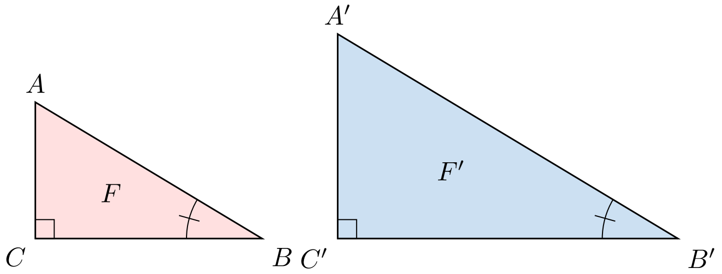
SIMILAR TRIANGLES

A ANGLE-ANGLE SIMILARITY

A.1 CHOOSING MATHEMATICAL ARGUMENTATION

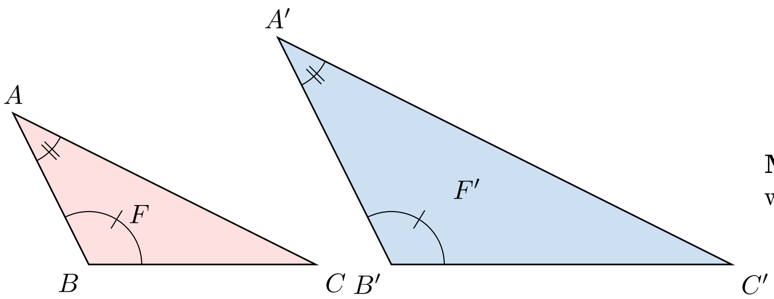
MCQ 1: Choose the correct mathematical argumentation for why the figures F and F' are similar.



- ☐ The triangles look the same.
- ☒ Both figures are right triangles with a common marked angle, so the triangles F and F' are similar.
- ☐ Both figures are right triangles, so the triangles F and F' are similar.
- ☐ Both triangles have the same marked angle, so the triangles F and F' are similar.

Answer: The correct argumentation is that both figures are right triangles (each has a right angle at C and C') and both triangles have the same marked angle ($\angle ABC = \angle A'B'C'$). By the Angle-Angle (AA) similarity criterion, triangles F and F' are similar.

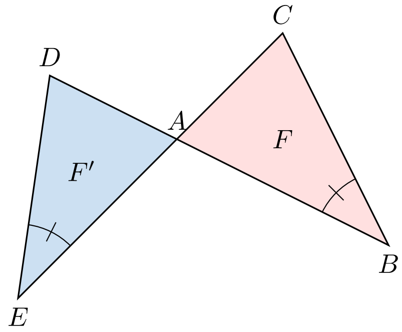
MCQ 2: Choose the correct mathematical argumentation for why the figures F and F' are similar.



- ☐ The triangles look the same.
- ☐ Both figures are right triangles with a common marked angle, so the triangles F and F' are similar.
- ☐ Both triangles have the same marked angle, so the triangles F and F' are similar.
- ☒ Both triangles have two marked angles in common, so the triangles F and F' are similar.

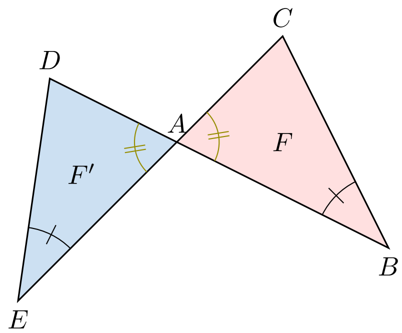
Answer: The correct argumentation is that both triangles have two marked angles in common ($\angle ABC = \angle A'B'C'$ and $\angle BAC = \angle B'A'C'$). By the Angle-Angle (AA) similarity criterion, triangles F and F' are similar.

MCQ 3: Choose the correct mathematical argumentation for why the figures F and F' are similar.

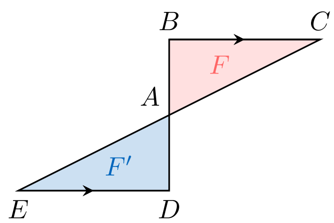


- ☐ The triangles look the same.
- ☒ Both triangles have a common marked angle and a pair of vertically opposite angles, so the triangles F and F' are similar.
- ☐ Both triangles have the same marked angle, so the triangles F and F' are similar.
- ☐ Both figures have a pair of vertically opposite angles, so the triangles F and F' are similar.

