SEQUENCES

A NUMERICAL SEQUENCE

Definition Numerical Sequence

A **numerical sequence** is an ordered list of numbers $(u_0, u_1, u_2, \dots)$ defined by a rule.

n	0	1	2	...
u_n	u_0	u_1	u_2	...

The number u_n is called the **nth term** of the sequence.

Ex: What is u_4 of this sequence?

n	0	1	2	3	4	5	...
u_n	3	5	7	9	11	13	...

Answer: $u_4 = 11$.

B DEFINITION USING A RECURSIVE RULE

Discover: Let's consider a sequence where the first term is 2, and each term is obtained by adding 3 to the previous term. The terms are:

$$\overset{+3 \quad +3 \quad +3}{(\ 2\ ,\ 5\ ,\ 8\ ,\ 11\ ,\ \dots)}$$
$$\begin{matrix} \| & \| & \| & \| \\ u_0 & u_1 & u_2 & u_3 \end{matrix}$$

We observe:

- $5 = 2{+}3 \longrightarrow u_1 = u_0{+}3 \longrightarrow u_{0+1} = u_0 + 3$
- $8 = 5{+}3 \longrightarrow u_2 = u_1{+}3 \longrightarrow u_{1+1} = u_1 + 3$
- $11 = 8{+}3 \longrightarrow u_3 = u_2{+}3 \longrightarrow u_{2+1} = u_2 + 3$
- $\vdots$
- So the rule is $u_{n+1} = u_n + 3$

$$\overset{+3 \quad +3 \quad +3 \qquad\qquad +3}{(\ u_0\ ,\ u_1\ ,\ u_2\ ,\ u_3\ ,\ \dots\ ,\ u_n\ ,\ u_{n+1}\ ,\ \dots)}$$

Definition Recursive Rule

A sequence can be defined by:

- the **first term** (starting number): u_0
- a **recursive rule** that tells how to obtain each term from the previous one:

$$u_{n+1} = \text{expression in } u_n$$

Ex: Write the recursive rule when each term is obtained by adding 2 to the previous term.

Answer:

$$u_{n+1} = u_n + 2$$

C DEFINITION USING AN EXPLICIT RULE

Definition Explicit Rule

A sequence can also be defined by an **explicit rule** (or **explicit formula**), which gives a direct formula for the nth term in terms of n:

$$u_n = \text{expression in } n$$

Ex: Consider the sequence defined by the explicit formula: $u_n = 3n + 2$.
Write the first five terms of this sequence.

Answer:

- For $n = 0$:

$$\begin{aligned} u_0 &= 3 \times 0 + 2 \\ &= 0 + 2 \\ &= 2 \end{aligned}$$

- For $n = 1$:

$$\begin{aligned} u_1 &= 3 \times 1 + 2 \\ &= 3 + 2 \\ &= 5 \end{aligned}$$

- For $n = 2$:

$$\begin{aligned} u_2 &= 3 \times 2 + 2 \\ &= 6 + 2 \\ &= 8 \end{aligned}$$

- For $n = 3$:

$$\begin{aligned} u_3 &= 3 \times 3 + 2 \\ &= 9 + 2 \\ &= 11 \end{aligned}$$

- For $n = 4$:

$$\begin{aligned} u_4 &= 3 \times 4 + 2 \\ &= 12 + 2 \\ &= 14 \end{aligned}$$

So the first five terms are: $2,\ 5,\ 8,\ 11,\ 14$.