SEQUENCES

A NUMERICAL SEQUENCE

A.1 FINDING u_n

Ex 1:

n	1	2	3	4	5	6
u_n	3	5	7	9	11	13

What is u_4?

[ ]

Ex 2:

n	1	2	3	4	5	6
u_n	2	6	12	20	30	42

What is u_5?

[ ]

Ex 3:

n	1	2	3	4	5	6	7	8
u_n	4	9	16	25	36	49	64	81

What is u_7?

[ ]

Ex 4:

n	1	2	3	4	5	6	7	8
u_n	1	3	7	15	31	63	127	255

What is u_8?

[ ]

A.2 FINDING u_n IN AN ARITHMETIC SEQUENCE

Ex 5: What is u_6 for this sequence?

n	1	2	3	4	5	6
u_n	3	5	7	9	11	[ ]

Ex 6: What is u_6 for this sequence?

n	1	2	3	4	5	6
u_n	3	8	13	18	23	[ ]

Ex 7: What is u_5 for this sequence?

n	1	2	3	4	5
u_n	20	18	16	14	[ ]

Ex 8: What is u_6 for this sequence?

n	1	2	3	4	5	6
u_n	80	70	60	50	40	[ ]

B DEFINITION USING A RECURSIVE RULE

B.1 CALCULATING THE FIRST TERMS

Ex 9: Write the sequence defined by: the first term is 7, and each term is obtained by adding 4 to the previous term.

([ ], [ ], [ ], [ ], [ ], ...)

Ex 10: Write the sequence defined by: the first term is 1, and each term is obtained by multiplying the previous term by 2.

([ ], [ ], [ ], [ ], [ ], ...)

Ex 11: Write the sequence defined by: the first term is 10, and each term is obtained by subtracting 5 from the previous term.

([ ], [ ], [ ], [ ], [ ], ...)

Ex 12: Write the sequence defined by: the first term is 2.5, and each term is obtained by adding 0.5 to the previous term.

([ ], [ ], [ ], [ ], [ ], ...)

B.2 CALCULATING THE FIRST TERMS

Ex 13: Calculate the first terms of the sequence defined by:

$$u_0 = 3 \text{ and } u_{n+1} = u_n + 4.$$

- $u_1 =$ [ ]
- $u_2 =$ [ ]
- $u_3 =$ [ ]
- $u_4 =$ [ ]
- $u_5 =$ [ ]

Ex 14: Calculate the first terms of the sequence defined by:

$$u_0 = 3 \text{ and } u_{n+1} = 2u_n.$$

- $u_1 =$ [ ]
- $u_2 =$ [ ]
- $u_3 =$ [ ]
- $u_4 =$ [ ]
- $u_5 =$ [ ]

Ex 15: Calculate the first terms of the sequence defined by:

$$u_0 = 12 \text{ and } u_{n+1} = u_n - 10.$$

- $u_1 =$ [ ]
- $u_2 =$ [ ]
- $u_3 =$ [ ]
- $u_4 =$ [ ]

- $u_5 =$ ______

Ex 16: Calculate the first terms of the sequence defined by:

$$u_0 = 64 \text{ and } u_{n+1} = \frac{u_n}{2}.$$

- $u_1 =$ ______
- $u_2 =$ ______
- $u_3 =$ ______
- $u_4 =$ ______
- $u_5 =$ ______

B.3 IDENTIFYING THE RECURSIVE RULE

Ex 17: Given the sequence: $(3, 5, 7, 9, 11, 13, \ldots)$

- The first term is ______.
- The rule is ☐ Add ☐ Subtract ☐ Multiply ☐ Divide ______

Ex 18: Given the sequence: $(60, 55, 50, 45, 40, 35, \ldots)$

- The first term is ______.
- The rule is ☐ Add ☐ Subtract ☐ Multiply ☐ Divide ______

Ex 19: Given the sequence: $(64, 32, 16, 8, 4, 2, \ldots)$

- The first term is ______.
- The rule is ☐ Add ☐ Subtract ☐ Multiply ☐ Divide ______

Ex 20: Given the sequence: $(1, 10, 100, 1\,000, 10\,000, \ldots)$

- The first term is ______.
- The rule is ☐ Add ☐ Subtract ☐ Multiply ☐ Divide ______

B.4 IDENTIFYING THE RECURSIVE RULE

Ex 21: Given the sequence: $(3, 5, 7, 9, 11, 13, \ldots)$, what is its recursive rule?

- $u_0 =$ ______.
- $u_{n+1} =$ ______

Ex 22: Given the sequence: $(100, 90, 80, 70, 60, \ldots)$, what is its recursive rule?

- $u_0 =$ ______.
- $u_{n+1} =$ ______

Ex 23: Given the sequence: $(2, 6, 18, 54, 162, \ldots)$, what is its recursive rule?

