SEQUENCES

A NUMERICAL SEQUENCE

Definition Numerical Sequence

A **numerical sequence** is an ordered list of numbers $(u_0, u_1, u_2, \dots)$ defined by a rule.

n	0	1	2	...
u_n	u_0	u_1	u_2	...

The number u_n is called the **nth term** of the sequence.

Ex: What is u_4 of this sequence?

n	0	1	2	3	4	5	...
u_n	3	5	7	9	11	13	...

Answer: $u_4 = 11$.

B DEFINITION USING A RECURSIVE RULE

Definition Recursive Rule

A sequence can be defined by:

- the **first term** (starting number), and
- a **recursive rule** that tells how to obtain each term from the previous one.

Ex: Write the sequence defined by: the first term is 2, and each term is obtained by adding 3 to the previous term.

Answer:

$$(\ 2\ \overset{+3}{\rightarrow}\ 5\ \overset{+3}{\rightarrow}\ 8\ \overset{+3}{\rightarrow}\ 11\ \overset{+3}{\rightarrow}\ 14\ ,\ \dots)$$

C DEFINITION USING AN EXPLICIT RULE

Definition Explicit Rule

A sequence can also be defined by an **explicit rule** (or **explicit formula**), which gives a direct formula for the nth term in terms of n:

$$u_n = \text{expression in } n$$

Ex: Consider the sequence defined by the explicit formula: $u_n = 3n + 2$.
Write the first five terms of this sequence.

Answer:

- For $n = 0$:

$$\begin{aligned} u_0 &= 3 \times 0 + 2 \\ &= 0 + 2 \\ &= 2 \end{aligned}$$

- For $n = 1$:

$$\begin{aligned} u_1 &= 3 \times 1 + 2 \\ &= 3 + 2 \\ &= 5 \end{aligned}$$

- For $n = 2$:

$$\begin{aligned} u_2 &= 3 \times 2 + 2 \\ &= 6 + 2 \\ &= 8 \end{aligned}$$

- For $n = 3$:

$$\begin{aligned} u_3 &= 3 \times 3 + 2 \\ &= 9 + 2 \\ &= 11 \end{aligned}$$

- For $n = 4$:

$$\begin{aligned} u_4 &= 3 \times 4 + 2 \\ &= 12 + 2 \\ &= 14 \end{aligned}$$

So the first five terms are: $2,\ 5,\ 8,\ 11,\ 14$.