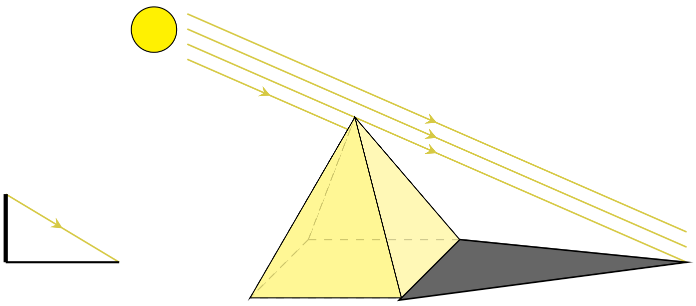
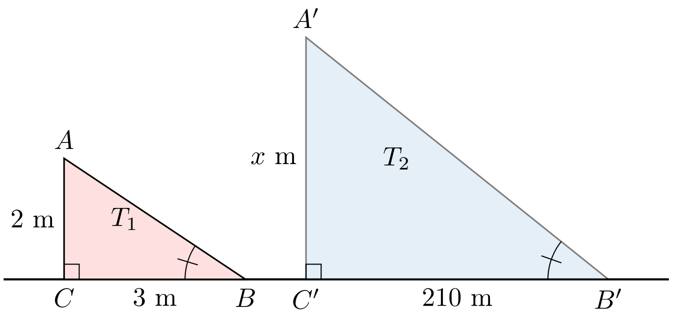
SIMILAR TRIANGLES

A ANGLE-ANGLE SIMILARITY

Discover: Thales, an ancient Greek mathematician, devised a clever method to measure the height of the Great Pyramid of Cheops. One sunny day, he observed that the pyramid cast a shadow 210 meters long. At the same time, a 2-meter-tall broom handle, standing upright, cast a shadow 3 meters long. By comparing these shadows, can you determine the height of the pyramid?



Answer: The triangles formed by the broom handle and its shadow ($T_1 : \triangle ABC$) and the pyramid and its shadow ($T_2 : \triangle A'B'C'$) are similar.



The ratios of corresponding sides are equal:

$$\frac{\text{height of } T_2}{\text{shadow of } T_2} = \frac{\text{height of } T_1}{\text{shadow of } T_1}$$

$$\frac{x}{210} = \frac{2}{3}$$

Solving for x:

$$x = 210 \times \frac{2}{3} = \frac{420}{3} = 140$$

Thus, the height of the Great Pyramid of Cheops is 140 meters.

Proposition Angle-Angle Similarity

If two angles of one triangle are equal to two angles of another triangle, then the two triangles are similar.

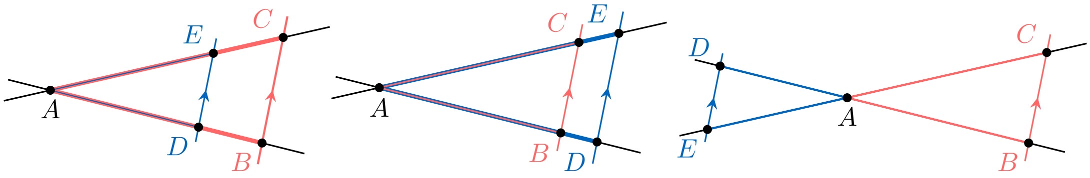
B THALES'S THEOREM

Theorem Thales's Theorem

Let $\triangle ABC$ be a triangle, with a point D on the line $\overleftrightarrow{AB}$ and a point E on the line $\overleftrightarrow{AC}$.
If the line $\overleftrightarrow{DE}$ is parallel to the line $\overleftrightarrow{BC}$, then the triangles $\triangle ABC$ and $\triangle ADE$ are similar:

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

Thales's Configurations: Key Figures



Each red triangle is similar to the blue triangle.

Proof

Since the line $\overleftrightarrow{DE}$ is parallel to the line $\overleftrightarrow{BC}$, the angles $\angle ADE$ and $\angle ABC$ are corresponding angles and therefore equal. Additionally, $\angle DAE$ and $\angle BAC$ are the same angle at vertex A. Thus, by the Angle-Angle (AA) similarity criterion, triangles $\triangle ABC$ and $\triangle ADE$ are similar. Therefore, the ratios of their corresponding sides are equal:

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$