SEQUENCES

A NUMERICAL SEQUENCE

A.1 FINDING u_n

Ex 1:

n	1	2	3	4	5	6
u_n	3	5	7	9	11	13

What is u_4?

$\boxed{9}$

Answer: $u_4 = 9$.

Ex 2:

n	1	2	3	4	5	6
u_n	2	6	12	20	30	42

What is u_5?

$\boxed{30}$

Answer: $u_5 = 30$.

Ex 3:

n	1	2	3	4	5	6	7	8
u_n	4	9	16	25	36	49	64	81

What is u_7?

$\boxed{64}$

Answer: $u_7 = 64$.

Ex 4:

n	1	2	3	4	5	6	7	8
u_n	1	3	7	15	31	63	127	255

What is u_8?

$\boxed{255}$

Answer: $u_8 = 255$.

A.2 FINDING u_n IN AN ARITHMETIC SEQUENCE

Ex 5: What is u_6 for this sequence?

n	1	2	3	4	5	6
u_n	3	5	7	9	11	$\boxed{13}$

Answer: $u_6 = 13$, because each term increases by 2.

n	1	2	3	4	5	6
u_n	3	5	7	9	11	13

+2 +2 +2 +2 +2

Ex 6: What is u_6 for this sequence?

n	1	2	3	4	5	6
u_n	3	8	13	18	23	$\boxed{28}$

Answer: $u_6 = 28$, because each term increases by 5.

n	1	2	3	4	5	6
u_n	3	8	13	18	23	28

+5 +5 +5 +5 +5

Ex 7: What is u_5 for this sequence?

n	1	2	3	4	5
u_n	20	18	16	14	$\boxed{12}$

Answer: $u_5 = 12$, because each term decreases by 2.

n	1	2	3	4	5
u_n	20	18	16	14	12

−2 −2 −2 −2

Ex 8: What is u_6 for this sequence?

n	1	2	3	4	5	6
u_n	80	70	60	50	40	$\boxed{30}$

Answer: $u_6 = 30$, because each term decreases by 10.

n	1	2	3	4	5	6
u_n	80	70	60	50	40	30

−10 −10 −10 −10 −10

B DEFINITION USING A RECURSIVE RULE

B.1 CALCULATING THE FIRST TERMS

Ex 9: Write the sequence defined by: the first term is 7, and each term is obtained by adding 4 to the previous term.

$(\boxed{7}, \boxed{11}, \boxed{15}, \boxed{19}, \boxed{23}, \dots)$

Answer:

$$7 \xrightarrow{+4} 11 \xrightarrow{+4} 15 \xrightarrow{+4} 19 \xrightarrow{+4} 23$$

The sequence is: $(7, 11, 15, 19, 23, \dots)$.

Ex 10: Write the sequence defined by: the first term is 1, and each term is obtained by multiplying the previous term by 2.

$(\boxed{1}, \boxed{2}, \boxed{4}, \boxed{8}, \boxed{16}, \dots)$

Answer:

$$1 \xrightarrow{\times 2} 2 \xrightarrow{\times 2} 4 \xrightarrow{\times 2} 8 \xrightarrow{\times 2} 16$$

The sequence is: $(1, 2, 4, 8, 16, \dots)$.

Ex 11: Write the sequence defined by: the first term is 10, and each term is obtained by subtracting 5 from the previous term.

$(\boxed{10}, \boxed{5}, \boxed{0}, \boxed{-5}, \boxed{-10}, \dots)$

Answer:

$$10 \xrightarrow{-5} 5 \xrightarrow{-5} 0 \xrightarrow{-5} -5 \xrightarrow{-5} -10$$

The sequence is: $(10, 5, 0, -5, -10, \dots)$.

Ex 12: Write the sequence defined by: the first term is 2.5, and each term is obtained by adding 0.5 to the previous term.

$(\boxed{2.5}, \boxed{3}, \boxed{3.5}, \boxed{4}, \boxed{4.5}, \dots)$

Answer:

$$2.5 \xrightarrow{+0.5} 3 \xrightarrow{+0.5} 3.5 \xrightarrow{+0.5} 4 \xrightarrow{+0.5} 4.5$$

The sequence is: $(2.5, 3, 3.5, 4, 4.5, \dots)$.

