### A NUMERICAL SEQUENCE

#### A.1 FINDING $u_n$

Ex 1:

n	1	2	3	4	5	6
$u_n$	3	5	7	9	11	13

What is  $u_4$ ?

9

Answer:  $u_4 = 9$ .

Ex 2:

n	1	2	3	4	5	6
$u_n$	2	6	12	20	30	42

What is  $u_5$ ?

30

Answer:  $u_5 = 30$ .

Ex 3:

n	1	2	3	4	5	6	7	8
$u_n$	4	9	16	<b>25</b>	36	49	64	81

What is  $u_7$ ?

64

Answer:  $u_7 = 64$ .

Ex 4:

n	1	2	3	4	5	6	7	8
$u_n$	1	3	7	15	31	63	127	255

What is  $u_8$ ?

255

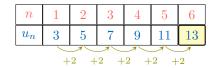
Answer:  $u_8 = 255$ .

#### A.2 FINDING $u_n$ IN AN ARITHMETIC SEQUENCE

**Ex 5:** What is  $u_6$  for this sequence?

	n	1	2	3	4	5	6
ĺ	$u_n$	3	5	7	9	11	13

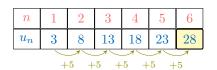
Answer:  $u_6 = 13$ , because each term increases by 2.



**Ex 6:** What is  $u_6$  for this sequence?

n	1	2	3	4	5	6
$u_n$	3	8	13	18	23	28

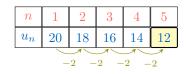
Answer:  $u_6 = 28$ , because each term increases by 5.



**Ex 7:** What is  $u_5$  for this sequence?

n	1	2	3	4	5
$u_n$	20	18	16	14	12

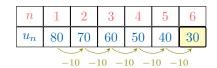
Answer:  $u_5 = 12$ , because each term decreases by 2.



**Ex 8:** What is  $u_6$  for this sequence?

n	1	2	3	4	5	6
$u_n$	80	70	<b>60</b>	<b>50</b>	<b>40</b>	30

Answer:  $u_6 = 30$ , because each term decreases by 10.



#### B DEFINITION USING A RECURSIVE RULE

#### **B.1 CALCULATING THE FIRST TERMS**

**Ex 9:** Write the sequence defined by: the first term is 7, and each term is obtained by adding 4 to the previous term.

$$(7, 11, 15, 19, 23, \dots)$$

Answer:

$$7 \xrightarrow{+4} 11 \xrightarrow{+4} 15 \xrightarrow{+4} 19 \xrightarrow{+4} 23$$

The sequence is: (7, 11, 15, 19, 23, ...).

Ex 10: Write the sequence defined by: the first term is 1, and each term is obtained by multiplying the previous term by 2.

$$(1, 2, 4, 8, 16, \dots)$$

Answer:

$$1 \xrightarrow{\times 2} 2 \xrightarrow{\times 2} 4 \xrightarrow{\times 2} 8 \xrightarrow{\times 2} 16$$

The sequence is: (1, 2, 4, 8, 16, ...).

Ex 11: Write the sequence defined by: the first term is 10, and each term is obtained by subtracting 5 from the previous term.

$$(10, 5, 0, -5, -10, \dots)$$

Answer:

$$10 \xrightarrow{-5} 5 \xrightarrow{-5} 0 \xrightarrow{-5} -5 \xrightarrow{-5} -10$$

The sequence is: (10, 5, 0, -5, -10, ...).

Ex 12: Write the sequence defined by: the first term is 2.5, and each term is obtained by adding 0.5 to the previous term.

$$(2.5, 3, 3.5, 4, 4.5, \dots)$$

Answer:

$$2.5 \xrightarrow{+0.5} 3 \xrightarrow{+0.5} 3.5 \xrightarrow{+0.5} 4 \xrightarrow{+0.5} 4.5$$

The sequence is: (2.5, 3, 3.5, 4, 4.5, ...).

#### **B.2 CALCULATING THE FIRST TERMS**

Ex 13: Calculate the first terms of the sequence defined by:

$$u_0 = 3$$
 and  $u_{n+1} = u_n + 4$ .