Answer: The correct argumentation is that both triangles have a common marked angle ($\angle CBA = \angle AED$) and a pair of vertically opposite angles ($\angle BAC = \angle EAD$ at vertex A), which are equal. By the Angle-Angle (AA) similarity criterion, triangles F and F' are similar.

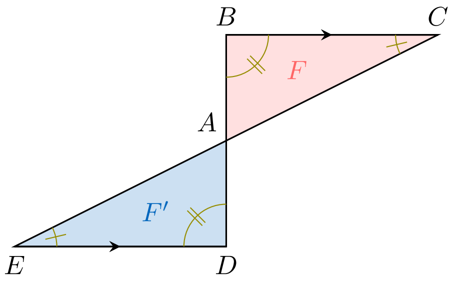


MCQ 4: Choose the correct mathematical argumentation for why the figures F and F' are similar.



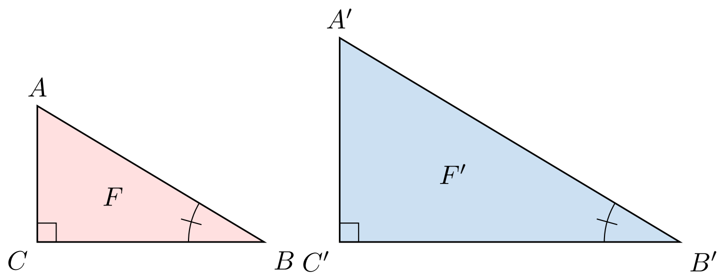
- ☐ The triangles look the same.
- ☐ Both triangles have a common marked angle and a pair of vertically opposite angles, so the triangles F and F' are similar.
- ☒ Since the lines are parallel, the corresponding angles in the two triangles are equal. So, the triangles F and F' are similar.
- ☐ Both figures have a pair of vertically opposite angles, so the triangles F and F' are similar.

Answer: The correct argumentation is "Since the lines are parallel, the corresponding angles in the two triangles are equal ($\angle ABC = \angle ADE$, $\angle BCA = \angle AED$). So, the triangles F and F' are similar." . By the Angle-Angle (AA) similarity criterion, triangles F and F' are similar.



A.2 WRITING MATHEMATICAL ARGUMENTATION

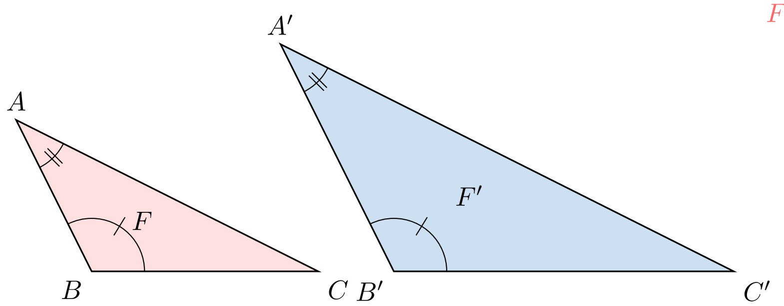
Ex 5: Justify with mathematical argumentation why the figures F and F' are similar.



Students should write a response such as: "Both figures are right triangles with a common marked angle, so triangles F and F' are similar by the Angle-Angle (AA) criterion." or "$\angle ABC = \angle A'B'C'$ and $\angle ACB = \angle A'C'B'$, so triangles F and F' are similar by the Angle-Angle (AA) criterion." The most important aspect is that they state that the two triangles have two equal angles, which ensures similarity by the AA criterion.

Answer: Both figures are right triangles (each has a right angle at C and C') and both triangles have the same marked angle ($\angle ABC = \angle A'B'C'$). By the Angle-Angle (AA) similarity criterion, triangles F and F' are similar.

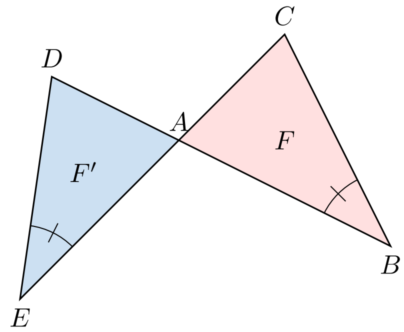
Ex 6: Justify with mathematical argumentation why the figures F and F' are similar.



