SEQUENCES

A NUMERICAL SEQUENCE

A.1 FINDING u_n

Ex 1:

n	1	2	3	4	5	6
u_n	3	5	7	9	11	13

What is u_4?

$$\boxed{9}$$

Answer: $u_4 = 9$.

Ex 2:

n	1	2	3	4	5	6
u_n	2	6	12	20	30	42

What is u_5?

$$\boxed{30}$$

Answer: $u_5 = 30$.

Ex 3:

n	1	2	3	4	5	6	7	8
u_n	4	9	16	25	36	49	64	81

What is u_7?

$$\boxed{64}$$

Answer: $u_7 = 64$.

Ex 4:

n	1	2	3	4	5	6	7	8
u_n	1	3	7	15	31	63	127	255

What is u_8?

$$\boxed{255}$$

Answer: $u_8 = 255$.

A.2 FINDING u_n IN AN ARITHMETIC SEQUENCE

Ex 5: What is u_6 for this sequence?

n	1	2	3	4	5	6
u_n	3	5	7	9	11	13

Answer: $u_6 = 13$, because each term increases by 2.

n	1	2	3	4	5	6
u_n	3	5	7	9	11	13

+2 +2 +2 +2 +2

Ex 6: What is u_6 for this sequence?

n	1	2	3	4	5	6
u_n	3	8	13	18	23	28

Answer: $u_6 = 28$, because each term increases by 5.

n	1	2	3	4	5	6
u_n	3	8	13	18	23	28

+5 +5 +5 +5 +5

Ex 7: What is u_5 for this sequence?

n	1	2	3	4	5
u_n	20	18	16	14	12

Answer: $u_5 = 12$, because each term decreases by 2.

n	1	2	3	4	5
u_n	20	18	16	14	12

-2 -2 -2 -2

Ex 8: What is u_6 for this sequence?

n	1	2	3	4	5	6
u_n	80	70	60	50	40	30

Answer: $u_6 = 30$, because each term decreases by 10.

n	1	2	3	4	5	6
u_n	80	70	60	50	40	30

-10 -10 -10 -10 -10

B DEFINITION USING A RECURSIVE RULE

B.1 CALCULATING THE FIRST TERMS

Ex 9: Write the sequence defined by: the first term is 7, and each term is obtained by adding 4 to the previous term.

$$(\boxed{7}, \boxed{11}, \boxed{15}, \boxed{19}, \boxed{23}, \dots)$$

Answer:

$$7 \xrightarrow{+4} 11 \xrightarrow{+4} 15 \xrightarrow{+4} 19 \xrightarrow{+4} 23$$

The sequence is: $(7, 11, 15, 19, 23, \dots)$.

Ex 10: Write the sequence defined by: the first term is 1, and each term is obtained by multiplying the previous term by 2.

$$(\boxed{1}, \boxed{2}, \boxed{4}, \boxed{8}, \boxed{16}, \dots)$$

Answer:

$$1 \xrightarrow{\times 2} 2 \xrightarrow{\times 2} 4 \xrightarrow{\times 2} 8 \xrightarrow{\times 2} 16$$

The sequence is: $(1, 2, 4, 8, 16, \dots)$.

Ex 11: Write the sequence defined by: the first term is 10, and each term is obtained by subtracting 5 from the previous term.

$$(\boxed{10}, \boxed{5}, \boxed{0}, \boxed{-5}, \boxed{-10}, \dots)$$

Answer:

$$10 \xrightarrow{-5} 5 \xrightarrow{-5} 0 \xrightarrow{-5} -5 \xrightarrow{-5} -10$$

The sequence is: $(10, 5, 0, -5, -10, \dots)$.

Ex 12: Write the sequence defined by: the first term is 2.5, and each term is obtained by adding 0.5 to the previous term.

$$(\boxed{2.5}, \boxed{3}, \boxed{3.5}, \boxed{4}, \boxed{4.5}, \dots)$$

Answer:

$$2.5 \xrightarrow{+0.5} 3 \xrightarrow{+0.5} 3.5 \xrightarrow{+0.5} 4 \xrightarrow{+0.5} 4.5$$

The sequence is: $(2.5, 3, 3.5, 4, 4.5, \dots)$.

