finding a number that, when multiplied by itself, gives the original number. For example,



3 squared is 9, so the square root of 9 is 3.



The **square root** of 9, written as $\sqrt{9}$, is the side length of a square with area 9.

Definition Square root

The **square root** of a, written $\sqrt{a}$, is the **positive number** which, when squared, gives a:

$$\left(\sqrt{a}\right)^2 = \sqrt{a} \times \sqrt{a} = a$$

Ex: Find $\sqrt{25}$.

Answer: Since $5 \times 5 = 25$, $\sqrt{25} = 5$.

Definition Perfect Squares

A **perfect square** is a number that is the result of squaring an integer.

Ex: The perfect squares of the first few integers are:
1, 4, 9, 16, 25, 36, 49, 64, and so on.

Definition Simplest square root Form

A square root is written in **simplest form** if the number under the square root sign is as small as possible.

B CALCULATING SQUARE ROOTS

It is easy to calculate the square root of a perfect square, but determining the square root of other numbers can be quite challenging.

Method Use a calculator

- Press the square root button $\boxed{\sqrt{}}$
- Enter the number
- Press the equals button $\boxed{=}$

Ex: Use a calculator to find $\sqrt{10}$ (round to 1 decimal place).

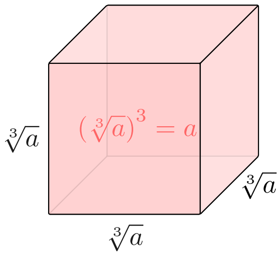
Answer: By entering $\sqrt{10}$ and pressing the equals button, the calculator displays: 3.16227766017.
So $\sqrt{10} \approx 3.2$.

C CUBE ROOT

Definition Cube Root

The **cube root** of a, written $\sqrt[3]{a}$, is the **number** which, when cubed, gives a:

$$\left(\sqrt[3]{a}\right)^3 = a$$



Ex: Find $\sqrt[3]{125}$.

Answer: As $5 \times 5 \times 5 = 125$, $\sqrt[3]{125} = 5$.

D LAWS OF SQUARE ROOTS

Proposition Law 1

For two positive numbers a and b:

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

Proof

$$\begin{aligned}\left(\sqrt{a} \cdot \sqrt{b}\right)^2 &= \left(\sqrt{a} \cdot \sqrt{b}\right) \cdot \left(\sqrt{a} \cdot \sqrt{b}\right) \\ &= \left(\sqrt{a} \cdot \sqrt{a}\right) \cdot \left(\sqrt{b} \cdot \sqrt{b}\right) \\ &= ab\end{aligned}$$

So, by definition, $\sqrt{a} \cdot \sqrt{b}$ is the square root of ab. Therefore, $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$.

Ex: Show that $\sqrt{6} = \sqrt{2} \cdot \sqrt{3}$.

Answer:

$$\begin{aligned}\sqrt{6} &= \sqrt{2 \times 3} \\ &= \sqrt{2} \cdot \sqrt{3}\end{aligned}$$

Proposition Law 2 (Perfect Square)

For a positive number a:

$$\sqrt{a^2} = a$$
$$\sqrt{a \cdot a} = a$$

Proof

$$\begin{aligned}\sqrt{a^2} &= \sqrt{a\cdot a}\\ &= \sqrt{a}\cdot\sqrt{a} \quad \text{(by Law 1)}\\ &= \left(\sqrt{a}\right)^2\\ &= a \quad \text{(by definition of the square root)}\end{aligned}$$

Ex: Find $\sqrt{25}$.

Answer:

$$\begin{aligned}\sqrt{25} &= \sqrt{5\times 5}\\ &= 5 \quad \text{(extract one number of the pair from the square root)}\end{aligned}$$

Proposition Law 3

For two positive numbers a and b:

$$\sqrt{a^2b} = a\sqrt{b}$$
$$\sqrt{a\cdot a\cdot b} = a\sqrt{b}$$

Proof

$$\begin{aligned}\sqrt{a^2b} &= \sqrt{a^2}\cdot\sqrt{b} \quad \text{(by Law 1)}\\ &= a\sqrt{b} \quad \text{(by Law 2)}\end{aligned}$$

Ex: Simplify $\sqrt{12} = 2\sqrt{3}$.

Answer:

$$\begin{aligned}\sqrt{12} &= \sqrt{2\times 2\times 3}\\ &= 2\sqrt{3} \quad \text{(extract one number of the pair from the square root)}\end{aligned}$$

Proposition Law 4

For two positive numbers a and b:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Proof

$$\begin{aligned}\left(\frac{\sqrt{a}}{\sqrt{b}}\right)^2 &= \frac{\sqrt{a}}{\sqrt{b}}\cdot\frac{\sqrt{a}}{\sqrt{b}}\\ &= \frac{\sqrt{a}\cdot\sqrt{a}}{\sqrt{b}\cdot\sqrt{b}}\\ &= \frac{a}{b}\end{aligned}$$

So, by definition, $\frac{\sqrt{a}}{\sqrt{b}}$ is the square root of $\frac{a}{b}$. Therefore, $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$.

Ex: Simplify $\dfrac{\sqrt{6}}{\sqrt{3}}$.

Answer:

$$\begin{aligned}\frac{\sqrt{6}}{\sqrt{3}} &= \sqrt{\frac{6}{3}}\\ &= \sqrt{2}\end{aligned}$$

E ALGEBRAIC OPERATIONS

Proposition Algebraic Operations

We can perform operations with square roots just as we do with ordinary numbers. In particular:

- We can add and subtract **like** square roots (i.e., same number under the root) in the same way that we add and subtract like algebraic terms.

- We can use the usual rules for expanding brackets.

Ex: Simplify: $2\sqrt{3}+4\sqrt{3}$

Answer:

$$\begin{aligned} 2\sqrt{3}+4\sqrt{3} &= (2+4)\sqrt{3} \quad \text{(factorisation)} \\ &= 6\sqrt{3} \end{aligned}$$