Students should write a response such as: "Both triangles have two marked angles in common, so triangles F and F' are similar by the Angle-Angle (AA) criterion." or "$\angle ABC = \angle A'B'C'$ and $\angle BAC = \angle B'A'C'$, so triangles F and F' are similar by the Angle-Angle (AA) criterion." The most important aspect is that they state that the two triangles have two equal angles, which ensures similarity by the AA criterion.

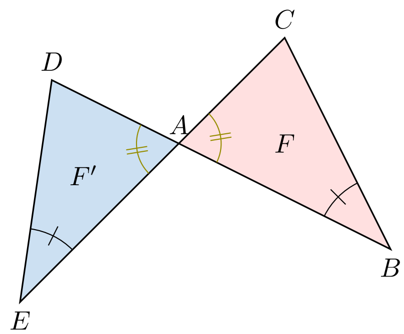
Answer: Both triangles have two marked angles in common ($\angle ABC = \angle A'B'C'$ and $\angle BAC = \angle B'A'C'$). By the Angle-Angle (AA) similarity criterion, triangles F and F' are similar.

Ex 7: Justify with mathematical argumentation why the figures F and F' are similar.

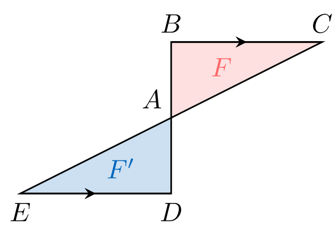


Students should write a response such as: "Both triangles have a common marked angle and a pair of vertically opposite angles, so triangles F and F' are similar by the Angle-Angle (AA) criterion." or "$\angle CBA = \angle AED$ and $\angle BAC = \angle EAD$, so triangles F and F' are similar by the Angle-Angle (AA) criterion." The most important aspect is that they state that the two triangles have two equal angles, which ensures similarity by the AA criterion.

Answer: Both triangles have a common marked angle ($\angle CBA = \angle AED$) and a pair of vertically opposite angles ($\angle BAC = \angle EAD$ at vertex A), which are equal. By the Angle-Angle (AA) similarity criterion, triangles F and F' are similar.

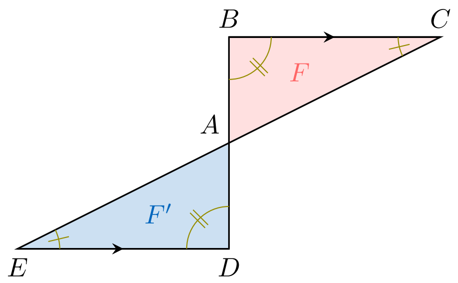


Ex 8: Justify with mathematical argumentation why the figures F and F' are similar.



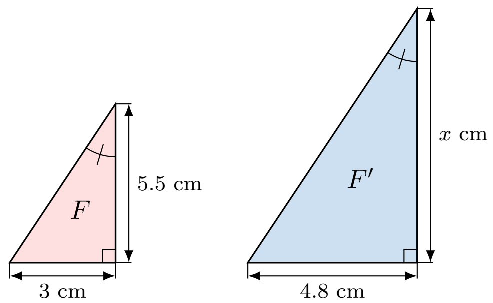
Students should write a response such as: "Since line $\overline{BC}$ is parallel to line $\overline{ED}$, the corresponding angles are equal, so triangles F and F' are similar by the Angle-Angle (AA) criterion." or "$\angle ABC = \angle ADE$ and $\angle BCA = \angle AED$, so triangles F and F' are similar by the Angle-Angle (AA) criterion." The most important aspect is that they state that the two triangles have two equal angles, which ensures similarity by the AA criterion.

Answer: Since line $\overleftrightarrow{BC}$ is parallel to line $\overleftrightarrow{ED}$, the corresponding angles are equal ($\angle ABC = \angle ADE$, $\angle BCA = \angle AED$). By the Angle-Angle (AA) similarity criterion, triangles F and F' are similar.



A.3 FINDING UNKNOWN LENGTHS IN SIMILAR TRIANGLES

Ex 9:



Find x

$x =$ 8.8

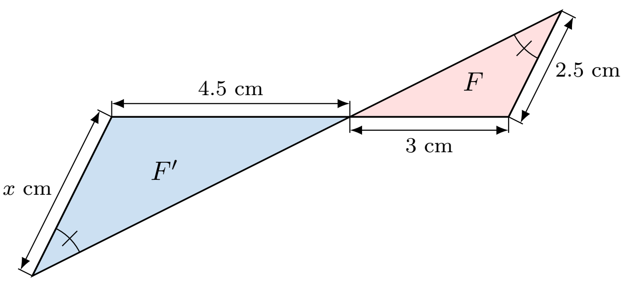
Answer:

1.
   - Both figures are right triangles (each has a right angle).
   - Both triangles have the same marked angle.