B.2 IDENTIFYING THE RECURSIVE RULE

Ex 13: Given the sequence: $(3, 5, 7, 9, 11, 13, \dots)$

- The first term is $\boxed{3}$.
- The rule is $\boxed{\textbf{Add}}$ $\boxed{2}$

Answer:

- The first term is 3.
- The rule is add 2:

$$3 \xrightarrow{+2} 5 \xrightarrow{+2} 7 \xrightarrow{+2} 9 \xrightarrow{+2} 11 \xrightarrow{+2} 13$$

Ex 14: Given the sequence: $(60, 55, 50, 45, 40, 35, \dots)$

- The first term is $\boxed{60}$.
- The rule is $\boxed{\textbf{Subtract}}$ $\boxed{5}$

Answer:

- The first term is 60.
- The rule is subtract 5:

$$60 \xrightarrow{-5} 55 \xrightarrow{-5} 50 \xrightarrow{-5} 45 \xrightarrow{-5} 40 \xrightarrow{-5} 35$$

Ex 15: Given the sequence: $(64, 32, 16, 8, 4, 2, \dots)$

- The first term is $\boxed{64}$.
- The rule is $\boxed{\textbf{Divide}}$ $\boxed{2}$

Answer:

- The first term is 64.
- The rule is divide by 2:

$$64 \xrightarrow{\div 2} 32 \xrightarrow{\div 2} 16 \xrightarrow{\div 2} 8 \xrightarrow{\div 2} 4 \xrightarrow{\div 2} 2$$

Ex 16: Given the sequence: $(1, 10, 100, 1\,000, 10\,000, \dots)$

- The first term is $\boxed{1}$.
- The rule is $\boxed{\textbf{Multiply}}$ $\boxed{10}$

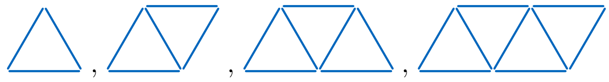
Answer:

- The first term is 1.
- The rule is multiply by 10:

$$1 \xrightarrow{\times 10} 10 \xrightarrow{\times 10} 100 \xrightarrow{\times 10} 1\,000 \xrightarrow{\times 10} 10\,000$$

B.3 IDENTIFYING THE RECURSIVE RULE IN GEOMETRIC PATTERNS

Ex 17: Observe the following pattern made with sticks:



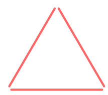
Fill in the table below:

Diagram number	1	2	3	4
Number of sticks	$\boxed{3}$	$\boxed{5}$	$\boxed{7}$	$\boxed{9}$

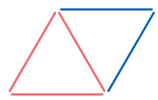
What rule can you find for the number of sticks?
Start with $\boxed{3}$ sticks. Add $\boxed{2}$ sticks for the next diagram.

Answer:

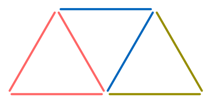
- For diagram number 1, the number of sticks is 3.



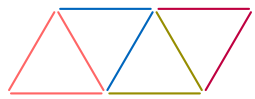
- For diagram number 2, the number of sticks is 5.



- For diagram number 3, the number of sticks is 7.

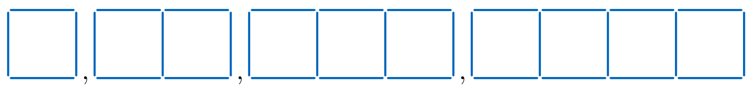


- For diagram number 4, the number of sticks is 9.



- **Rule:** Start with 3 sticks, and add 2 sticks for the next diagram.

Ex 18: Observe the following pattern made with sticks:



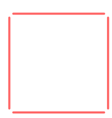
Fill in the table below:

Diagram number	1	2	3	4
Number of sticks	$\boxed{4}$	$\boxed{7}$	$\boxed{10}$	$\boxed{13}$

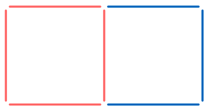
What rule can you find for the number of sticks?
Start with $\boxed{4}$ sticks. Add $\boxed{3}$ sticks for the next diagram.

Answer:

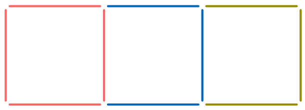
- For diagram number 1, the number of sticks is 4.



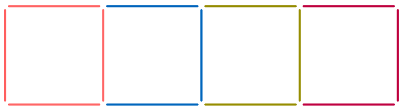
- For diagram number 2, the number of sticks is 7.



- For diagram number 3, the number of sticks is 10.

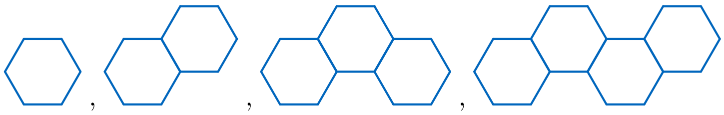


- For diagram number 4, the number of sticks is 13.