- $u_1 = \boxed{7}$
- $u_2 = \boxed{11}$
- $u_3 = \boxed{15}$
- $u_4 = \boxed{19}$
- $u_5 = 23$

Answer:

$$3 \xrightarrow{+4} 7 \xrightarrow{+4} 11 \xrightarrow{+4} 15 \xrightarrow{+4} 19 \xrightarrow{+4} 23$$

- $u_1 = u_0 + 4 = 3 + 4 = 7$
- $u_2 = u_1 + 4 = 7 + 4 = 11$
- $u_3 = u_2 + 4 = 11 + 4 = 15$
- $u_4 = u_3 + 4 = 15 + 4 = 19$
- $u_5 = u_4 + 4 = 19 + 4 = 23$

Ex 14: Calculate the first terms of the sequence defined by:

$$u_0 = 3$$
 and  $u_{n+1} = 2u_n$ .

- $u_1 = 6$
- $u_2 = \boxed{12}$
- $u_3 = 24$
- $u_4 = \boxed{48}$
- $u_5 = 96$

Answer:

$$3 \xrightarrow{\times 2} 6 \xrightarrow{\times 2} 12 \xrightarrow{\times 2} 24 \xrightarrow{\times 2} 48 \xrightarrow{\times 2} 96$$

- $u_1 = 2u_0 = 2 \times 3 = 6$
- $u_2 = 2u_1 = 2 \times 6 = 12$
- $u_3 = 2u_2 = 2 \times 12 = 24$
- $u_4 = 2u_3 = 2 \times 24 = 48$
- $u_5 = 2u_4 = 2 \times 48 = 96$

**Ex 15:** Calculate the first terms of the sequence defined by:  $u_0 = 12$  and  $u_{n+1} = u_n - 10$ .

- $u_1 = \boxed{2}$
- $u_2 = \boxed{-8}$
- $u_3 = \boxed{-18}$
- $u_4 = \boxed{-28}$
- $u_5 = \boxed{-38}$

Answer:

$$12 \xrightarrow{-10} 2 \xrightarrow{-10} -8 \xrightarrow{-10} -18 \xrightarrow{-10} -28 \xrightarrow{-10} -38$$

- $u_1 = u_0 10 = 12 10 = 2$
- $u_2 = u_1 10 = 2 10 = -8$
- $u_3 = u_2 10 = -8 10 = -18$
- $u_4 = u_3 10 = -18 10 = -28$
- $u_5 = u_4 10 = -28 10 = -38$

Ex 16: Calculate the first terms of the sequence defined by:

$$u_0 = 64$$
 and  $u_{n+1} = \frac{u_n}{2}$ .

- $u_1 = 32$
- $u_2 = 16$
- $u_3 = 8$
- $u_4 = \boxed{4}$
- $u_5 = \boxed{2}$

Answer:

$$64 \xrightarrow{\div 2} 32 \xrightarrow{\div 2} 16 \xrightarrow{\div 2} 8 \xrightarrow{\div 2} 4 \xrightarrow{\div 2} 2$$

- $u_1 = \frac{u_0}{2} = \frac{64}{2} = 32$
- $u_2 = \frac{u_1}{2} = \frac{32}{2} = 16$
- $u_3 = \frac{u_2}{2} = \frac{16}{2} = 8$
- $u_4 = \frac{u_3}{2} = \frac{8}{2} = 4$
- $u_5 = \frac{u_4}{2} = \frac{4}{2} = 2$

#### **B.3 IDENTIFYING THE RECURSIVE RULE**

**Ex 17:** Given the sequence: (3, 5, 7, 9, 11, 13, ...)

- The first term is  $\boxed{3}$ .
- The rule is Add 2

Answer:

- The first term is 3.
- The rule is add 2:

$$3 \xrightarrow{+2} 5 \xrightarrow{+2} 7 \xrightarrow{+2} 9 \xrightarrow{+2} 11 \xrightarrow{+2} 13$$

**Ex 18:** Given the sequence: (60, 55, 50, 45, 40, 35, ...)

- The first term is 60.
- The rule is Subtract 5

Answer:

- The first term is 60.
- The rule is subtract 5:

$$60 \xrightarrow{-5} 55 \xrightarrow{-5} 50 \xrightarrow{-5} 45 \xrightarrow{-5} 40 \xrightarrow{-5} 35$$

**Ex 19:** Given the sequence: (64, 32, 16, 8, 4, 2, ...)

- The first term is 64.
- The rule is Divide 2

Answer:

- The first term is 64.
- The rule is divide by 2:

$$64 \xrightarrow{\div 2} 32 \xrightarrow{\div 2} 16 \xrightarrow{\div 2} 8 \xrightarrow{\div 2} 4 \xrightarrow{\div 2} 2$$

**Ex 20:** Given the sequence: (1, 10, 100, 1000, 10000, ...)

- The first term is 1.
- The rule is **Multiply** 10

Answer:

- The first term is 1.
- The rule is multiply by 10:

$$1 \xrightarrow{\times 10} 10 \xrightarrow{\times 10} 100 \xrightarrow{\times 10} 1000 \xrightarrow{\times 10} 10000$$

#### **B.4 IDENTIFYING THE RECURSIVE RULE**

**Ex 21:** Given the sequence: (3, 5, 7, 9, 11, 13, ...), what is its recursive rule?

- $u_0 = \boxed{3}$ .
- $\bullet \ u_{n+1} = \boxed{u_n + 2}$

Answer:

- The first term is  $u_0 = 3$ .
- The rule is add 2:

$$(3,5,7,9,\ldots,u_n,u_{n+1},\ldots)$$

$$u_{n+1} = u_n + 2$$

Ex 22: Given the sequence: (100, 90, 80, 70, 60, ...), what is its recursive rule?

- $u_0 = \boxed{100}$ .
- $\bullet \ u_{n+1} = \boxed{u_n 10}$

Answer:

- The first term is  $u_0 = 100$ .
- The rule is subtract 10:

$$(100, 90, 80, 70, \dots, u_n, u_{n+1}, \dots)$$

$$u_{n+1} = u_n - 10$$

**Ex 23:** Given the sequence:  $(2, 6, 18, 54, 162, \dots)$ , what is its recursive rule?

- $u_0 = \boxed{2}$ .
- $\bullet \ u_{n+1} = \boxed{3 \times u_n}$

Answer:

- The first term is  $u_0 = 2$ .
- The rule is multiply by 3:

$$(2,6,18,54,\ldots,u_n,u_{n+1},\ldots)$$

$$u_{n+1} = 3 \times u_n$$

**Ex 24:** Given the sequence: (8, 4, 2, 1, 0.5, 0.25, ...), what is its recursive rule?

- $u_0 = 8$ .
- $\bullet \ u_{n+1} = \boxed{u_n \div 2}$

Answer:

- The first term is  $u_0 = 8$ .
- The rule is divide by 2:

$$(8,4,2,1,0.5,\ldots,u_n,u_{n+1},\ldots)$$

$$u_{n+1} = \frac{u_n}{2}$$

#### **B.5 MODELING REAL SITUATIONS**

**Ex 25:** A scientist observes a culture of bacteria. At the start, there are  $u_0 = 5$  bacteria in a petri dish. Each day, the number of bacteria doubles.

Let  $u_n$  be the number of bacteria at the day n. What are the first three terms of the sequence  $(u_n)$ ?

- $u_1 = \boxed{10}$  bacteria
- $u_2 = \boxed{20}$  bacteria
- $u_3 = \boxed{40}$  bacteria

What is its recursive rule?

$$u_{n+1} = \boxed{2 \times u_n}$$

Answer: The number of bacteria doubles each day:

- $u_1 = 2 \times u_0 = 2 \times 5 = 10$
- $u_2 = 2 \times u_1 = 2 \times 10 = 20$
- $u_3 = 2 \times u_2 = 2 \times 20 = 40$

The rule is multiply by 2:

$$u_{n+1} = 2 \times u_n$$

**Ex 26:** Each day, I walk more and more steps. On day 0, I walk  $u_0 = 1000$  steps. Each day, I walk 500 steps more than the day before.

Let  $u_n$  be the number of steps I have walked at the end of day n. What are the first three terms of the sequence  $(u_n)$ ?