   So the triangles F and F' are similar.
2. The ratios of the corresponding sides are equals:

$$\frac{x}{5.5} = \frac{4.8}{3}$$
$$\therefore \quad x \times 3 = 5.5 \times 4.8 \quad \text{(cross multiplication)}$$
$$\therefore \quad x = \frac{5.5 \times 4.8}{3}$$
$$\therefore \quad x = 8.8$$

Ex 10:



Find x (use a calculator).

$x =$ 3.75

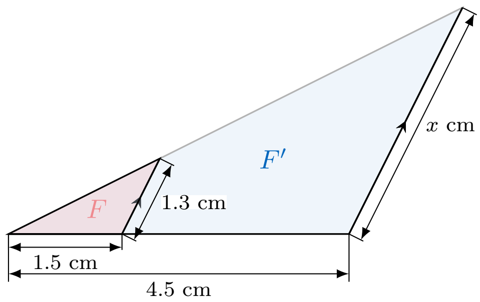
Answer:

1.
   - They share vertically opposite angles, which are equal.
   - They have a common marked angle.

   By the Angle-Angle (AA) similarity criterion, triangles F and F' are similar.
2. The ratios of corresponding sides are equal:

$$\frac{x}{2.5} = \frac{4.5}{3}$$
$$x \times 3 = 2.5 \times 4.5 \quad \text{(cross multiplication)}$$
$$x = \frac{2.5 \times 4.5}{3}$$
$$x = \frac{11.25}{3}$$
$$x = 3.75$$

Ex 11:



Find x.

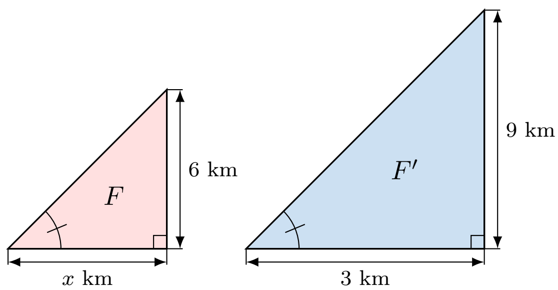
$x =$ 3.9

Answer:

1. Since the lines are parallel, the corresponding angles in the two triangles are equal. So, the triangles F and F' are similar.
2. The ratios of corresponding sides are equal:

$$\frac{x}{1.3} = \frac{4.5}{1.5}$$
$$x \times 1.5 = 1.3 \times 4.5 \quad \text{(cross multiplication)}$$
$$x = \frac{1.3 \times 4.5}{1.5}$$
$$x = \frac{5.85}{1.5}$$
$$x = 3.9$$

Ex 12:



Find x.

$$x = \boxed{2}$$

Answer:

1. • They have right angles.
   • They have a common marked angle.

   By the Angle-Angle (AA) similarity criterion, triangles F and F' are similar.

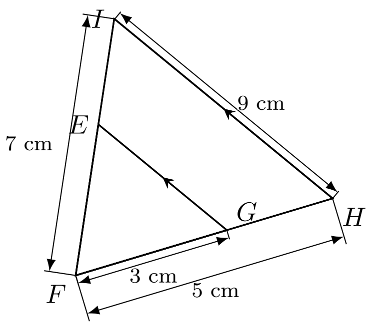
2. The ratios of corresponding sides are equal:

$$\frac{3}{x} = \frac{9}{6}$$
$$x \times 9 = 3 \times 6 \quad \text{(cross multiplication)}$$
$$x = \frac{3 \times 6}{9} \quad \text{(dividing by 9)}$$
$$x = \frac{18}{9}$$
$$x = 2$$

B THALES'S THEOREM

B.1 APPLYING THALES'S THEOREM WITHOUT JUSTIFICATION

Ex 13: The lines $\overleftrightarrow{GH}$ and $\overleftrightarrow{EI}$ intersect at F, and the lines $\overleftrightarrow{GE}$ and $\overleftrightarrow{HI}$ are parallel. Given $FG = 3$ cm, $FH = 5$ cm, $FI = 7$ cm, and $HI = 9$ cm:



Calculate the lengths FE and EG.

$$FE = \boxed{4.2} \text{ cm and } EG = \boxed{5.4} \text{ cm.}$$

Answer:

1. Since the lines $\overleftrightarrow{GH}$ and $\overleftrightarrow{EI}$ intersect at F, and $\overleftrightarrow{GE} \parallel \overleftrightarrow{HI}$, by Thales's theorem, triangles $\triangle FGE$ and $\triangle FHI$ are similar.

