SEQUENCES

A NUMERICAL SEQUENCE

Definition Numerical Sequence

A **numerical sequence** is an ordered list of numbers $(u_0, u_1, u_2, \dots)$ defined by a rule.

n	0	1	2	...
u_n	u_0	u_1	u_2	...

The number u_n is called the **nth term** of the sequence.

Ex: What is u_4 of this sequence?

n	0	1	2	3	4	5	...
u_n	3	5	7	9	11	13	...

Answer: $u_4 = 11$.

B DEFINITION USING A RECURSIVE RULE

Definition Recursive Rule

A sequence can be defined by:

- the **first term** (starting number): u_0

- a **recursive rule** that tells how to obtain each term from the previous one:

$$u_{n+1} = \text{expression in } u_n$$

Ex: Write the recursive rule when each term is obtained by adding 2 to the previous term.

Answer:

$$u_{n+1} = u_n + 2$$

C DEFINITION USING AN EXPLICIT RULE

Definition Explicit Rule

A sequence can also be defined by an **explicit rule** (or **explicit formula**), which gives a direct formula for the nth term in terms of n:

$$u_n = \text{expression in } n$$

Ex: Consider the sequence defined by the explicit formula: $u_n = 3n + 2$.
Write the first five terms of this sequence.

Answer:

- For $n = 0$:

$$
\begin{aligned}
u_0 &= 3 \times 0 + 2 \\
&= 0 + 2 \\
&= 2
\end{aligned}
$$

- For $n = 1$:

$$
\begin{aligned}
u_1 &= 3 \times 1 + 2 \\
&= 3 + 2 \\
&= 5
\end{aligned}
$$

- For $n = 2$:

$$
\begin{aligned}
u_2 &= 3 \times 2 + 2 \\
&= 6 + 2 \\
&= 8
\end{aligned}
$$

- For $n = 3$:
$$u_3 = 3 \times 3 + 2$$
$$= 9 + 2$$
$$= 11$$

- For $n = 4$:
$$u_4 = 3 \times 4 + 2$$
$$= 12 + 2$$
$$= 14$$

So the first five terms are: 2, 5, 8, 11, 14.