- **Rule:** Start with 4 sticks, and add 3 sticks for the next diagram.

Ex 19: Observe the following pattern made with sticks:



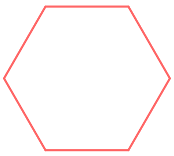
Fill in the table below:

Diagram number	1	2	3	4
Number of sticks	6	11	16	21

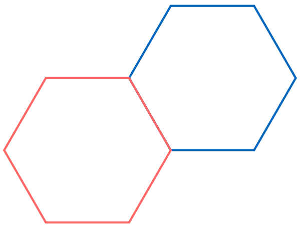
What rule can you find for the number of sticks?
Start with 6 sticks. Add 5 sticks for the next diagram.

Answer:

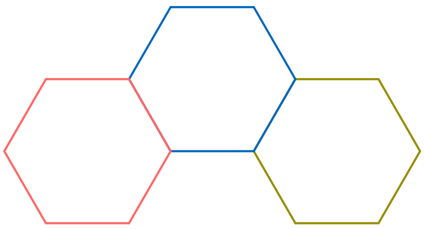
- **Diagram 1 :** For 1 hexagon, the number of sticks is 6.



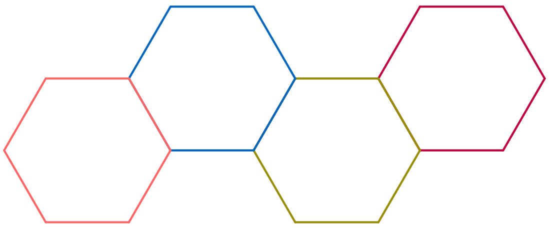
- **Diagram 2 :** For 2 hexagons, the number of sticks is 11.



- **Diagram 3 :** For 3 hexagons, the number of sticks is 16.



- **Diagram 4 :** For 4 hexagons, the number of sticks is 21.



- **Rule:** Start with 6 sticks, and add 5 sticks for each additional hexagon.

Ex 20: Observe the following pattern made with sticks:

Fill in the table below:

Diagram number	1	2	3	4
Number of sticks	4	6	8	10

What rule can you find for the number of sticks?
Start with 4 sticks. Add 2 sticks for the next diagram.

Answer:

- For diagram number 1, the number of sticks is 4.

- For diagram number 2, the number of sticks is 6.

- For diagram number 3, the number of sticks is 8.

- For diagram number 4, the number of sticks is 10.

- **Rule:** Start with 4 sticks, and add 2 sticks for each new diagram.

B.4 MODELING REAL SITUATIONS WITH SEQUENCES

Ex 21: A scientist observes a culture of bacteria. At the start, there are $u_0 = 5$ bacteria in a petri dish. Each day, the number of bacteria doubles.
Let u_n be the number of bacteria at the day n. What are the first five terms of the sequence (u_n)?

- $u_1 =$ 10 bacteria
- $u_2 =$ 20 bacteria

- $u_3 = \boxed{40}$ bacteria
- $u_4 = \boxed{80}$ bacteria
- $u_5 = \boxed{160}$ bacteria

Answer: The number of bacteria doubles each day:

- $u_1 = 2 \times u_0 = 2 \times 5 = 10$
- $u_2 = 2 \times u_1 = 2 \times 10 = 20$
- $u_3 = 2 \times u_2 = 2 \times 20 = 40$
- $u_4 = 2 \times u_3 = 2 \times 40 = 80$
- $u_5 = 2 \times u_4 = 2 \times 80 = 160$

Ex 22: Let u_n be the number of steps I have walked at the end of day n.On day 0, I walk $u_0 = 1000$ steps. Each day, I walk 500 more steps than the previous day.
What are the first terms of the sequence (u_n)?

- $u_1 = \boxed{1500}$ steps
- $u_2 = \boxed{2000}$ steps
- $u_3 = \boxed{2500}$ steps
- $u_4 = \boxed{3000}$ steps
- $u_5 = \boxed{3500}$ steps

Answer: The sequence increases by 500 steps each day.

- $u_1 = u_0 + 500 = 1000 + 500 = 1500$ steps
- $u_2 = u_1 + 500 = 1500 + 500 = 2000$ steps
- $u_3 = u_2 + 500 = 2000 + 500 = 2500$ steps
- $u_4 = u_3 + 500 = 2500 + 500 = 3000$ steps
- $u_5 = u_4 + 500 = 3000 + 500 = 3500$ steps

Ex 23: Suppose I deposit \$100 in a savings account. Each year, my amount is multiplied by 1.1 (that is, it increases by 10% every year).
Let u_n be the amount of money in the account after n years. What are the first five terms of the sequence (u_n)?