2. The ratios of corresponding sides are equal:

$$\frac{FH}{FG} = \frac{FI}{FE} = \frac{HI}{EG}$$
$$\frac{5}{3} = \frac{7}{FE} = \frac{9}{EG}$$

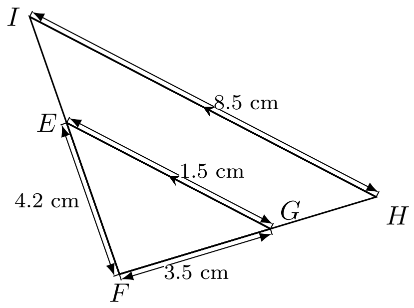
   • For FE:

$$\frac{7}{FE} = \frac{5}{3}$$
$$FE \times 5 = 7 \times 3 \quad \text{(cross multiplication)}$$
$$FE = \frac{7 \times 3}{5}$$
$$FE = 4.2\,\text{cm}$$

   • For EG:

$$\frac{9}{EG} = \frac{5}{3}$$
$$EG \times 5 = 9 \times 3 \quad \text{(cross multiplication)}$$
$$EG = \frac{9 \times 3}{5}$$
$$EG = 5.4\,\text{cm}$$

Ex 14: The lines $\overleftrightarrow{GH}$ and $\overleftrightarrow{EI}$ intersect at F, and the lines $\overleftrightarrow{GE}$ and $\overleftrightarrow{HI}$ are parallel. Given $FG = 3.5$ cm, $FE = 4.2$ cm, $EG = 1.5$ cm, and $HI = 8.5$ cm:



Calculate the lengths FI and FH.

$$FI = \boxed{7.14} \text{ cm and } FH = \boxed{5.95} \text{ cm.}$$

Answer:

1. Since the lines $\overleftrightarrow{GH}$ and $\overleftrightarrow{EI}$ intersect at F, and $\overleftrightarrow{GE} \parallel \overleftrightarrow{HI}$, by Thales's theorem, triangles $\triangle FGE$ and $\triangle FHI$ are similar.

2. The ratios of corresponding sides are equal:

$$\frac{FH}{FG} = \frac{FI}{FE} = \frac{HI}{EG}$$
$$\frac{FH}{3.5} = \frac{FI}{4.2} = \frac{8.5}{1.5} = \frac{17}{3}$$

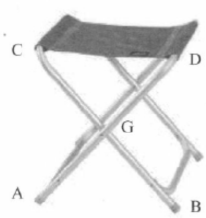
   • For FI:

$$\frac{FI}{4.2} = \frac{17}{3}$$
$$FI = 4.2 \times \frac{17}{3}$$
$$FI = 7.14\,\text{cm}$$

   • For FH:

$$\frac{FH}{3.5} = \frac{17}{3}$$
$$FH = 3.5 \times \frac{17}{3}$$
$$FH = 5.95\,\text{cm}$$

Ex 15: A folding stool is modeled geometrically with segments $\overline{CB}$ and $\overline{AD}$ for the metal frame and segment $\overline{CD}$ for the fabric seat. Given $CG = DG = 30$ cm, $AG = BG = 45$ cm, and $AB = 51$ cm, and knowing that the seat $\overleftrightarrow{CD}$ is parallel to the ground represented by $\overleftrightarrow{AB}$:



Determine the length of the seat CD.

$$CD = \boxed{34} \text{ cm}$$

Answer:

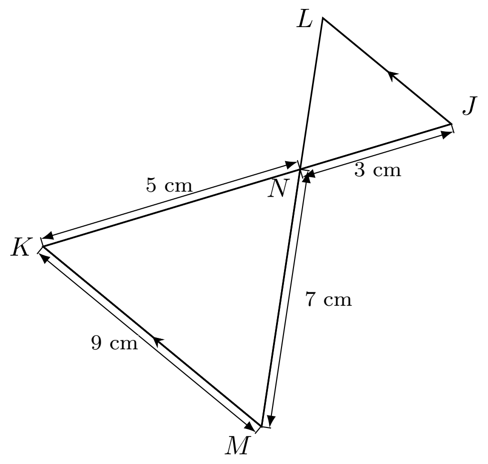
1. Since the lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ intersect at G, and $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, by Thales's theorem, triangles $\triangle GAB$ and $\triangle GCD$ are similar.