- $u_1 = |1500|$  steps
- $u_2 = 2000$  steps
- $u_3 = |2500|$  steps

What is its recursive rule?

$$u_{n+1} = \boxed{u_n + 500}$$

Answer: The number of steps increases by 500 each day:

- $u_1 = u_0 + 500 = 1000 + 500 = 1500$
- $u_2 = u_1 + 500 = 1500 + 500 = 2000$
- $u_3 = u_2 + 500 = 2000 + 500 = 2500$

The recursive rule is:

$$u_{n+1} = u_n + 500$$

Ex 27: Suppose I deposit \$100 in a savings account. Each year, my amount is multiplied by 1.1 (that is, it increases by 10% each year).

Let  $u_n$  be the amount of money in the account after n years. What are the first three terms of the sequence  $(u_n)$ ?

•  $u_0 = \boxed{100}$  dollars

- $u_1 = \boxed{110}$  dollars
- $u_2 = \boxed{121}$  dollars

What is its recursive rule?

$$u_{n+1} = \boxed{1.1 \times u_n}$$

Answer: The amount increases by 10% each year, so it is multiplied by 1.1:

- $u_0 = 100$
- $u_1 = 1.1 \times u_0 = 1.1 \times 100 = 110$
- $u_2 = 1.1 \times u_1 = 1.1 \times 110 = 121$

The recursive rule is:

$$u_{n+1} = 1.1 \times u_n$$

**Ex 28:** At the start, I have  $u_0 = 20$  dollars. Each week, my parents give me \$10 more.

Let  $u_n$  be the amount of money I have at the beginning of week n. What are the first three terms of the sequence  $(u_n)$ ?

- $u_1 = \boxed{30}$  dollars
- $u_2 = \boxed{40}$  dollars
- $u_3 = \boxed{50}$  dollars

What is its recursive rule?

$$u_{n+1} = \boxed{u_n + 10}$$

Answer: The sequence increases by \$10 each week:

- $u_1 = u_0 + 10 = 20 + 10 = 30$
- $u_2 = u_1 + 10 = 30 + 10 = 40$
- $u_3 = u_2 + 10 = 40 + 10 = 50$

The recursive rule is:

$$u_{n+1} = u_n + 10$$

#### C DEFINITION USING AN EXPLICIT RULE

## C.1 CALCULATING TERMS FROM AN EXPLICIT FORMULA

**Ex 29:** Consider the sequence defined by the explicit formula:  $u_n = 3n + 2$ .

Write the first four terms of this sequence.

- $u_0 = \boxed{2}$
- $u_1 = 5$
- $u_2 = \boxed{8}$
- $u_3 = |11|$

Answer:

• For n=0:

$$u_0 = 3 \times 0 + 2$$
  
= 0 + 2  
= 2

• For n = 1:

$$u_1 = 3 \times 1 + 2$$
  
= 3 + 2  
= 5

• For n=2:

$$u_2 = 3 \times 2 + 2$$
  
= 6 + 2  
= 8

• For n=3:

$$u_3 = 3 \times 3 + 2$$
  
= 9 + 2  
= 11

So the first four terms are: 2, 5, 8, 11.

**Ex 30:** Consider the sequence defined by the explicit formula:  $u_n = -10n + 100$ .

Write the first four terms of this sequence.

• 
$$u_0 = 100$$

• 
$$u_1 = \boxed{90}$$

• 
$$u_2 = \boxed{80}$$

• 
$$u_3 = \boxed{70}$$

Answer:

• For n = 0:

$$u_0 = -10 \times 0 + 100$$
  
= 0 + 100  
= 100

• For n = 1:

$$u_1 = -10 \times 1 + 100$$
  
= -10 + 100  
= 90

• For n=2:

$$u_2 = -10 \times 2 + 100$$
$$= -20 + 100$$
$$= 80$$

• For n = 3:

$$u_3 = -10 \times 3 + 100$$
$$= -30 + 100$$
$$= 70$$

So the first four terms are: 100, 90, 80, 70.