- $u_0 = \boxed{100}$ dollars
- $u_1 = \boxed{110}$ dollars
- $u_2 = \boxed{121}$ dollars
- $u_3 = \boxed{133.1}$ dollars
- $u_4 = \boxed{146.41}$ dollars

Answer: The amount increases by 10% each year, so it is multiplied by 1.1 each time:

- $u_0 = 100$
- $u_1 = 1.1 \times u_0 = 1.1 \times 100 = 110$
- $u_2 = 1.1 \times u_1 = 1.1 \times 110 = 121$
- $u_3 = 1.1 \times u_2 = 1.1 \times 121 = 133.1$
- $u_4 = 1.1 \times u_3 = 1.1 \times 133.1 = 146.41$

Ex 24: Let u_n be the amount of money I have at the beginning of week n.At the start, I have $u_0 = 20$ dollars. At the end of each week, my parents give me \$ 10 more.
What are the first terms of the sequence (u_n)?

- $u_1 = \boxed{30}$ dollars
- $u_2 = \boxed{40}$ dollars
- $u_3 = \boxed{50}$ dollars
- $u_4 = \boxed{60}$ dollars
- $u_5 = \boxed{70}$ dollars

Answer: The sequence increases by \$ 10 each week.

- $u_1 = u_0 + 10 = 20 + 10 = 30$ dollars
- $u_2 = u_1 + 10 = 30 + 10 = 40$ dollars
- $u_3 = u_2 + 10 = 40 + 10 = 50$ dollars
- $u_4 = u_3 + 10 = 50 + 10 = 60$ dollars
- $u_5 = u_4 + 10 = 60 + 10 = 70$ dollars

C DEFINITION USING AN EXPLICIT RULE

C.1 CALCULATING TERMS FROM AN EXPLICIT FORMULA

Ex 25: Consider the sequence defined by the explicit formula: $u_n = 3n + 2$.
Write the first four terms of this sequence.

- $u_0 = \boxed{2}$
- $u_1 = \boxed{5}$
- $u_2 = \boxed{8}$
- $u_3 = \boxed{11}$

Answer:

- For $n = 0$:
$$\begin{aligned} u_0 &= 3 \times 0 + 2 \\ &= 0 + 2 \\ &= 2 \end{aligned}$$
- For $n = 1$:
$$\begin{aligned} u_1 &= 3 \times 1 + 2 \\ &= 3 + 2 \\ &= 5 \end{aligned}$$
- For $n = 2$:
$$\begin{aligned} u_2 &= 3 \times 2 + 2 \\ &= 6 + 2 \\ &= 8 \end{aligned}$$

- For $n = 3$:

$$\begin{aligned} u_3 &= 3 \times 3 + 2 \\ &= 9 + 2 \\ &= 11 \end{aligned}$$

So the first four terms are: $2,\ 5,\ 8,\ 11$.

Ex 26: Consider the sequence defined by the explicit formula: $u_n = -10n + 100$.
Write the first four terms of this sequence.

- $u_0 = \boxed{100}$
- $u_1 = \boxed{90}$
- $u_2 = \boxed{80}$
- $u_3 = \boxed{70}$

Answer:

- For $n = 0$:

$$\begin{aligned} u_0 &= -10 \times 0 + 100 \\ &= 0 + 100 \\ &= 100 \end{aligned}$$

- For $n = 1$:

$$\begin{aligned} u_1 &= -10 \times 1 + 100 \\ &= -10 + 100 \\ &= 90 \end{aligned}$$

- For $n = 2$:

$$\begin{aligned} u_2 &= -10 \times 2 + 100 \\ &= -20 + 100 \\ &= 80 \end{aligned}$$

- For $n = 3$:

$$\begin{aligned} u_3 &= -10 \times 3 + 100 \\ &= -30 + 100 \\ &= 70 \end{aligned}$$

So the first four terms are: $100,\ 90,\ 80,\ 70$.

Ex 27: Consider the sequence defined by the explicit formula: $u_n = n^2 + 2$.
Write the first four terms of this sequence.