2. The ratios of corresponding sides are equal:

$$\frac{GD}{GA} = \frac{GC}{GB} = \frac{CD}{AB}$$
$$\frac{30}{45} = \frac{30}{45} = \frac{CD}{51}$$
$$CD = \frac{51 \times 30}{45}$$
$$CD = 34\,\text{cm}$$

The length of the seat is 34 cm.

Ex 16: The lines $\overleftrightarrow{JK}$ and $\overleftrightarrow{LM}$ intersect at N, and the lines $\overleftrightarrow{JL}$ and $\overleftrightarrow{KM}$ are parallel. Given $JN = 3$ cm, $NK = 5$ cm, $LM = 7$ cm, and $KM = 9$ cm:



Calculate the lengths NL and LJ.

$$NL = \boxed{4.2} \text{ cm and } LJ = \boxed{5.4} \text{ cm.}$$

Answer:

1. Since the lines $\overleftrightarrow{JK}$ and $\overleftrightarrow{LM}$ intersect at N, and $\overleftrightarrow{JL} \parallel \overleftrightarrow{KM}$, by Thales's theorem, triangles $\triangle NJL$ and $\triangle NKM$ are similar.

2. The ratios of corresponding sides are equal:

$$\frac{NK}{NJ} = \frac{NM}{NL} = \frac{KM}{LJ}$$
$$\frac{5}{3} = \frac{7}{NL} = \frac{9}{LJ}$$

- For NL:

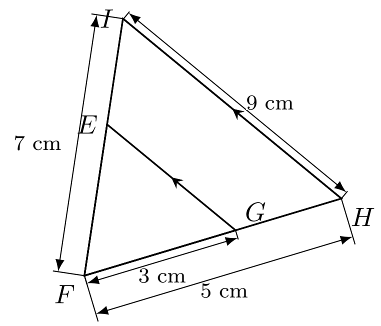
$$\frac{7}{NL} = \frac{5}{3}$$
$$NL \times 5 = 7 \times 3 \quad \text{(cross multiplication)}$$
$$NL = \frac{7 \times 3}{5}$$
$$NL = 4.2\,\text{cm}$$

- For LJ:

$$\frac{9}{LJ} = \frac{5}{3}$$
$$LJ \times 5 = 9 \times 3 \quad \text{(cross multiplication)}$$
$$LJ = \frac{9 \times 3}{5}$$
$$LJ = 5.4\,\text{cm}$$

B.2 APPLYING THALES'S THEOREM

Ex 17: The lines $\overleftrightarrow{GH}$ and $\overleftrightarrow{EI}$ intersect at F, and the lines $\overleftrightarrow{GE}$ and $\overleftrightarrow{HI}$ are parallel. Given $FG = 3$ cm, $FH = 5$ cm, $FI = 7$ cm, and $HI = 9$ cm:



Calculate the lengths FE and EG. Justify.

Students should apply Thales's theorem, stating that $\triangle FGE \sim \triangle FHI$ due to parallel lines $\overleftrightarrow{GE} \parallel \overleftrightarrow{HI}$, and use the ratios of corresponding sides to find FE and EG. A complete response might be: "Since $\overleftrightarrow{GE} \parallel \overleftrightarrow{HI}$ and $\overleftrightarrow{GH}$ intersects $\overleftrightarrow{EI}$ at F, $\triangle FGE \sim \triangle FHI$ by Thales's theorem. The ratio of corresponding sides is $\frac{FH}{FG} = \frac{5}{3}$. Thus, $\frac{FI}{FE} = \frac{7}{FE} = \frac{5}{3}$, so $FE = \frac{7 \times 3}{5} = 4.2$ cm, and $\frac{HI}{EG} = \frac{9}{EG} = \frac{5}{3}$, so $EG = \frac{9 \times 3}{5} = 5.4$ cm."