B.2 IDENTIFYING THE RECURSIVE RULE

Ex 13: Given the sequence: $(3, 5, 7, 9, 11, 13, \dots)$

- The first term is $\boxed{3}$.
- The rule is $\boxed{\textbf{Add}}\ \boxed{2}$

Answer:

- The first term is 3.
- The rule is add 2:
$$3 \xrightarrow{+2} 5 \xrightarrow{+2} 7 \xrightarrow{+2} 9 \xrightarrow{+2} 11 \xrightarrow{+2} 13$$

Ex 14: Given the sequence: $(60, 55, 50, 45, 40, 35, \dots)$

- The first term is $\boxed{60}$.
- The rule is $\boxed{\textbf{Subtract}}\ \boxed{5}$

Answer:

- The first term is 60.
- The rule is subtract 5:
$$60 \xrightarrow{-5} 55 \xrightarrow{-5} 50 \xrightarrow{-5} 45 \xrightarrow{-5} 40 \xrightarrow{-5} 35$$

Ex 15: Given the sequence: $(64, 32, 16, 8, 4, 2, \dots)$

- The first term is $\boxed{64}$.
- The rule is $\boxed{\textbf{Divide}}\ \boxed{2}$

Answer:

- The first term is 64.
- The rule is divide by 2:
$$64 \xrightarrow{\div 2} 32 \xrightarrow{\div 2} 16 \xrightarrow{\div 2} 8 \xrightarrow{\div 2} 4 \xrightarrow{\div 2} 2$$

Ex 16: Given the sequence: $(1, 10, 100, 1\,000, 10\,000, \dots)$

- The first term is $\boxed{1}$.
- The rule is $\boxed{\textbf{Multiply}}\ \boxed{10}$

Answer:

- The first term is 1.
- The rule is multiply by 10:
$$1 \xrightarrow{\times 10} 10 \xrightarrow{\times 10} 100 \xrightarrow{\times 10} 1\,000 \xrightarrow{\times 10} 10\,000$$

B.3 IDENTIFYING THE RECURSIVE RULE IN GEOMETRIC PATTERNS

Ex 17: Observe the following pattern made with sticks:

Fill in the table below:

Diagram number	1	2	3	4
Number of sticks	3	5	7	9

What rule can you find for the number of sticks?
Start with $\boxed{3}$ sticks. Add $\boxed{2}$ sticks for the next diagram.
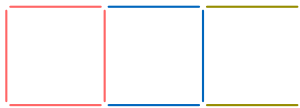
Answer:

- For diagram number 1, the number of sticks is 3.

- For diagram number 2, the number of sticks is 5.

- For diagram number 3, the number of sticks is 7.

- For diagram number 4, the number of sticks is 9.

- **Rule:** Start with 3 sticks, and add 2 sticks for the next diagram.

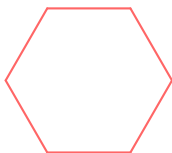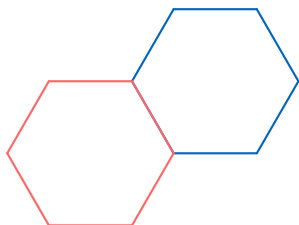**Ex 18:** Observe the following pattern made with sticks:

Fill in the table below:

Diagram number	1	2	3	4
Number of sticks	4	7	10	13

What rule can you find for the number of sticks?
Start with $\boxed{4}$ sticks. Add $\boxed{3}$ sticks for the next diagram.

Answer:

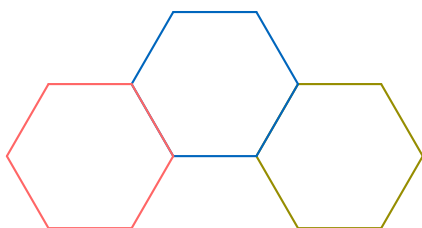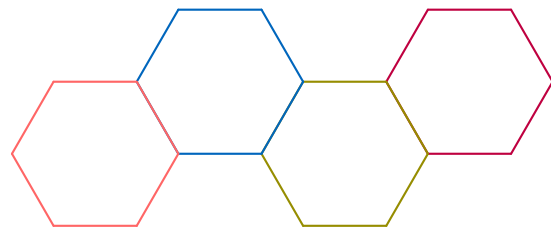- For diagram number 1, the number of sticks is 4.

- For diagram number 2, the number of sticks is 7.

- For diagram number 3, the number of sticks is 10.

- For diagram number 4, the number of sticks is 13.

- **Rule:** Start with 4 sticks, and add 3 sticks for the next diagram.

Ex 19: Observe the following pattern made with sticks:

Fill in the table below:

Diagram number	1	2	3	4
Number of sticks	6	11	16	21

What rule can you find for the number of sticks?
Start with 6 sticks. Add 5 sticks for the next diagram.

Answer:

- **Diagram 1 :** For 1 hexagon, the number of sticks is 6.

- **Diagram 2 :** For 2 hexagons, the number of sticks is 11.