- $u_0 = \boxed{2}$
- $u_1 = \boxed{3}$
- $u_2 = \boxed{6}$
- $u_3 = \boxed{11}$

Answer:

- For $n = 0$:

$$\begin{aligned} u_0 &= 0^2 + 2 \\ &= 0 + 2 \\ &= 2 \end{aligned}$$

- For $n = 1$:

$$\begin{aligned} u_1 &= 1^2 + 2 \\ &= 1 + 2 \\ &= 3 \end{aligned}$$

- For $n = 2$:

$$\begin{aligned} u_2 &= 2^2 + 2 \\ &= 4 + 2 \\ &= 6 \end{aligned}$$

- For $n = 3$:

$$\begin{aligned} u_3 &= 3^2 + 2 \\ &= 9 + 2 \\ &= 11 \end{aligned}$$

So the first four terms are: $2,\ 3,\ 6,\ 11$.

Ex 28: Consider the sequence defined by the explicit formula: $u_n = (n + 1)n$.
Write the first four terms of this sequence.

- $u_0 = \boxed{0}$
- $u_1 = \boxed{2}$
- $u_2 = \boxed{6}$
- $u_3 = \boxed{12}$

Answer:

- For $n = 0$:

$$\begin{aligned} u_0 &= (0 + 1) \times 0 \\ &= 1 \times 0 \\ &= 0 \end{aligned}$$

- For $n = 1$:

$$\begin{aligned} u_1 &= (1 + 1) \times 1 \\ &= 2 \times 1 \\ &= 2 \end{aligned}$$

- For $n = 2$:

$$\begin{aligned} u_2 &= (2 + 1) \times 2 \\ &= 3 \times 2 \\ &= 6 \end{aligned}$$

- For $n = 3$:

$$\begin{aligned} u_3 &= (3 + 1) \times 3 \\ &= 4 \times 3 \\ &= 12 \end{aligned}$$

So the first four terms are: $0,\ 2,\ 6,\ 12$.

C.2 MODELING REAL SITUATIONS WITH SEQUENCES

Ex 29: You start with \$30 and each week your parent gives you \$10.
The amount of money you have after n weeks is given by the formula:

$$\begin{aligned} u_n &= \text{Initial Amount} + \text{Nbr weeks} \times \text{Amount received each week} \\ &= 30 + n \times 10 \\ &= 30 + 10n \end{aligned}$$

where u_n is the amount after n weeks. How much money will you have after 20 weeks?

$\boxed{230}$ dollars

Answer:

- After 20 weeks:
$$\begin{aligned} u_{20} &= 30 + 10 \times 20 \\ &= 30 + 200 \\ &= 230 \end{aligned}$$
So, after 20 weeks you will have \$230.

Ex 30: You deposit \$1 500 in a savings account that pays simple interest at a rate of 4% per year.
The amount of money in your account after n years is given by the formula:

$$\begin{aligned} u_n &= \text{Initial Amount} + \text{Nbr years} \times \text{Percentage of the initial amount} \\ &= 1\,500 + n \times 0.04 \times 1\,500 \\ &= 1\,500 + 60n \end{aligned}$$

where u_n is the amount after n years. What is your amount at year 20?

$\boxed{2700}$ dollars

Answer:

- At year 20:
$$\begin{aligned} u_{20} &= 1\,500 + 60 \times 20 \\ &= 1\,500 + 1\,200 \\ &= 2\,700 \end{aligned}$$
So, your amount at year 20 is \$2 700.

Ex 31: You start a stamp collection with 12 stamps. Each month, you add 4 new stamps to your collection.
The number of stamps you have after n months is given by the formula:

$$\begin{aligned} u_n &= \text{Initial number of stamps} + \text{Nbr months} \times \text{Stamps added per month} \\ &= 12 + n \times 4 \\ &= 12 + 4n \end{aligned}$$

where u_n is the number of stamps after n months. How many stamps will you have after 15 months?

$\boxed{72}$ stamps

Answer:

- After 15 months:
$$\begin{aligned} u_{15} &= 12 + 4 \times 15 \\ &= 12 + 60 \\ &= 72 \end{aligned}$$
So, after 15 months you will have 72 stamps.

Ex 32: A school plants 5 trees in its garden to start. Every year, they plant 3 new trees.
The total number of trees after n years is given by the formula:

$$\begin{aligned} u_n &= \text{Initial number of trees} + \text{Nbr years} \times \text{Trees planted per year} \\ &= 5 + n \times 3 \\ &= 5 + 3n \end{aligned}$$

where u_n is the number of trees after n years. How many trees will there be after 12 years?

$\boxed{41}$ trees

Answer:

- After 12 years:
$$\begin{aligned} u_{12} &= 5 + 3 \times 12 \\ &= 5 + 36 \\ &= 41 \end{aligned}$$
So, after 12 years there will be 41 trees.