Answer:

1. Since the lines $\overleftrightarrow{GH}$ and $\overleftrightarrow{EI}$ intersect at F, and $\overleftrightarrow{GE} \parallel \overleftrightarrow{HI}$, by Thales's theorem, triangles $\triangle FGE$ and $\triangle FHI$ are similar.

2. The ratios of corresponding sides are equal:

$$\frac{FH}{FG} = \frac{FI}{FE} = \frac{HI}{EG}$$
$$\frac{5}{3} = \frac{7}{FE} = \frac{9}{EG}$$

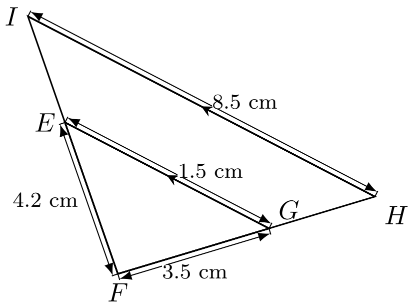
- For FE:

$$\frac{7}{FE} = \frac{5}{3}$$
$$FE \times 5 = 7 \times 3 \quad \text{(cross multiplication)}$$
$$FE = \frac{7 \times 3}{5}$$
$$FE = 4.2\,\text{cm}$$

- For EG:

$$\frac{9}{EG} = \frac{5}{3}$$
$$EG \times 5 = 9 \times 3 \quad \text{(cross multiplication)}$$
$$EG = \frac{9 \times 3}{5}$$
$$EG = 5.4\,\text{cm}$$

Ex 18: The lines $\overleftrightarrow{GH}$ and $\overleftrightarrow{EI}$ intersect at F, and the lines $\overleftrightarrow{GE}$ and $\overleftrightarrow{HI}$ are parallel. Given $FG = 3.5$ cm, $FE = 4.2$ cm, $EG = 1.5$ cm, and $HI = 8.5$ cm:



Calculate the lengths FI and FH. Justify.

Students should apply Thales's theorem, stating that $\triangle FGE \sim \triangle FHI$ due to parallel lines $\overleftrightarrow{GE} \parallel \overleftrightarrow{HI}$, and use the ratios of corresponding sides to find FI and FH. A complete response might be: "Since $\overleftrightarrow{GE} \parallel \overleftrightarrow{HI}$ and $\overleftrightarrow{GH}$ intersects $\overleftrightarrow{EI}$ at F, $\triangle FGE \sim \triangle FHI$ by Thales's theorem. The ratio of corresponding sides is $\frac{HI}{EG} = \frac{8.5}{1.5} = \frac{17}{3}$. Thus, $\frac{FI}{FE} = \frac{FI}{4.2} = \frac{17}{3}$, so $FI = 4.2 \times \frac{17}{3} = 7.14$ cm, and $\frac{FH}{FG} = \frac{FH}{3.5} = \frac{17}{3}$, so $FH = 3.5 \times \frac{17}{3} = 5.95$ cm."

Answer:

1. Since the lines $\overleftrightarrow{GH}$ and $\overleftrightarrow{EI}$ intersect at F, and $\overleftrightarrow{GE} \parallel \overleftrightarrow{HI}$, by Thales's theorem, triangles $\triangle FGE$ and $\triangle FHI$ are similar.

2. The ratios of corresponding sides are equal:

$$\frac{FH}{FG} = \frac{FI}{FE} = \frac{HI}{EG}$$
$$\frac{FH}{3.5} = \frac{FI}{4.2} = \frac{8.5}{1.5} = \frac{17}{3}$$

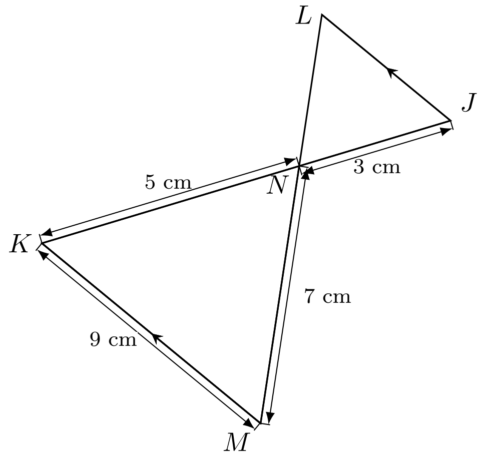
- For FI:

$$\frac{FI}{4.2} = \frac{17}{3}$$
$$FI = 4.2 \times \frac{17}{3}$$
$$FI = 7.14\,\text{cm}$$

- For FH:

$$\frac{FH}{3.5} = \frac{17}{3}$$
$$FH = 3.5 \times \frac{17}{3}$$
$$FH = 5.95\,\text{cm}$$

Ex 19: The lines $\overleftrightarrow{JK}$ and $\overleftrightarrow{LM}$ intersect at N, and the lines $\overleftrightarrow{JL}$ and $\overleftrightarrow{KM}$ are parallel. Given $JN = 3$ cm, $NK = 5$ cm, $LM = 7$ cm, and $KM = 9$ cm:



Calculate the lengths NL and LJ. Justify.