- **Diagram 3 :** For 3 hexagons, the number of sticks is 16.

- **Diagram 4 :** For 4 hexagons, the number of sticks is 21.

- **Rule:** Start with 6 sticks, and add 5 sticks for each additional hexagon.

Ex 20: Observe the following pattern made with sticks:

Fill in the table below:

Diagram number	1	2	3	4
Number of sticks	4	6	8	10

What rule can you find for the number of sticks?
Start with 4 sticks. Add 2 sticks for the next diagram.

Answer:

- For diagram number 1, the number of sticks is 4.

- For diagram number 2, the number of sticks is 6.

- For diagram number 3, the number of sticks is 8.

- For diagram number 4, the number of sticks is 10.

- **Rule:** Start with 4 sticks, and add 2 sticks for each new diagram.

B.4 MODELING REAL SITUATIONS WITH SEQUENCES

Ex 21: A scientist observes a culture of bacteria. At the start, there are $u_0 = 5$ bacteria in a petri dish. Each day, the number of bacteria doubles.
Let u_n be the number of bacteria at the day n. What are the first five terms of the sequence (u_n)?

- $u_1 = 10$ bacteria

- $u_2 = 20$ bacteria

- $u_3 = \boxed{40}$ bacteria

- $u_4 = \boxed{80}$ bacteria

- $u_5 = \boxed{160}$ bacteria

Answer: The number of bacteria doubles each day:

- $u_1 = 2 \times u_0 = 2 \times 5 = 10$

- $u_2 = 2 \times u_1 = 2 \times 10 = 20$

- $u_3 = 2 \times u_2 = 2 \times 20 = 40$

- $u_4 = 2 \times u_3 = 2 \times 40 = 80$

- $u_5 = 2 \times u_4 = 2 \times 80 = 160$

Ex 22: Let u_n be the number of steps I have walked at the end of day n. On day 0, I walk $u_0 = 1000$ steps. Each day, I walk 500 more steps than the previous day.
What are the first terms of the sequence (u_n)?

- $u_1 = \boxed{1500}$ steps

- $u_2 = \boxed{2000}$ steps

- $u_3 = \boxed{2500}$ steps

- $u_4 = \boxed{3000}$ steps

- $u_5 = \boxed{3500}$ steps

Answer: The sequence increases by 500 steps each day.

- $u_1 = u_0 + 500 = 1000 + 500 = 1500$ steps

- $u_2 = u_1 + 500 = 1500 + 500 = 2000$ steps

- $u_3 = u_2 + 500 = 2000 + 500 = 2500$ steps

- $u_4 = u_3 + 500 = 2500 + 500 = 3000$ steps

- $u_5 = u_4 + 500 = 3000 + 500 = 3500$ steps

Ex 23: Suppose I deposit \$100 in a savings account. Each year, my amount is multiplied by 1.1 (that is, it increases by 10% every year).
Let u_n be the amount of money in the account after n years. What are the first five terms of the sequence (u_n)?

- $u_0 = \boxed{100}$ dollars

- $u_1 = \boxed{110}$ dollars

- $u_2 = \boxed{121}$ dollars

- $u_3 = \boxed{133.1}$ dollars

- $u_4 = \boxed{146.41}$ dollars

Answer: The amount increases by 10% each year, so it is multiplied by 1.1 each time:

- $u_0 = 100$

- $u_1 = 1.1 \times u_0 = 1.1 \times 100 = 110$

- $u_2 = 1.1 \times u_1 = 1.1 \times 110 = 121$

- $u_3 = 1.1 \times u_2 = 1.1 \times 121 = 133.1$

- $u_4 = 1.1 \times u_3 = 1.1 \times 133.1 = 146.41$

Ex 24: Let u_n be the amount of money I have at the beginning of week n. At the start, I have $u_0 = 20$ dollars. At the end of each week, my parents give me \$ 10 more.
What are the first terms of the sequence (u_n)?

- $u_1 = \boxed{30}$ dollars

- $u_2 = \boxed{40}$ dollars

- $u_3 = \boxed{50}$ dollars

- $u_4 = \boxed{60}$ dollars

- $u_5 = \boxed{70}$ dollars

Answer: The sequence increases by \$ 10 each week.

- $u_1 = u_0 + 10 = 20 + 10 = 30$ dollars

- $u_2 = u_1 + 10 = 30 + 10 = 40$ dollars

- $u_3 = u_2 + 10 = 40 + 10 = 50$ dollars

- $u_4 = u_3 + 10 = 50 + 10 = 60$ dollars

- $u_5 = u_4 + 10 = 60 + 10 = 70$ dollars