- $u_0 =$ ______.
- $u_{n+1} =$ ______

Ex 24: Given the sequence: $(8, 4, 2, 1, 0.5, 0.25, \ldots)$, what is its recursive rule?

- $u_0 =$ ______.
- $u_{n+1} =$ ______

B.5 MODELING REAL SITUATIONS

Ex 25: A scientist observes a culture of bacteria. At the start, there are $u_0 = 5$ bacteria in a petri dish. Each day, the number of bacteria doubles.
Let u_n be the number of bacteria at the day n. What are the first three terms of the sequence (u_n)?

- $u_1 =$ ______ bacteria
- $u_2 =$ ______ bacteria
- $u_3 =$ ______ bacteria

What is its recursive rule?

$$u_{n+1} = \text{______}$$

Ex 26: Each day, I walk more and more steps. On day 0, I walk $u_0 = 1000$ steps. Each day, I walk 500 steps more than the day before.
Let u_n be the number of steps I have walked at the end of day n. What are the first three terms of the sequence (u_n)?

- $u_1 =$ ______ steps
- $u_2 =$ ______ steps
- $u_3 =$ ______ steps

What is its recursive rule?

$$u_{n+1} = \text{______}$$

Ex 27: Suppose I deposit \$100 in a savings account. Each year, my amount is multiplied by 1.1 (that is, it increases by 10% each year).
Let u_n be the amount of money in the account after n years. What are the first three terms of the sequence (u_n)?

- $u_0 = \square$ dollars
- $u_1 = \square$ dollars
- $u_2 = \square$ dollars

What is its recursive rule?

$$u_{n+1} = \square$$

Ex 28: At the start, I have $u_0 = 20$ dollars. Each week, my parents give me \$10 more.
Let u_n be the amount of money I have at the beginning of week n. What are the first three terms of the sequence (u_n)?

- $u_1 = \square$ dollars
- $u_2 = \square$ dollars
- $u_3 = \square$ dollars

What is its recursive rule?

$$u_{n+1} = \square$$

C DEFINITION USING AN EXPLICIT RULE

C.1 CALCULATING TERMS FROM AN EXPLICIT FORMULA

Ex 29: Consider the sequence defined by the explicit formula: $u_n = 3n + 2$.
Write the first four terms of this sequence.

- $u_0 = \square$
- $u_1 = \square$
- $u_2 = \square$
- $u_3 = \square$

Ex 30: Consider the sequence defined by the explicit formula: $u_n = -10n + 100$.
Write the first four terms of this sequence.

- $u_0 = \square$
- $u_1 = \square$
- $u_2 = \square$
- $u_3 = \square$

Ex 31: Consider the sequence defined by the explicit formula: $u_n = n^2 + 2$.
Write the first four terms of this sequence.

- $u_0 = \square$
- $u_1 = \square$
- $u_2 = \square$
- $u_3 = \square$

Ex 32: Consider the sequence defined by the explicit formula: $u_n = (n + 1)n$.
Write the first four terms of this sequence.

- $u_0 = \square$
- $u_1 = \square$
- $u_2 = \square$
- $u_3 = \square$

C.2 MODELING REAL SITUATIONS WITH SEQUENCES

Ex 33: You start with \$30 and each week your parent gives you \$10.
The amount of money you have after n weeks is given by the formula:

$$\begin{aligned} u_n &= \text{Initial Amount} + \text{Nbr weeks} \times \text{Amount received each week} \\ &= 30 + n \times 10 \\ &= 30 + 10n \end{aligned}$$

where u_n is the amount after n weeks. How much money will you have after 20 weeks?

$\square$ dollars

Ex 34: You deposit \$1 500 in a savings account that pays simple interest at a rate of 4% per year.
The amount of money in your account after n years is given by the formula:

$$\begin{aligned} u_n &= \text{Initial Amount} + \text{Nbr years} \times \text{Percentage of the initial amoun} \\ &= 1\,500 + n \times 0.04 \times 1\,500 \\ &= 1\,500 + 60n \end{aligned}$$

where u_n is the amount after n years. What is your amount at year 20?

$\square$ dollars

Ex 35: You start a stamp collection with 12 stamps. Each month, you add 4 new stamps to your collection.
The number of stamps you have after n months is given by the formula:

$$\begin{aligned} u_n &= \text{Initial number of stamps} + \text{Nbr months} \times \text{Stamps added per r} \\ &= 12 + n \times 4 \\ &= 12 + 4n \end{aligned}$$

where u_n is the number of stamps after n months. How many stamps will you have after 15 months?

$\square$ stamps

Ex 36: A school plants 5 trees in its garden to start. Every year, they plant 3 new trees.
The total number of trees after n years is given by the formula:

$$\begin{aligned} u_n &= \text{Initial number of trees} + \text{Nbr years} \times \text{Trees planted per year} \\ &= 5 + n \times 3 \\ &= 5 + 3n \end{aligned}$$

where u_n is the number of trees after n years. How many trees will there be after 12 years?

[ ] trees