Students should apply Thales's theorem, stating that $\triangle NJL \sim \triangle NKM$ due to parallel lines $\overleftrightarrow{JL} \parallel \overleftrightarrow{KM}$, and use the ratios of corresponding sides to find NL and LJ. A complete response might be: "Since $\overleftrightarrow{JL} \parallel \overleftrightarrow{KM}$ and $\overleftrightarrow{JK}$ intersects $\overleftrightarrow{LM}$ at N, $\triangle NJL \sim \triangle NKM$ by Thales's theorem. The ratio of corresponding sides is $\frac{NK}{NJ} = \frac{5}{3}$. Thus, $\frac{NM}{NL} = \frac{7}{NL} = \frac{5}{3}$, so $NL = \frac{7\times3}{5} = 4.2$ cm, and $\frac{KM}{LJ} = \frac{9}{LJ} = \frac{5}{3}$, so $LJ = \frac{9\times3}{5} = 5.4$ cm."

Answer:

1. Since the lines $\overleftrightarrow{JK}$ and $\overleftrightarrow{LM}$ intersect at N, and $\overleftrightarrow{JL} \parallel \overleftrightarrow{KM}$, by Thales's theorem, triangles $\triangle NJL$ and $\triangle NKM$ are similar.

2. The ratios of corresponding sides are equal:

$$\frac{NK}{NJ} = \frac{NM}{NL} = \frac{KM}{LJ}$$
$$\frac{5}{3} = \frac{7}{NL} = \frac{9}{LJ}$$

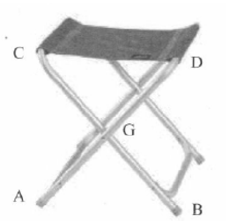
- For NL:

$$\frac{7}{NL} = \frac{5}{3}$$
$$NL \times 5 = 7 \times 3 \quad \text{(cross multiplication)}$$
$$NL = \frac{7 \times 3}{5}$$
$$NL = 4.2\,\text{cm}$$

- For LJ:

$$\frac{9}{LJ} = \frac{5}{3}$$
$$LJ \times 5 = 9 \times 3 \quad \text{(cross multiplication)}$$
$$LJ = \frac{9 \times 3}{5}$$
$$LJ = 5.4\,\text{cm}$$

Ex 20: A folding stool is modeled geometrically with segments $\overline{CB}$ and $\overline{AD}$ for the metal frame and segment $\overline{CD}$ for the fabric seat. Given $CG = DG = 30$ cm, $AG = BG = 45$ cm, and $AB = 51$ cm, and knowing that the seat $\overleftrightarrow{CD}$ is parallel to the ground represented by $\overleftrightarrow{AB}$:



Calculate the length of the seat CD. Justify.

Students should apply Thales's theorem, stating that $\triangle GAB \sim \triangle GCD$ due to parallel lines $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, and use the ratios of corresponding sides to find CD. A complete response might be: "Since $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{AD}$ intersects $\overleftrightarrow{BC}$ at G, $\triangle GAB \sim \triangle GCD$ by Thales's theorem. The ratio of corresponding sides is $\frac{GD}{GA} = \frac{30}{45} = \frac{2}{3}$. Thus, $\frac{CD}{AB} = \frac{2}{3}$, so $CD = \frac{2}{3} \times 51 = 34$ cm."

Answer:

1. Since the lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ intersect at G, and $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, by Thales's theorem, triangles $\triangle GAB$ and $\triangle GCD$ are similar.

2. The ratios of corresponding sides are equal:

$$\frac{GD}{GA} = \frac{GC}{GB} = \frac{CD}{AB}$$
$$\frac{30}{45} = \frac{30}{45} = \frac{CD}{51}$$
$$CD = \frac{51 \times 30}{45}$$
$$CD = 34\,\text{cm}$$

The length of the seat is 34 cm.