**Ex 31:** Consider the sequence defined by the explicit formula:  $u_n = n^2 + 2$ .

Write the first four terms of this sequence.

$$\bullet \ u_0 = \boxed{2}$$

• 
$$u_1 = \boxed{3}$$

• 
$$u_2 = 6$$

• 
$$u_3 = \boxed{11}$$

Answer:

• For n = 0:

$$u_0 = 0^2 + 2$$
$$= 0 + 2$$
$$= 2$$

• For n = 1:

$$u_1 = 1^2 + 2$$
$$= 1 + 2$$
$$= 3$$

• For n = 2:

$$u_2 = 2^2 + 2$$
$$= 4 + 2$$
$$= 6$$

• For n = 3:

$$u_3 = 3^2 + 2$$
$$= 9 + 2$$
$$= 11$$

So the first four terms are: 2, 3, 6, 11.

**Ex 32:** Consider the sequence defined by the explicit formula:  $u_n = (n+1)n$ .

Write the first four terms of this sequence.

• 
$$u_0 = \boxed{0}$$

• 
$$u_1 = \boxed{2}$$

• 
$$u_2 = \boxed{6}$$

• 
$$u_3 = \boxed{12}$$

Answer:

• For n = 0:

$$u_0 = (0+1) \times 0$$
$$= 1 \times 0$$
$$= 0$$

• For n = 1:

$$u_1 = (1+1) \times 1$$
$$= 2 \times 1$$
$$= 2$$

• For n = 2:

$$u_2 = (2+1) \times 2$$
$$= 3 \times 2$$
$$= 6$$

• For n = 3:

$$u_3 = (3+1) \times 3$$
$$= 4 \times 3$$
$$= 12$$

So the first four terms are: 0, 2, 6, 12.

# C.2 MODELING REAL SITUATIONS WITH SEQUENCES

 $\mathbf{Ex}$  33: You start with \$30 and each week your parent gives you \$10

The amount of money you have after n weeks is given by the formula:

 $u_n = \text{Initial Amount} + \text{Nbr weeks} \times \text{Amount received each week}$ 

$$=30+n\times10$$

$$= 30 + 10n$$

where  $u_n$  is the amount after n weeks. How much money will you have after 20 weeks?

230 dollars

Answer:

• After 20 weeks:

$$u_{20} = 30 + 10 \times 20$$
  
=  $30 + 200$   
=  $230$ 

So, after 20 weeks you will have \$230.

**Ex 34:** You deposit \$1 500 in a savings account that pays simple interest at a rate of 4% per year.

The amount of money in your account after n years is given by the formula:

 $u_n = \text{Initial Amount} + \text{Nbr years} \times \text{Percentage of the initial amount}$ 

$$= 1500 + n \times 0.04 \times 1500$$

$$= 1500 + 60n$$

where  $u_n$  is the amount after n years. What is your amount at year 20?

2700 dollars

Answer:

• At year 20:

$$u_{20} = 1500 + 60 \times 20$$
$$= 1500 + 1200$$
$$= 2700$$

So, your amount at year 20 is \$2,700.

Ex 35: You start a stamp collection with 12 stamps. Each month, you add 4 new stamps to your collection.

The number of stamps you have after n months is given by the formula:

 $u_n = \text{Initial number of stamps} + \text{Nbr months} \times \text{Stamps added per month}$ 

$$=12+n\times4$$

$$= 12 + 4n$$

where  $u_n$  is the number of stamps after n months. How many stamps will you have after 15 months?

72 stamps

Answer:

• After 15 months:

$$u_{15} = 12 + 4 \times 15$$
  
= 12 + 60  
= 72

So, after 15 months you will have 72 stamps.

**Ex 36:** A school plants 5 trees in its garden to start. Every year, they plant 3 new trees.

The total number of trees after n years is given by the formula:

 $u_n = \text{Initial number of trees} + \text{Nbr years} \times \text{Trees planted per year}$ 

$$=5+n\times3$$

$$= 5 + 3n$$

where  $u_n$  is the number of trees after n years. How many trees will there be after 12 years?

41 trees

Answer:

• After 12 years:

$$u_{12} = 5 + 3 \times 12$$
  
= 5 + 36  
= 41

So, after 12 years there will be 41 trees.